Physics Webinar Series, Department of Physics & Astronomy
National Institute of Technology (NIT), Rourkela, India
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Microscopic Approaches for Macroscopic Phenomena of Neutron Stars

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Tokyo Institute of Technology

東京工業大学
Tokyo Institute of Technology

Brief personal history

1988: Born in Tokyo, grew up in Satte City in Saitama


Cherry blossoms in Satte City

2003-2006: Inagakuen Public High School

2006-2010: Tokyo University of Science (BSc)

2010-2015: University of Tsukuba (MSc-PhD)

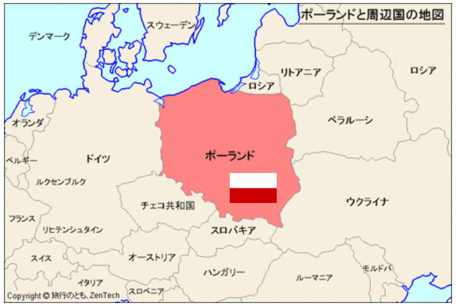
Apr. 2015-Aug. 2017: Warsaw University of Technology, Poland (Postdoc)

Sep. 2017-Dec. 2017: University of Washington, USA (Postdoc)

Jan. 2018-Mar. 2021: Niigata University (Assistant Professor, tenure track)

Apr. 2021-Present: Tokyo Institute of Technology (Associate Professor)

Main research field: Nuclear Theory



Hierarchy of Scales in the Universe

Macroscopic world

Microscopic world

Cosmic Eye - Universe Size Comparison: https://youtu.be/8Are9dDbW24

Hierarchy of Scales in the Universe

Neutron stars, NS merger, nucleosynthesis, GW, ...

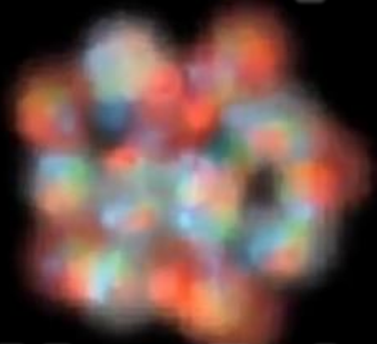
Macroscopic

(Nuclear)Astrophysics

Nuclear structure, Equation of State (EoS)
Superfluidity & Superconductivity
Reaction rates, Fission fragments, ...

Neutron-star structure, Star quakes, GW
Pulsar glitches, Cooling
Stellar evolution, Nucleosynthesis, ...

Microscopic



Nuclear Many-Body Problem

10 fm

Cosmic Eye - Universe Size Comparison: https://youtu.be/8Are9dDbW24

Hierarchy of Scales in the Universe

Neutron stars, NS merger, nucleosynthesis, GW, ...

Macroscopic

(Nuclear)Astrophysics

Nuclear structure, E
Superfluid
Reaction ra

ture, Star quakes, GW
Cooling
osynthesis, ...

Our Mission:

To establish a concrete microscopic foundation of macroscopic models

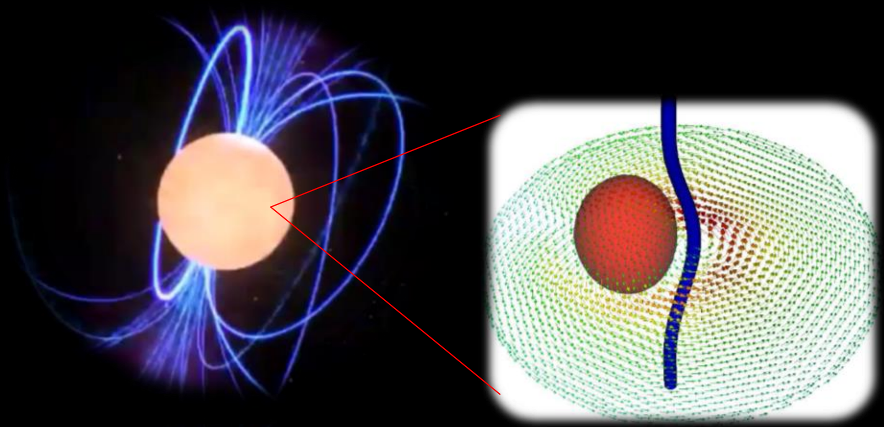
Nuclear Many-Body Problem

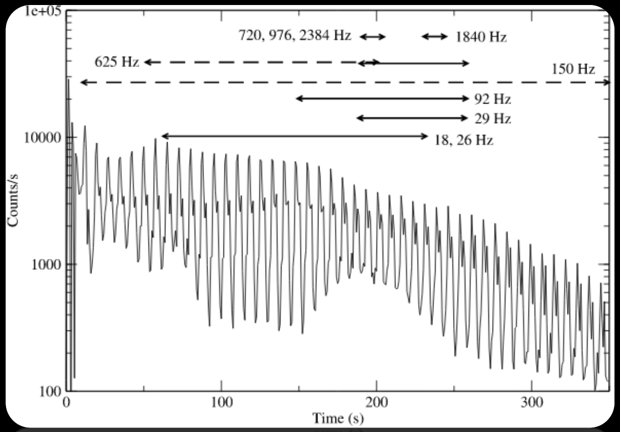
Cosmic Eye - Universe Size Comparison: https://youtu.be/8Are9dDbW24

Today, I will mainly talk about:

Dynamics of <u>quantum vortices</u> of superfluid neutrons

<u>Time-dependent band theory</u> for the inner crust of neutron stars





⟷ Neutron-star glitches

⟷ Quasi-periodic oscillations

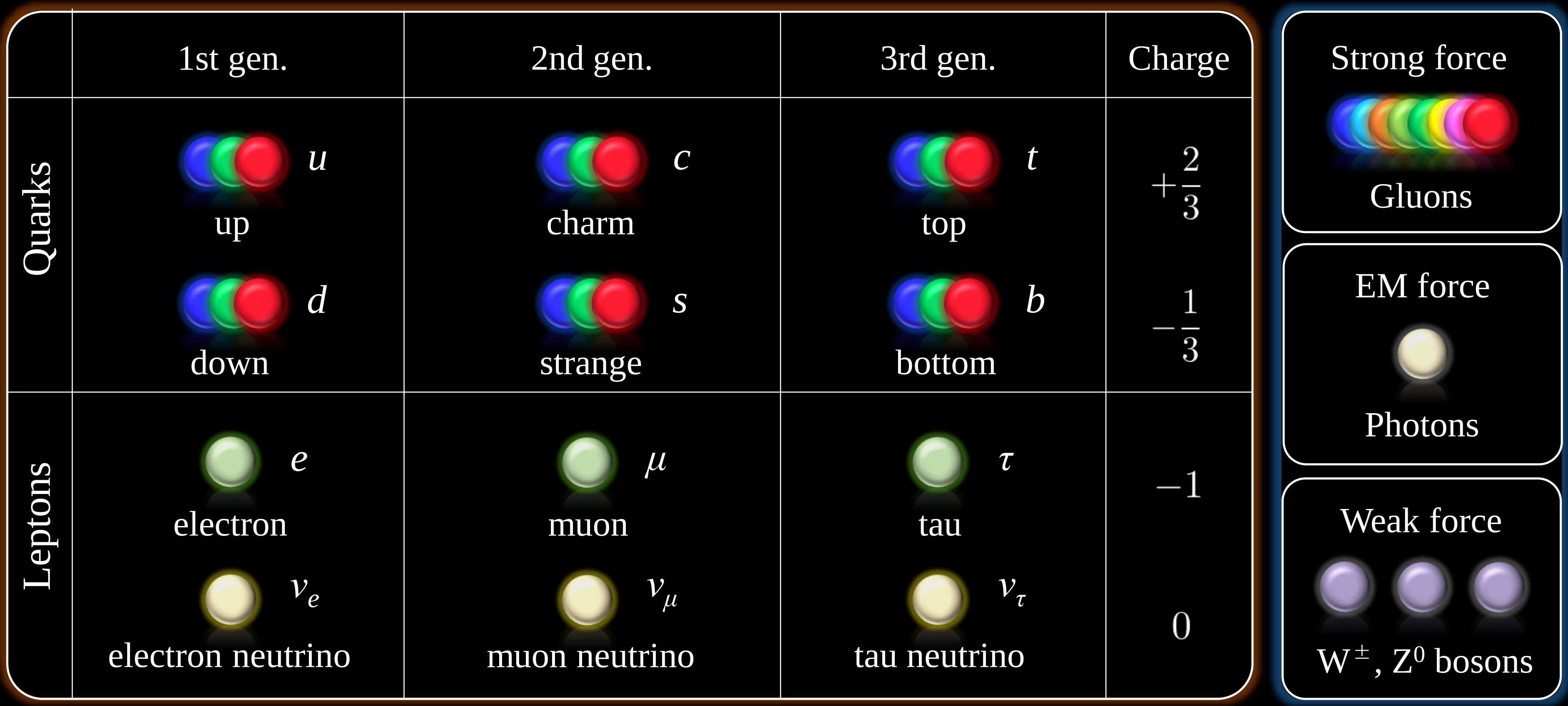
..and some related topics of nuclear physics

From **quarks** to atomic nuclei

Standard model of the elementary particles

- ✓ Elementary particles: fundamental particles **without structure**
- ✓ Four forces: **strong, weak, electromagnetic**, and gravitational forces
- ✓ Particle physics explores an **ultimate theory** of the Universe

Higgs boson



Fermions

	1st gen.	2nd gen.	3rd gen.	Charge
Quarks	u up	c charm	t top	$+\frac{2}{3}$
	d down	s strange	b bottom	$-\frac{1}{3}$
Leptons	e electron	μ muon	τ tau	-1
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	0

Gauge bosons

Strong force

Gluons

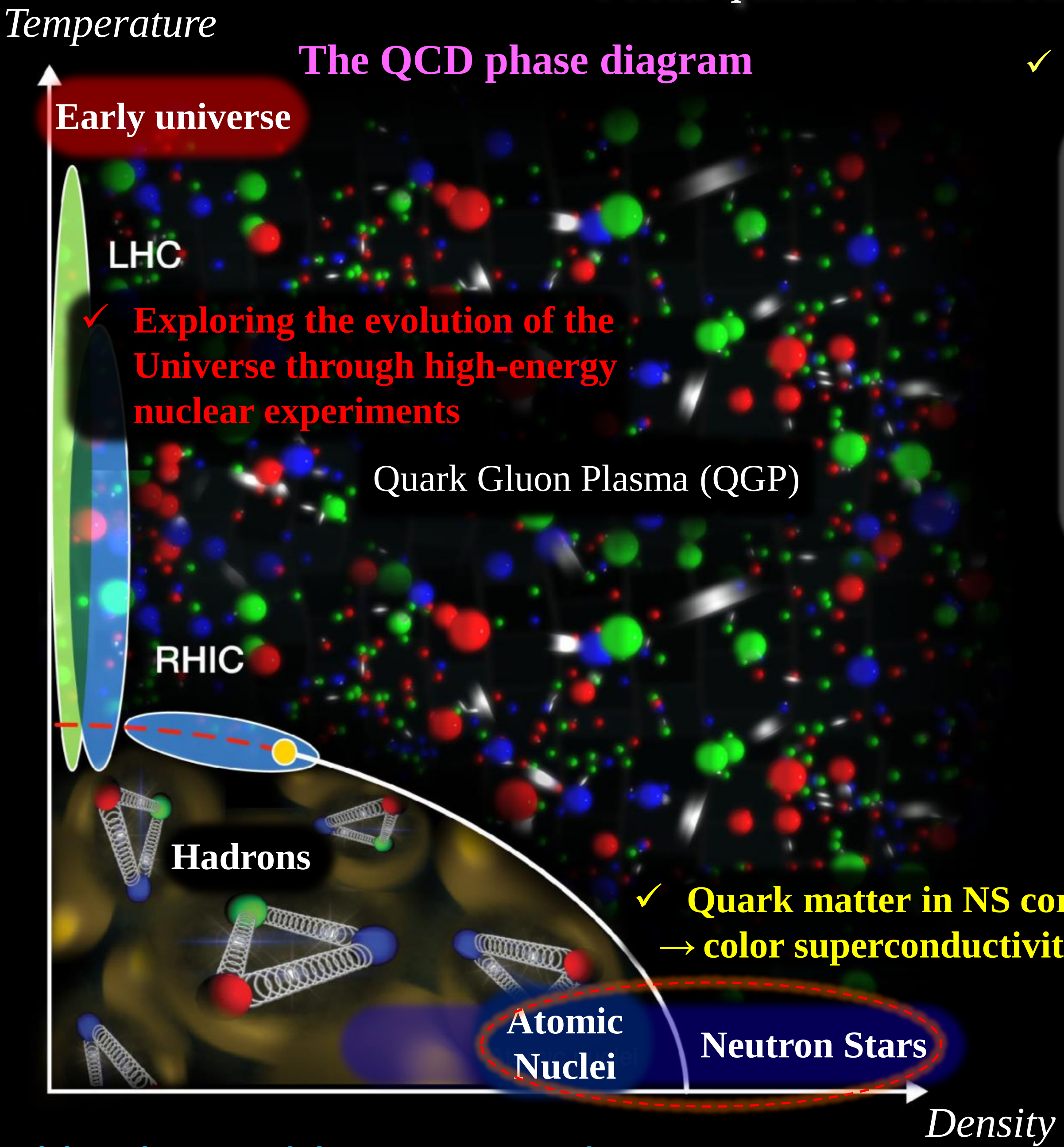
EM force

Photons

Weak force

$W^\pm$, Z^0 bosons

From quarks to hadrons



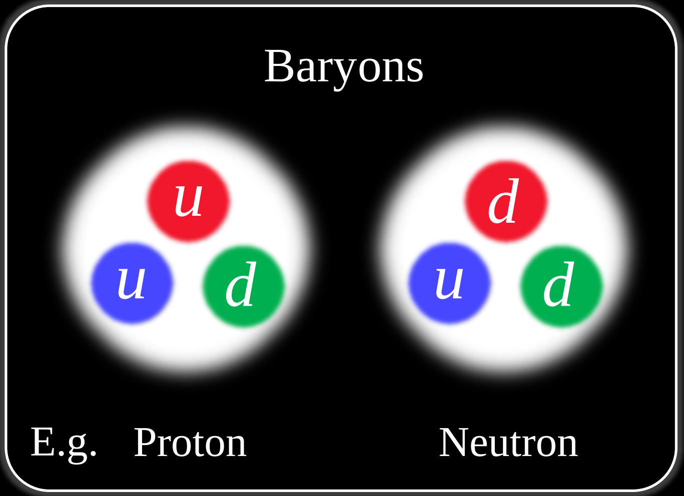
✓ **Exploring the evolution of the Universe through high-energy nuclear experiments**

✓ **Quark matter in NS core?**
→ color superconductivity

Left figure: https://www.bnl.gov/newsroom/news.php?a=24281

✓ **Hadrons:** composite particles of quarks

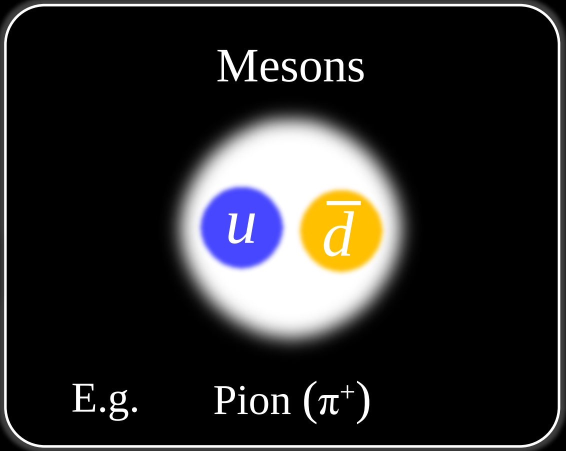
Baryons



E.g. Proton Neutron

➢ A nucleon (proton or neutron) is a baryon.

Mesons



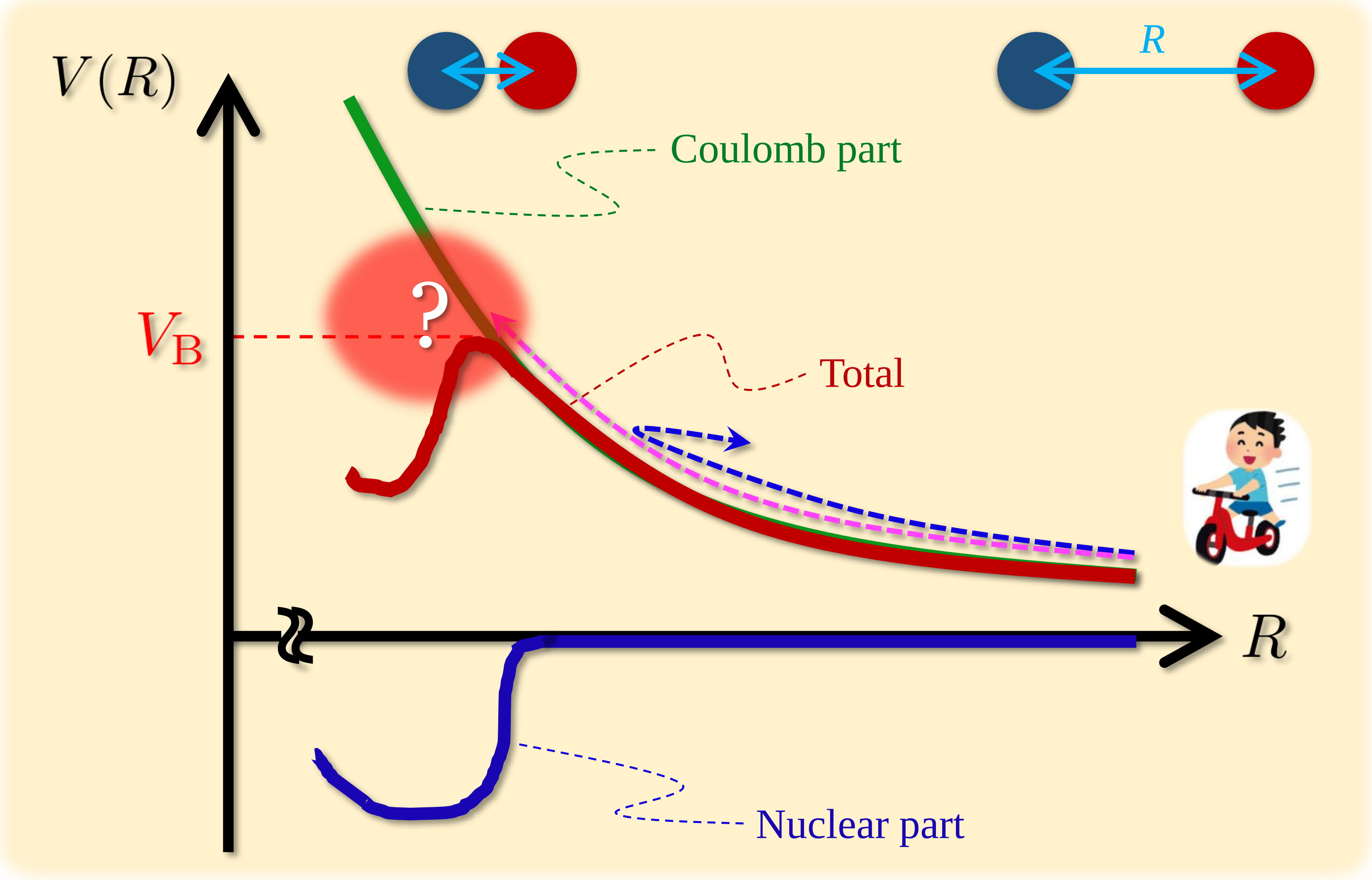
E.g. Pion (π^+)

Not this “high energy” !!

Animation: Brookhaven National Laboratory (http://www.bnl.gov/rhic/); https://youtu.be/Vyq_AYWctSo

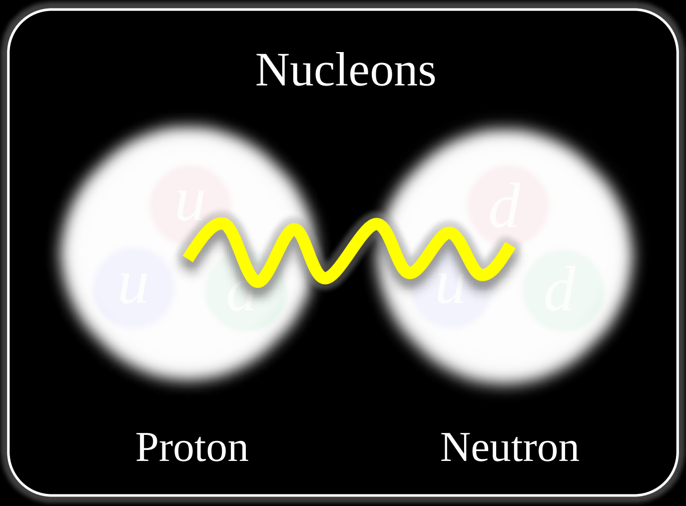
We collide two nuclei "gently"

and study quantum many-body dynamics of neutrons and protons



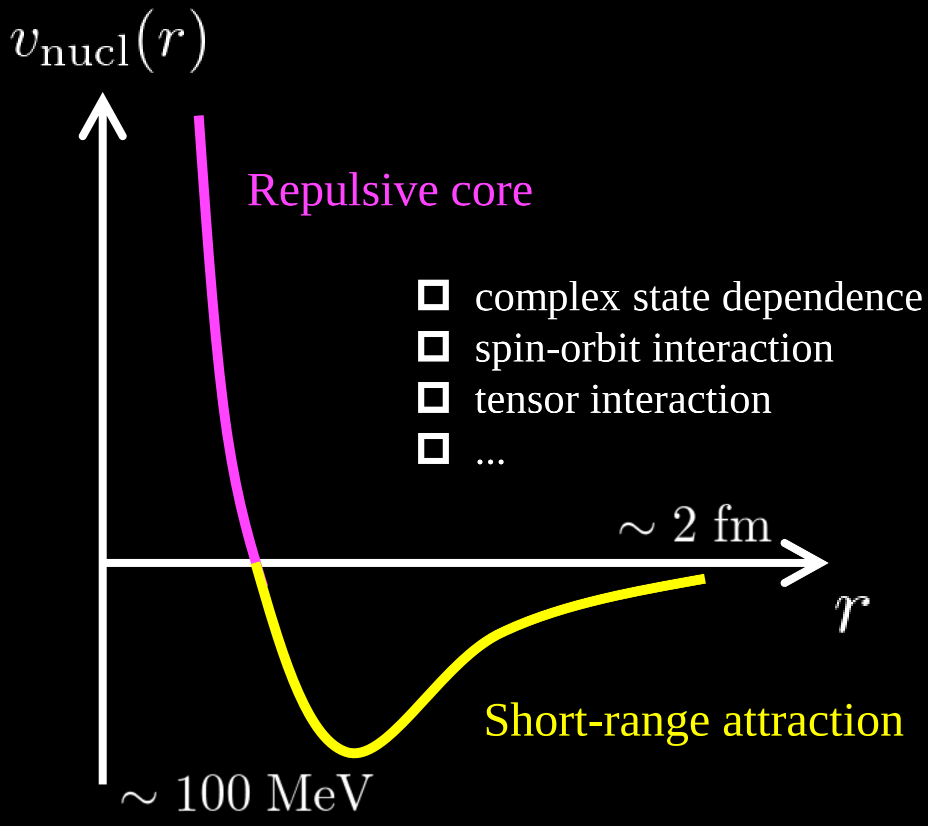
In “low-energy” nuclear physics,
we treat neutrons and protons as building blocks

What we study is:
A quantum many-body problem of fermions interacting through the nuclear force


Nucleons

Proton

Neutron



- ☐ complex state dependence
- ☐ spin-orbit interaction
- ☐ tensor interaction
- ☐ …

(TD)DFT in a tiny nutshell

What is DFT?

A theory which gives us access to the *exact* solution

Equivalent!
(for a special EDF)

$$\hat{H}\Psi(\boldsymbol{r}_1,\dots,\boldsymbol{r}_N) = E\Psi(\boldsymbol{r}_1,\dots,\boldsymbol{r}_N)$$

Kohn-Sham equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + v_{\text{KS}}[\rho(\boldsymbol{r})]\right]\phi_i(\boldsymbol{r}) = \varepsilon_i\,\phi_i(\boldsymbol{r})$$

EDF

This is the key!

$$v_{\text{KS}}[\rho(\boldsymbol{r})] = \frac{\delta\mathcal{E}[\rho]}{\delta\rho} \qquad \rho(\boldsymbol{r}) = \sum_{i=1}^{N}|\phi_i(\boldsymbol{r})|^2$$

Energy can also be written as a functional of density

$$E[\rho] = \big\langle \underline{\Psi[\rho]} \big| \hat{H} \big| \underline{\Psi[\rho]} \big\rangle$$

w.f. is a functional of density

P. Hohenberg and W. Kohn, Phys. Rev. B **136**, 864 (1964)

CAUTION!

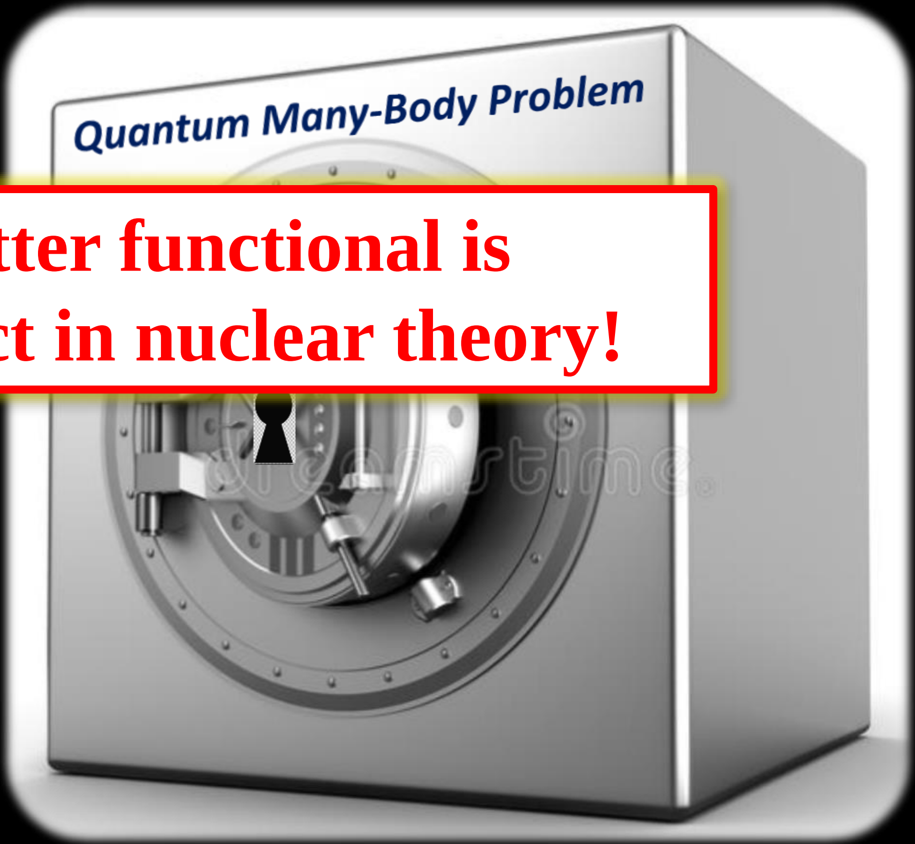
The existence was proven, but its shape is unknown..

“Inverse Kohn-Sham”

CAUTION!

The existence was proven, but its shape is unknown..

“Inverse Kohn-Sham”

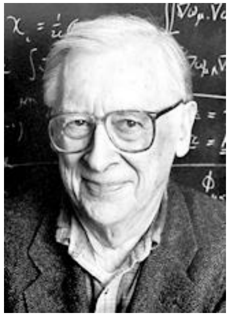


Developing a better functional is an important subject in nuclear theory!

Great Success of the Density Functional Theory

The Nobel Prize in Chemistry 1998

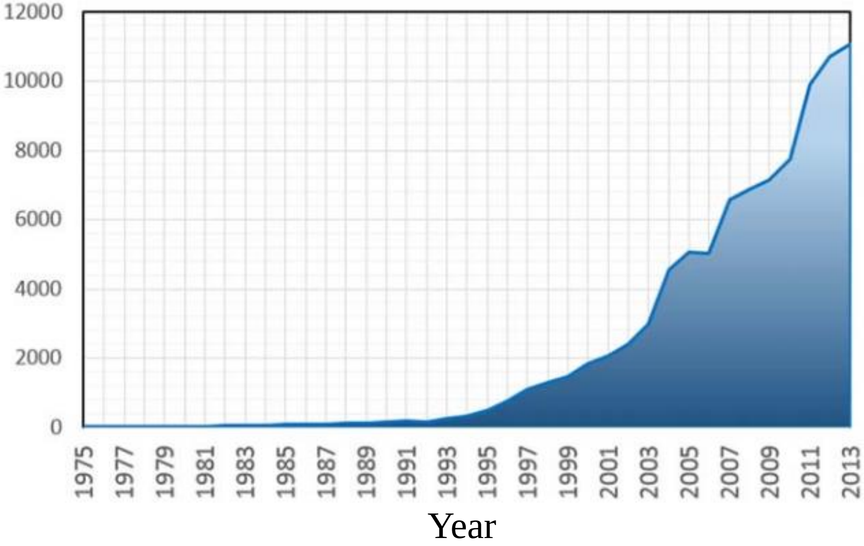
Walter Kohn



John Pople

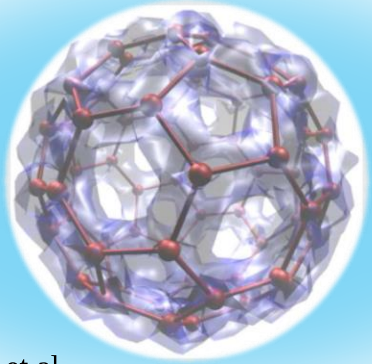
©https://www.nobelprize.org

Number of publications with “DFT” (till 2013)



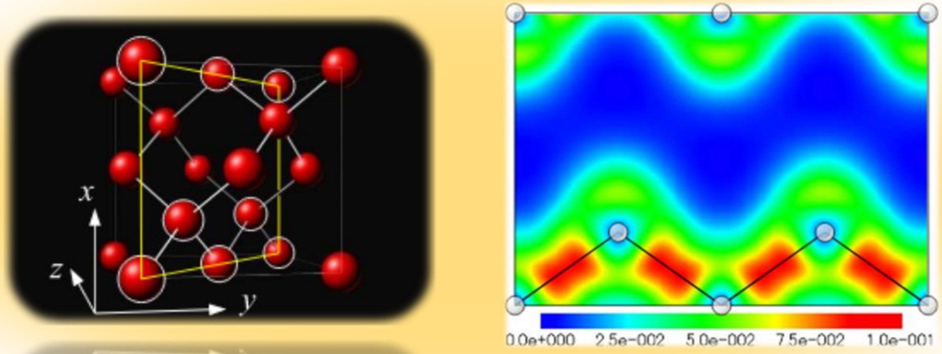
A. Galano and J.R. Alvarez-Idadoy, J. Compt. Chem. **35**, 2019 (2014)

Fullerene: C_{60}



C-Z. Gao et al.,
J. Phys. B: At. Mol. Opt. Phys. **48**, 105102 (2015)

Si crystal

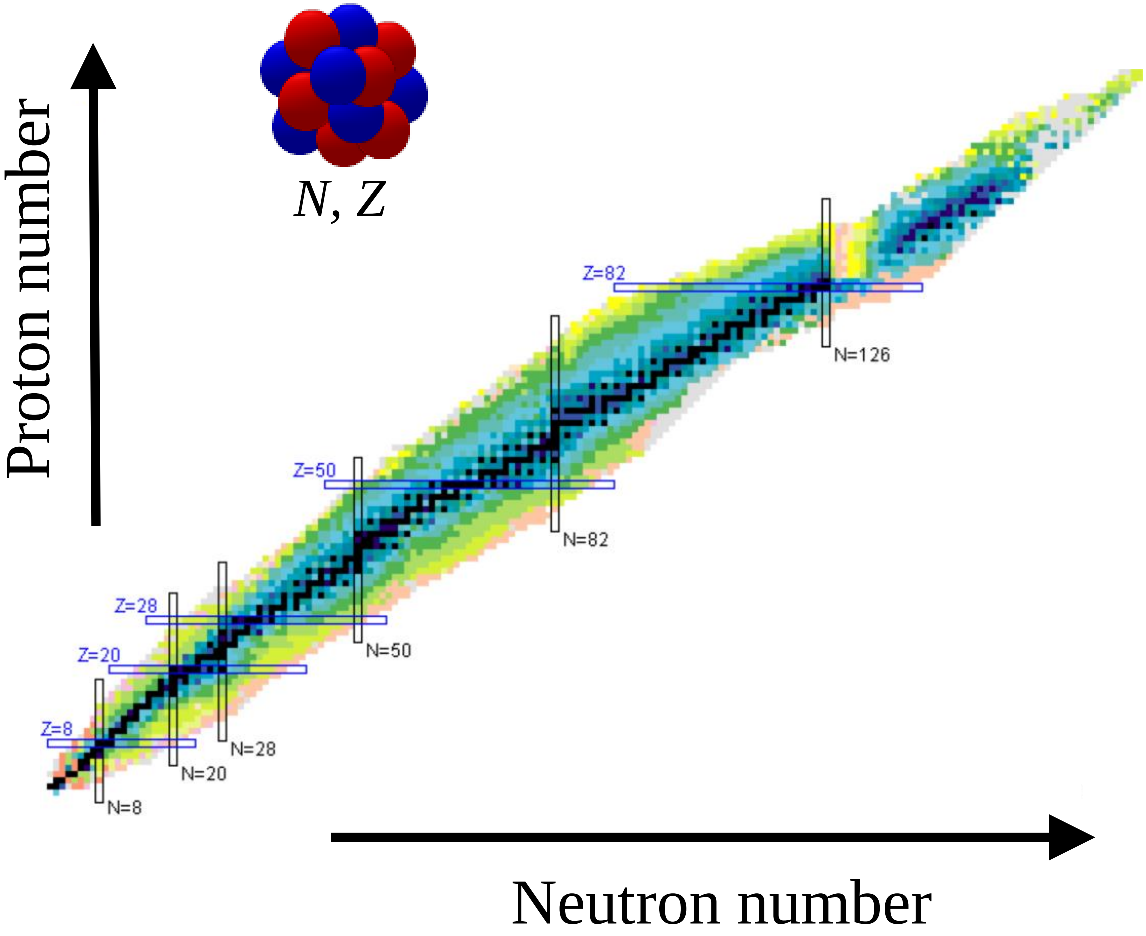


Y. Shinohara, K. Yabana, Y. Kawashita, J.-I. Iwata, T. Otobe, and G. F. Bertsch,
Phys. Rev. B **82**, 155110 (2010)

The seminal papers on DFT {
- P. Hohenberg and W. Kohn, Phys. Rev. **136**, B864 (1964) ➔ **19,015 citations!**
- W. Kohn and L.J. Sham, Phys. Rev. **140**, A1133 (1965) ➔ **24,384 citations!**

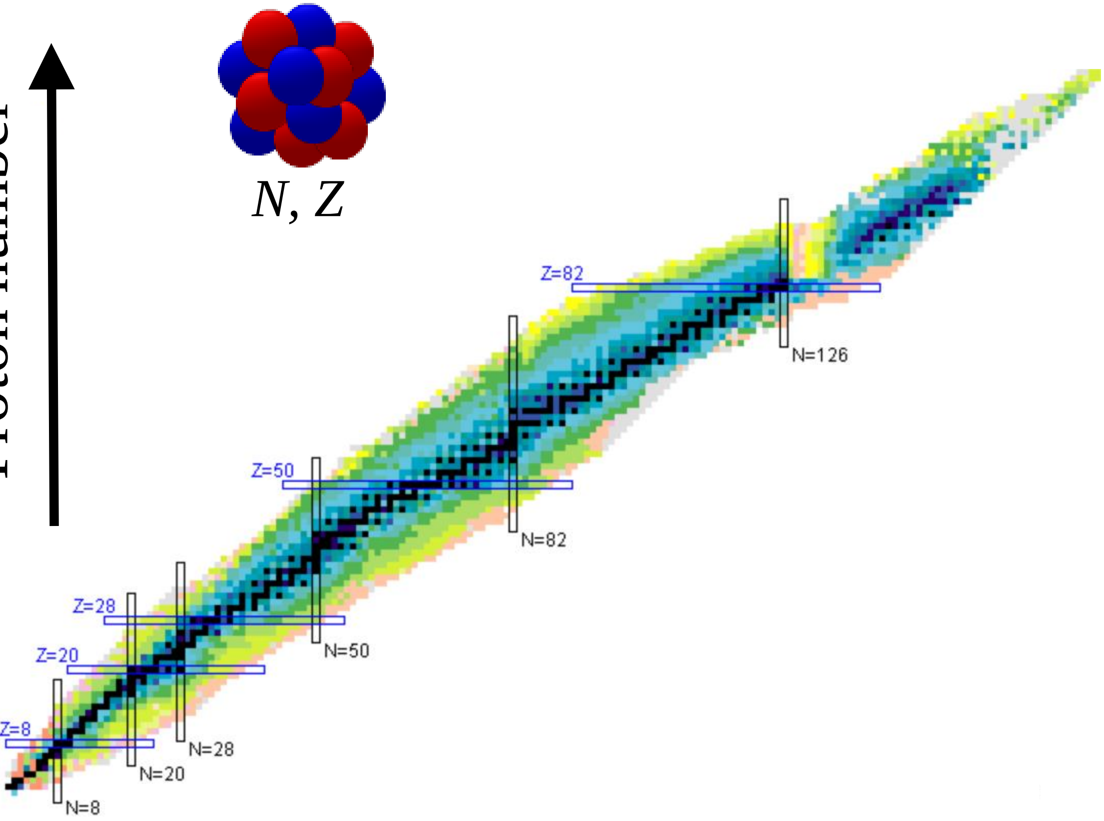
DFT in Nuclear Physics

All nuclei can be described with a *single* EDF



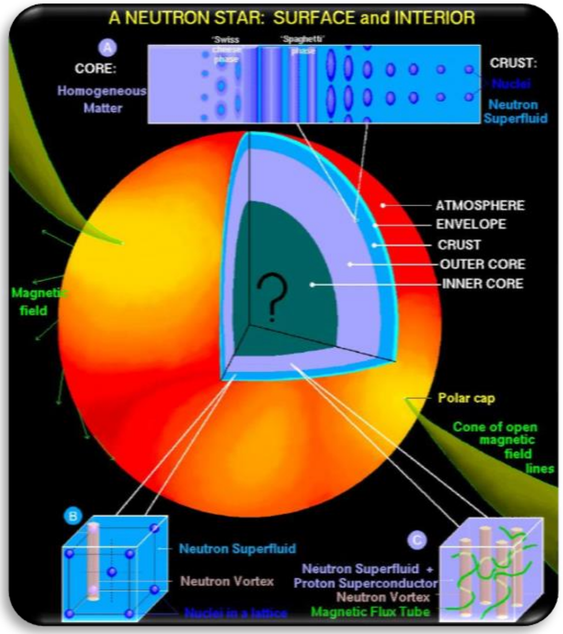
N, Z

All nuclei can be described with a *single* EDF



Proton number

Neutron star

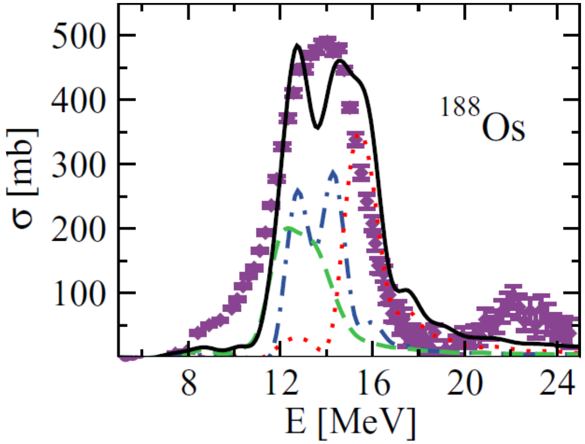


Neutron number

TDDFT in Nuclear Physics

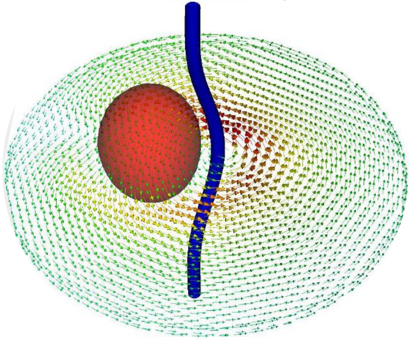
TDDFT is a versatile tool!!

IVGDR



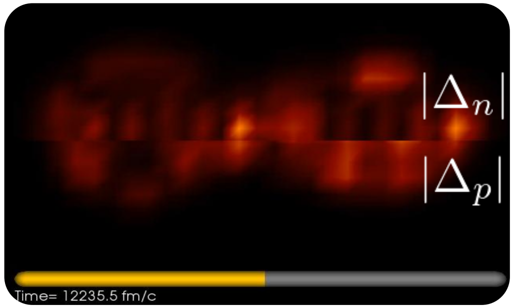
Phys. Rev. C **84**, 051309(R) (2011)
I. Stetcu, A. Bulgac, P. Magierski, and K.J. Roche

Vortex-nucleus dynamics



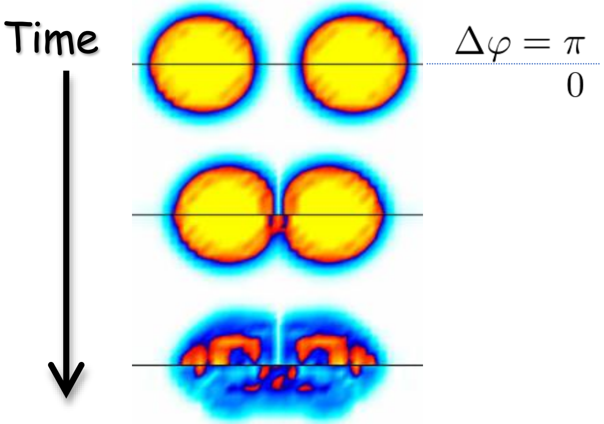
Phys. Rev. Lett. **117**, 232701 (2016)
G. Wlazłowski, K.S., P. Magierski, A. Bulgac, and M.M. Forbes

Induced fission of ^{240}Pu



Phys. Rev. Lett. **116**, 122504 (2016)
A. Bulgac, P. Magierski, K.J. Roche, and I. Stetcu

Low-energy heavy-ion reactions



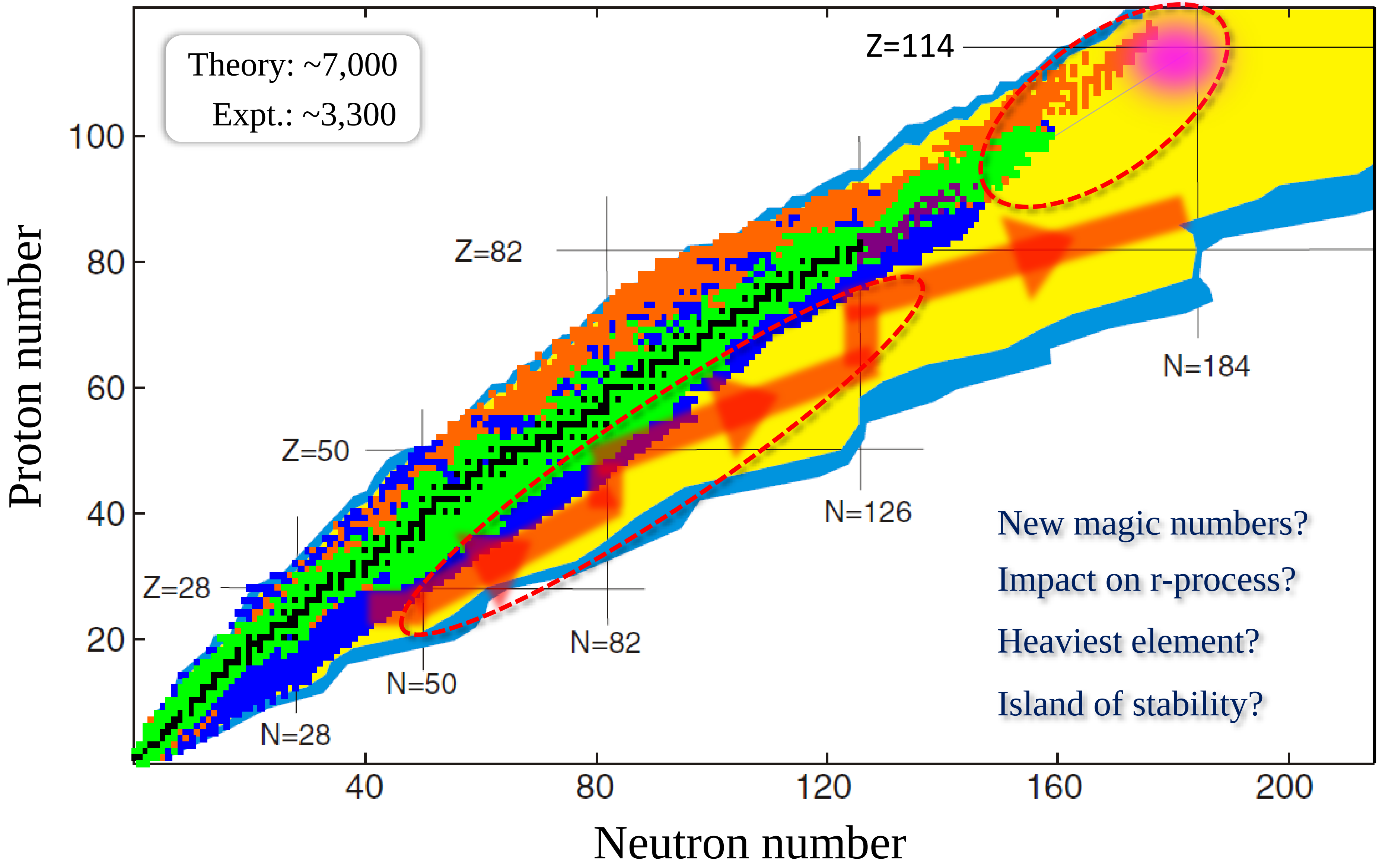
Phys. Rev. Lett. **119**, 042501 (2017)
P. Magierski, K.S., and G. Wlazłowski

At the frontiers in nuclear physics I:

Voyage towards the limit of nuclear existence

The “map” of atomic nuclei: the nuclear chart

(a.k.a. the Segrè chart)



New magic numbers?

Impact on r-process?

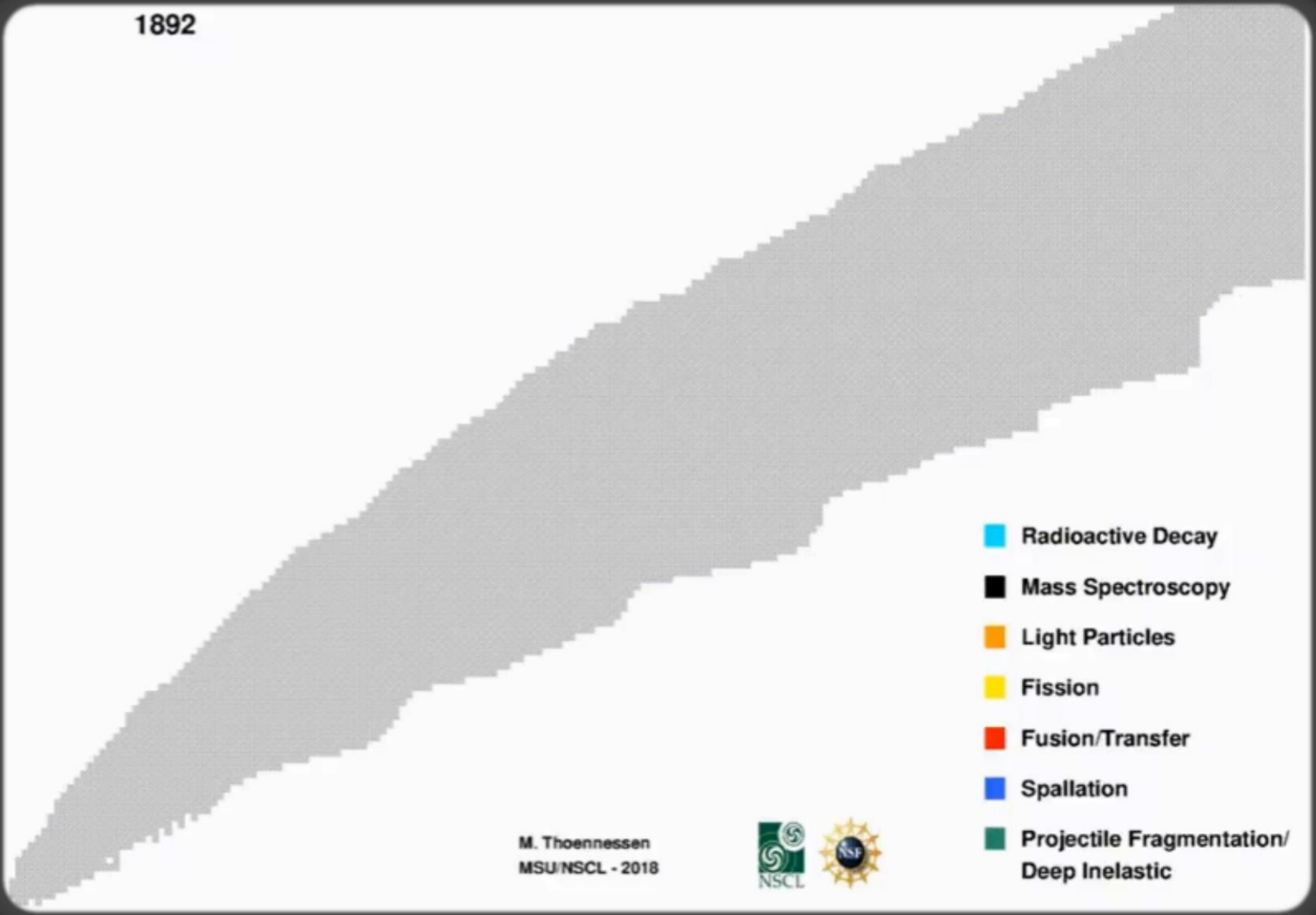
Heaviest element?

Island of stability?

M. Thoennessen, Rep. Prog. Phys. **76**, 056301 (2013)

Movie from “Discovery of Nuclides Project” by Michael Thoennessen

https://people.nscl.msu.edu/~thoennes/isotopes/



https://people.nscl.msu.edu/~thoennes/isotopes/2018-Isotope-Movie.mp4

The continent of stability has been well explored..

The Great Wall of China by Stacy Funderburke @shutterstock

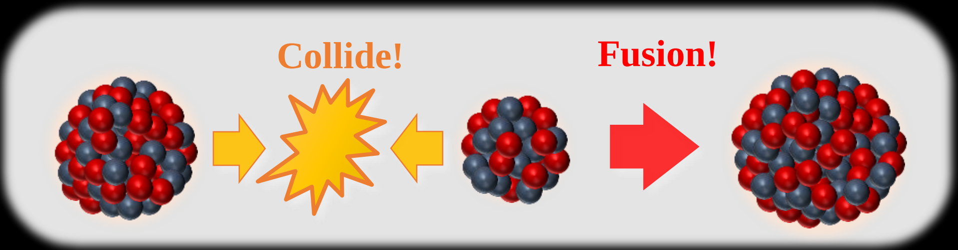
Now we are sailing towards the edge of the nuclear landscape..

Stable nuclei: 288
Experiment: ~3300
Theory: ~7000-10000

- ☐ drip lines
- ☐ shell structure
- ☐ deformation
- ☐ skin, halo
- ☐ nuclear matter properties
- ☐ nucleosynthesis
- ☐ ...

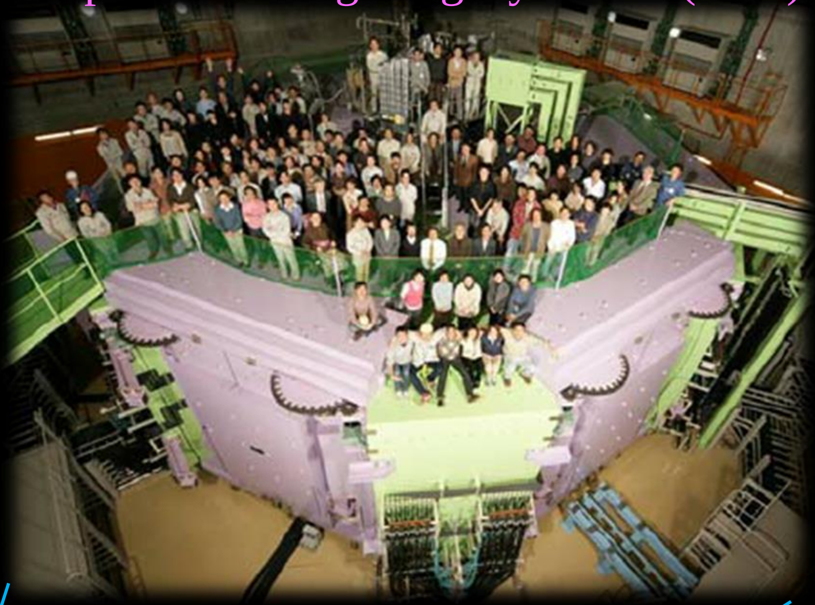
“Flat Earth” by iStock; “Fanciful view of ship sailing over edge of Earth” by Georgia Studies Images

Nucleosynthesis at accelerator facilities

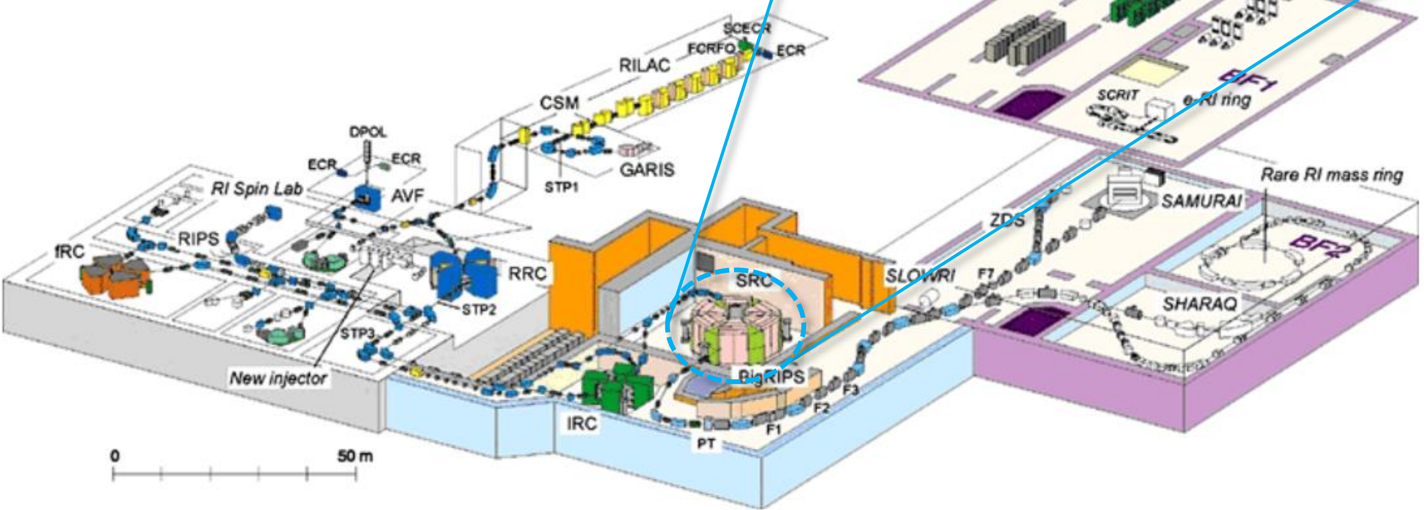


The world-leading factory of unstable nuclei!

Superconducting Ring Cyclotron (SRC)



RI Beam Factory (RIBF) @RIKEN



At the frontiers in nuclear physics II:

Physics of **“Neutron Stars”**

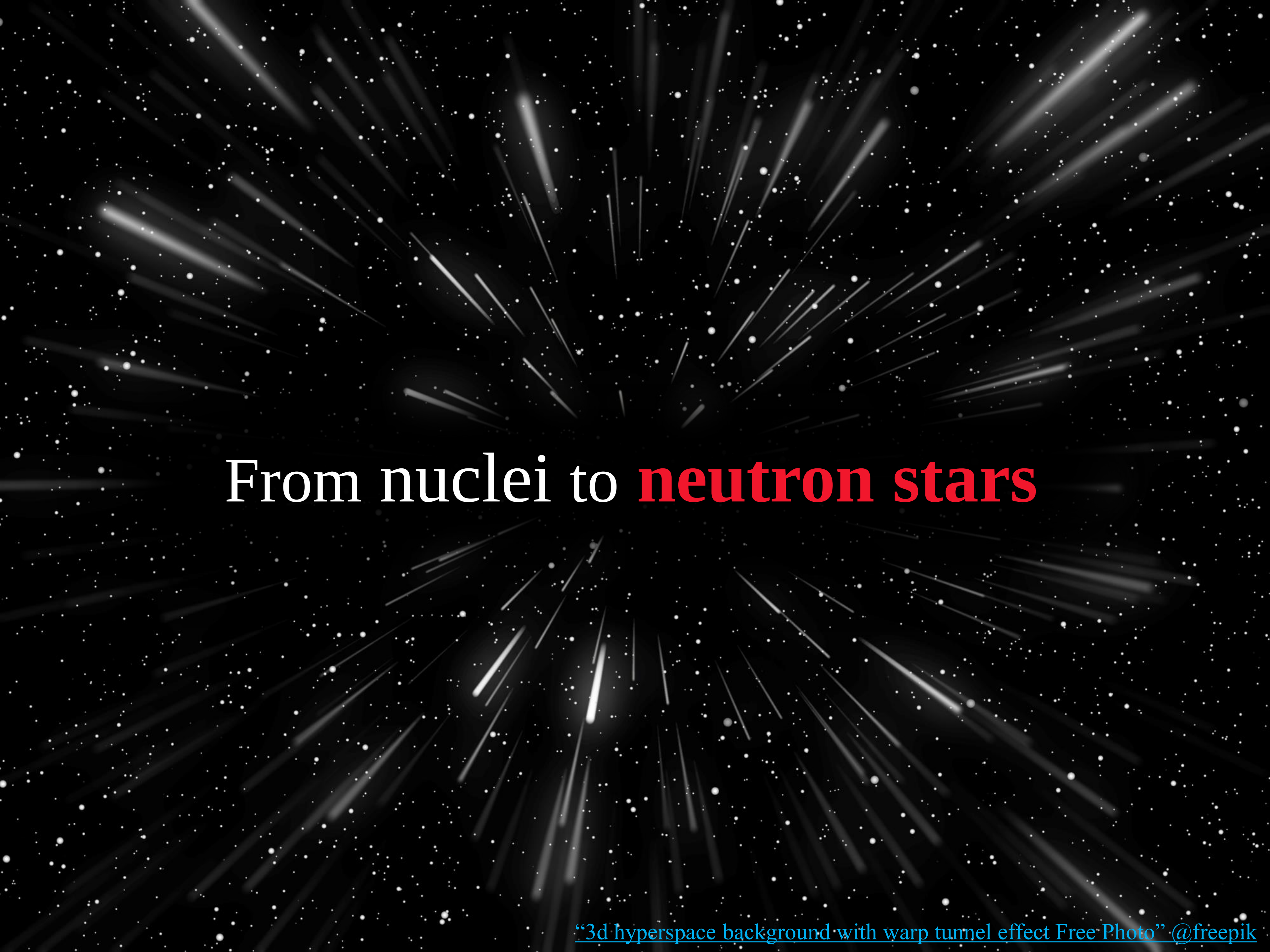
→ also relevant to:

condensed matter physics, solid-state physics,
as well as astrophysics

Now we are sailing towards the edge of the nuclear landscape..

Let's leave the planet
of finite nuclei!

“Flat Earth” by iStock; “Star Wars Spaceship” by Valérian Pierret @Artstation

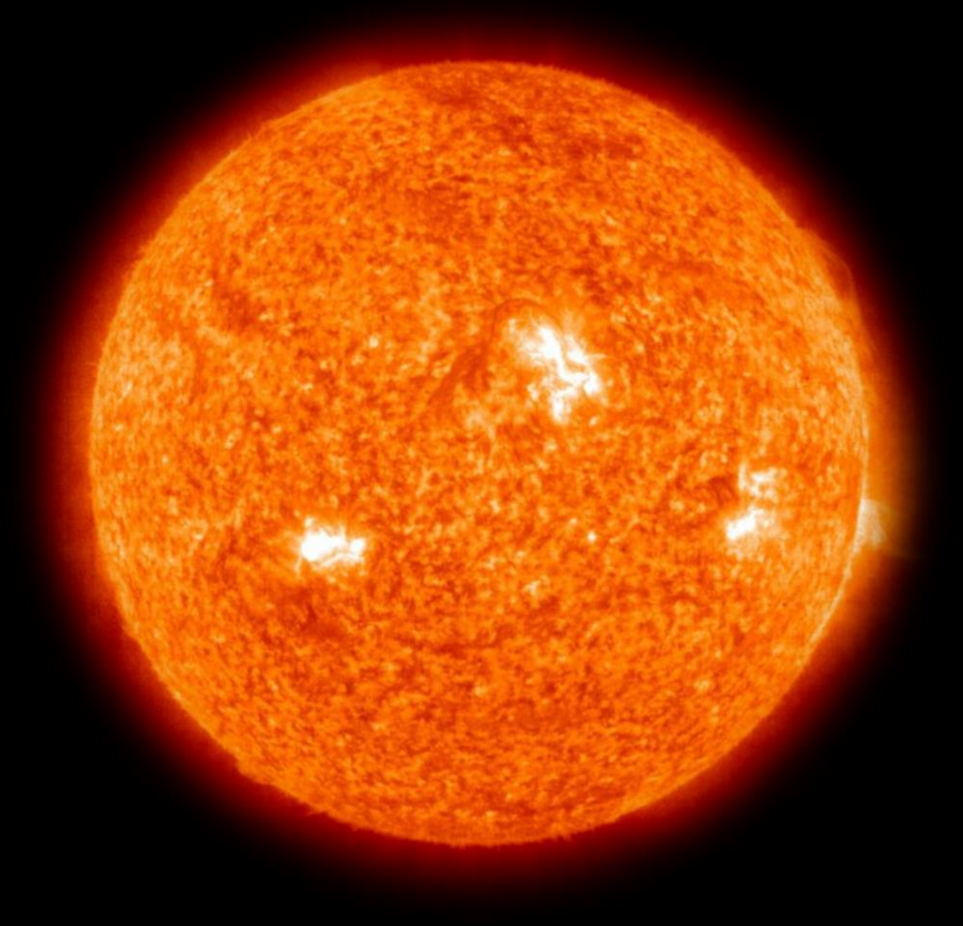


From nuclei to **neutron stars**

“3d hyperspace background with warp tunnel effect Free Photo” @freepik

The Sun

- Radius: ~7×10^8m (~109 times bigger than Earth)
- Mass: ~2×10^{30}kg (~330 thousands times heavier than Earth)
- Central temp.: ~10 million ℃
- Surface temp.: ~5000 ℃

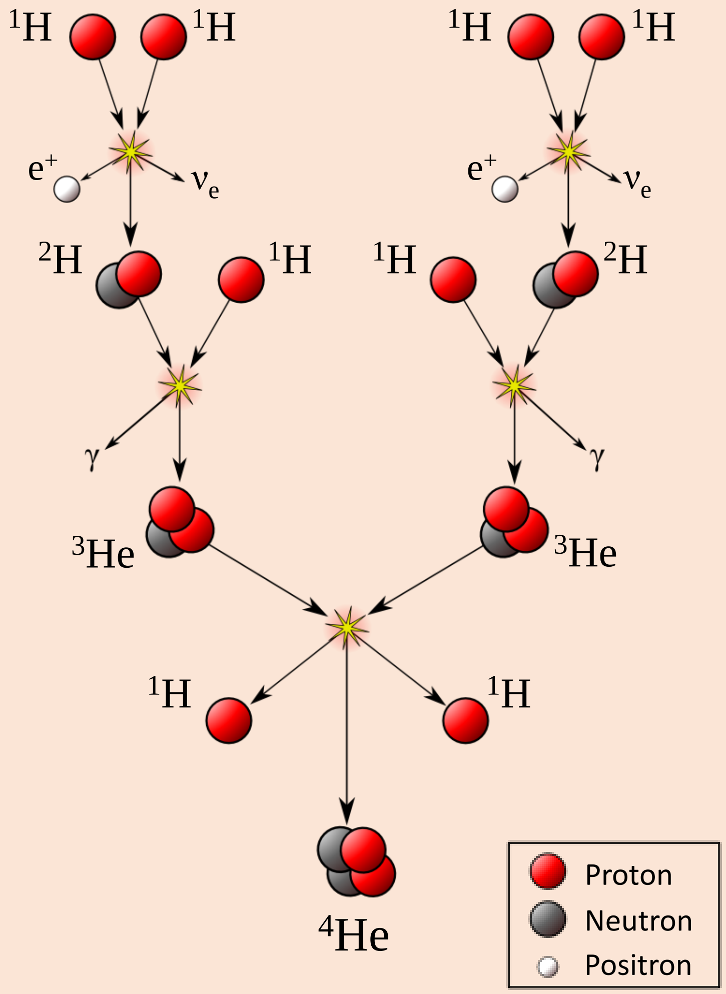


Stars shine due to nuclear fusion reactions

Picture: https://en.wikipedia.org/wiki/Sun

Energy source of the Sun: Nuclear fusion

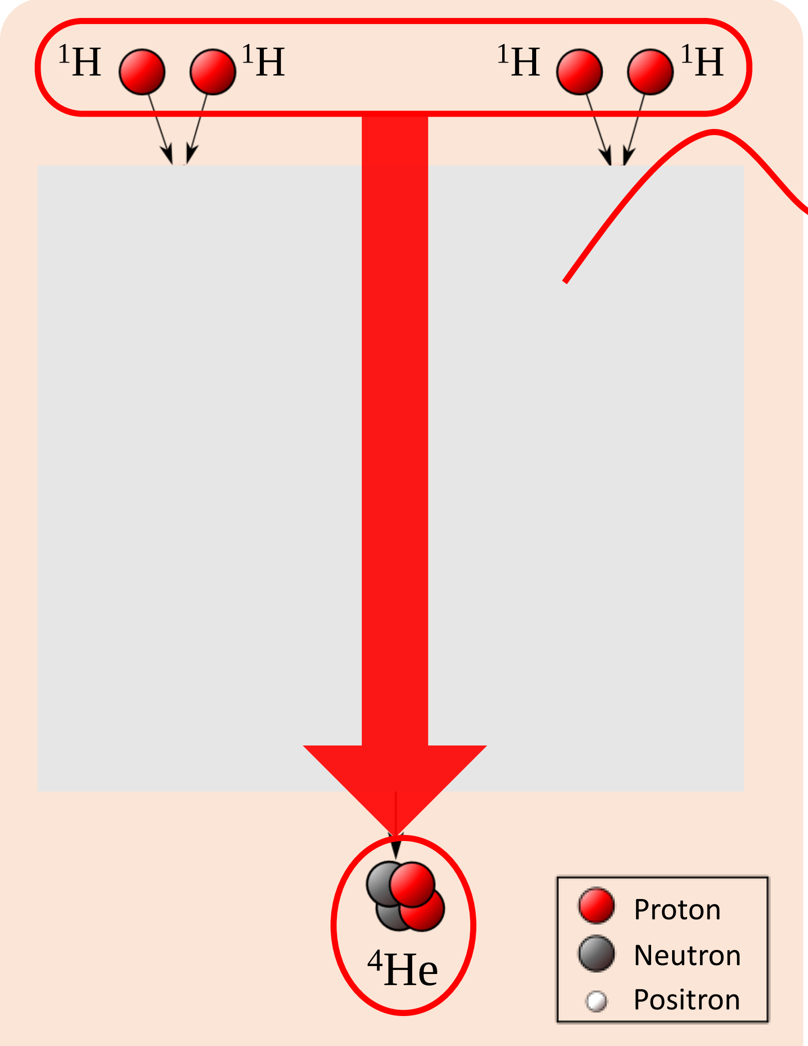
Proton-proton (p-p) chain



$$4p + 2e^- \rightarrow {}^4\mathrm{He} + 6\,\gamma + 2\nu_e + 26.65\,\mathrm{MeV}$$

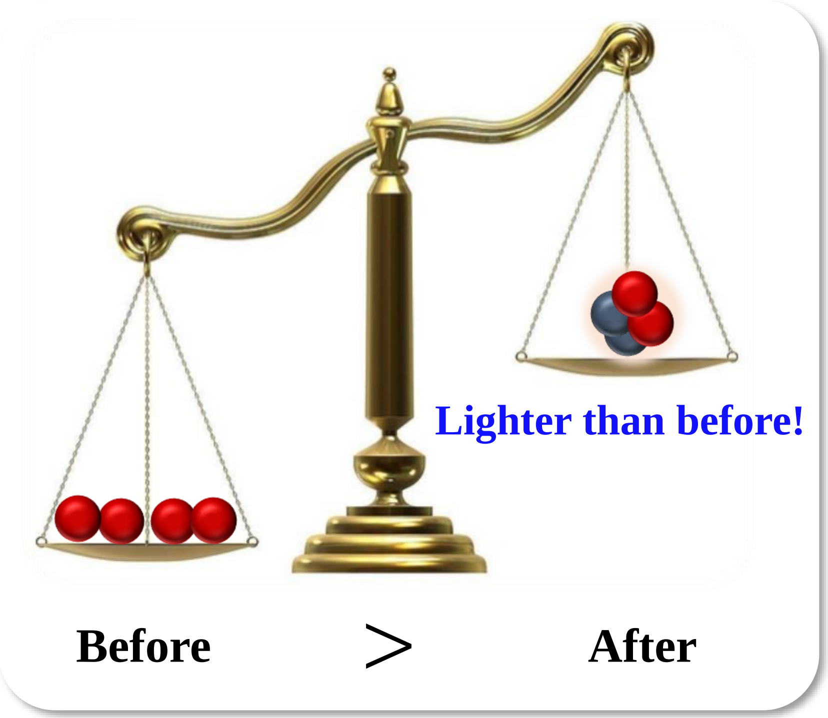
Energy source of the Sun: Nuclear fusion

Proton-proton (p-p) chain



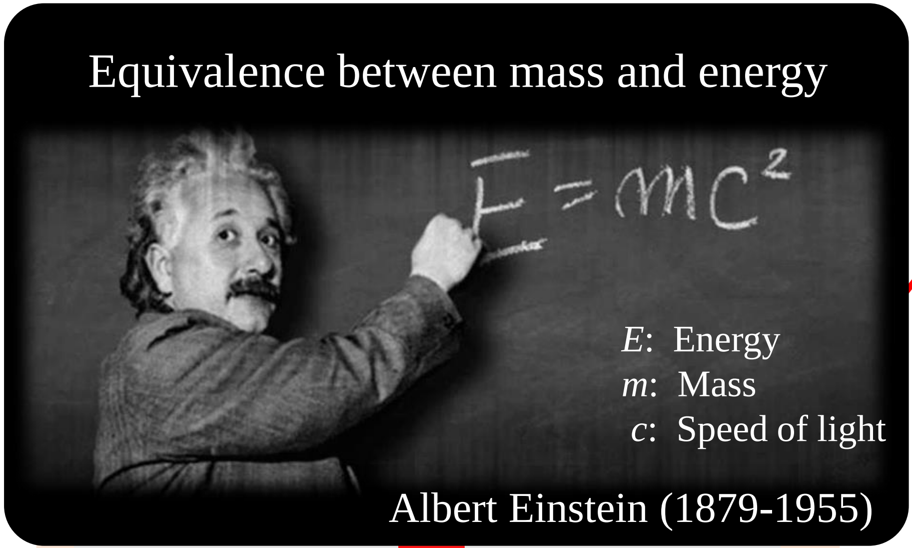
$$4p + 2e^- \rightarrow {}^4He + 6\,\gamma + 2\nu_e + 26.65\,\mathrm{MeV}$$

What happened:
Four protons became a helium nucleus!



Energy source of the Sun: Nuclear fusion

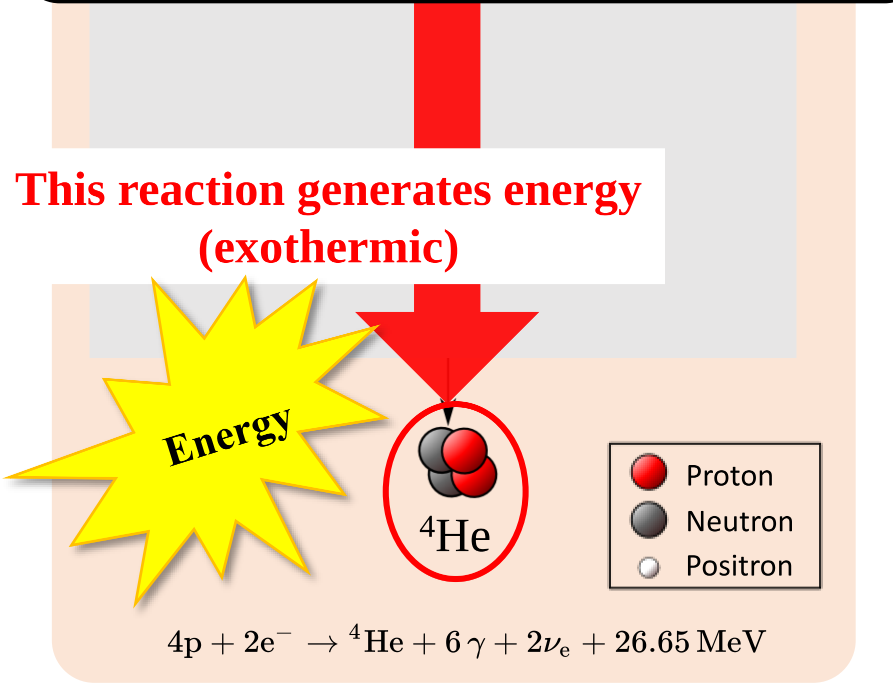
Equivalence between mass and energy



E: Energy
m: Mass
c: Speed of light

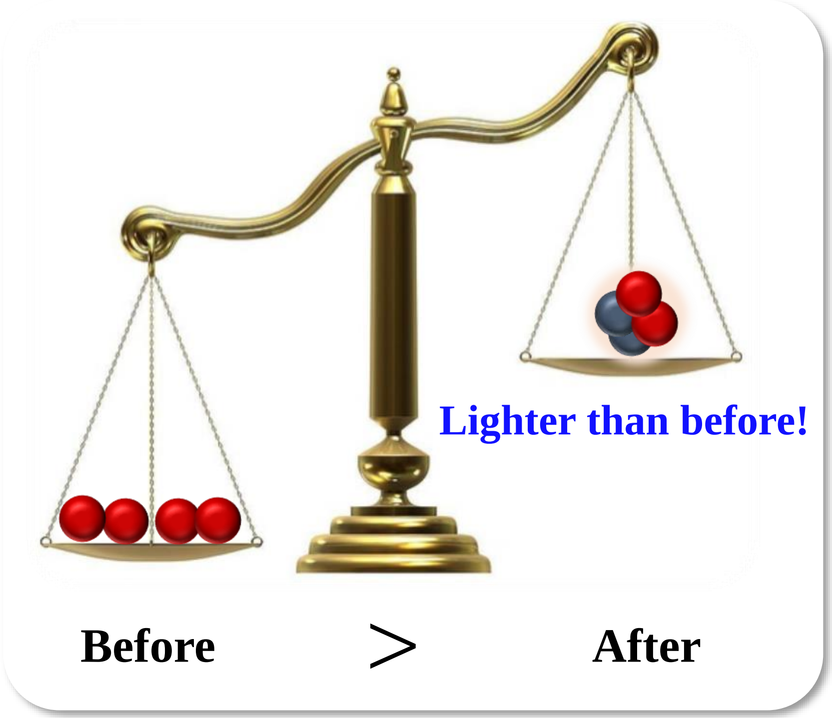
Albert Einstein (1879-1955)

This reaction generates energy (exothermic)



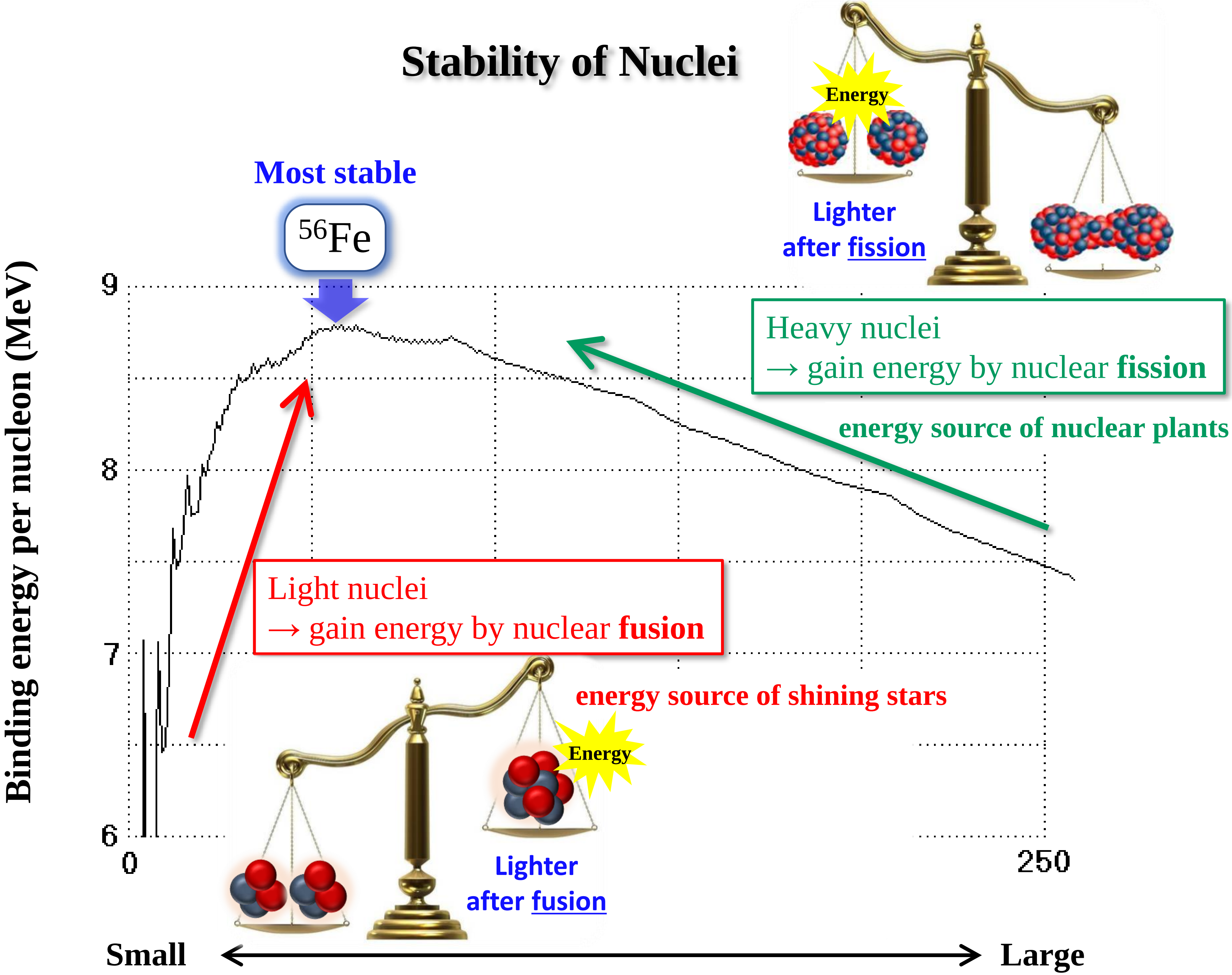
$$4\mathrm{p} + 2\mathrm{e}^- \to {}^4\mathrm{He} + 6\,\gamma + 2\nu_\mathrm{e} + 26.65\,\mathrm{MeV}$$

What happened:
Four protons became a helium nucleus!



Quantum tunneling allows for overcoming the Coulomb barrier!

Stability of Nuclei



Most stable

^{56}Fe

Heavy nuclei
→ gain energy by nuclear **fission**

energy source of nuclear plants

Lighter
after fission

Light nuclei
→ gain energy by nuclear **fusion**

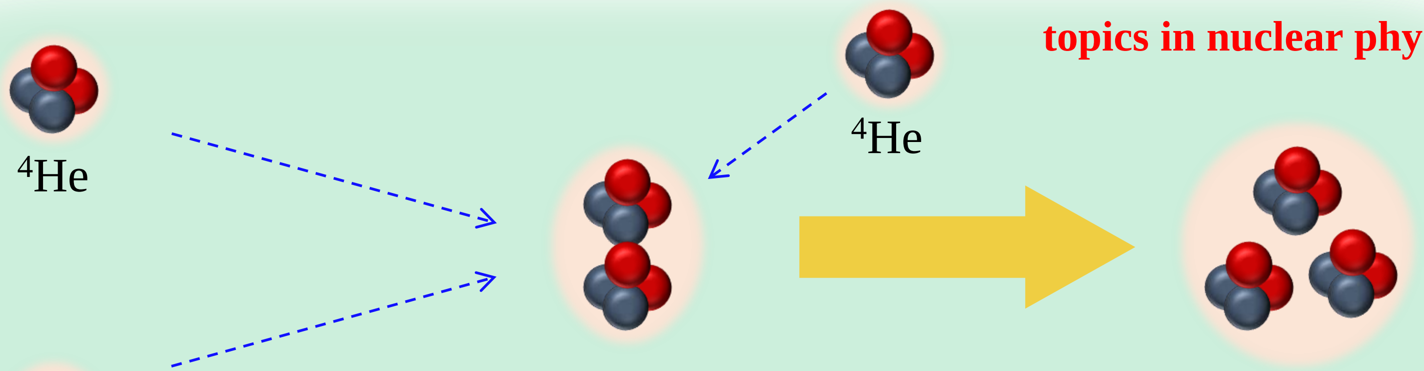
energy source of shining stars

Lighter
after fusion

In massive stars, reactions should proceed further, but..

“Triple-alpha reaction”

- ➢ **Cluster structure is one of hot topics in nuclear physics today!**



^{4}He

^{4}He

^{8}Be
(Proton × 4, Neutron × 4)

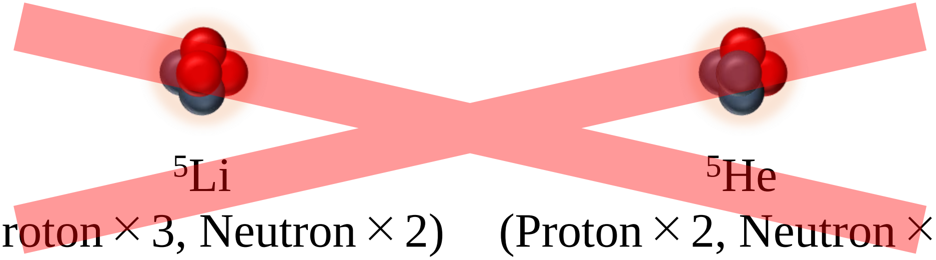
^{12}C*
(Proton × 6, Neutron × 6)

^{4}He
(Proton × 2, Neutron × 2)

Unstable (decays in ~10^{-17} sec)

Hoyle **predicted** an excited state with 3α **cluster structure** near the 3α threshold!

Neither ^{5}Li nor ^{5}Be can be an alternative



^{5}Li
(Proton × 3, Neutron × 2)

^{5}He
(Proton × 2, Neutron × 3)

Unstable (decays in ~10^{-22} sec)

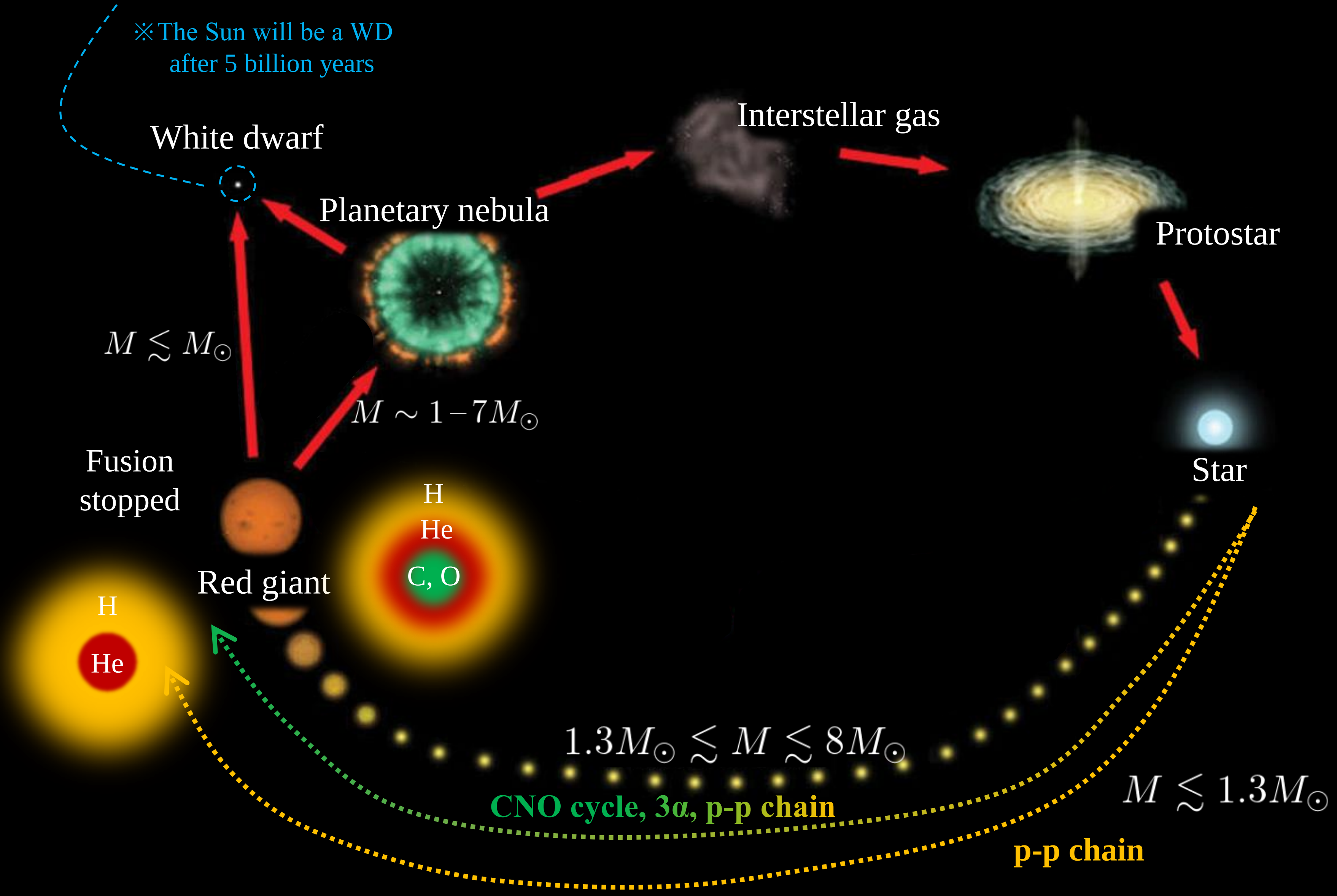
F. Hoyle (1915-2001)

Fred Hoyle: https://en.wikipedia.org/wiki/Fred_Hoyle

Life cycle of a light star

The size is like Earth,
but with the solar mass

※The Sun will be a WD
after 5 billion years



White dwarf

Planetary nebula

Interstellar gas

Protostar

Star

$M \lesssim M_\odot$

$M \sim 1-7M_\odot$

Fusion
stopped

Red giant

H
He
C, O

H
He

$1.3M_\odot \lesssim M \lesssim 8M_\odot$

CNO cycle, 3α, p-p chain

$M \lesssim 1.3M_\odot$

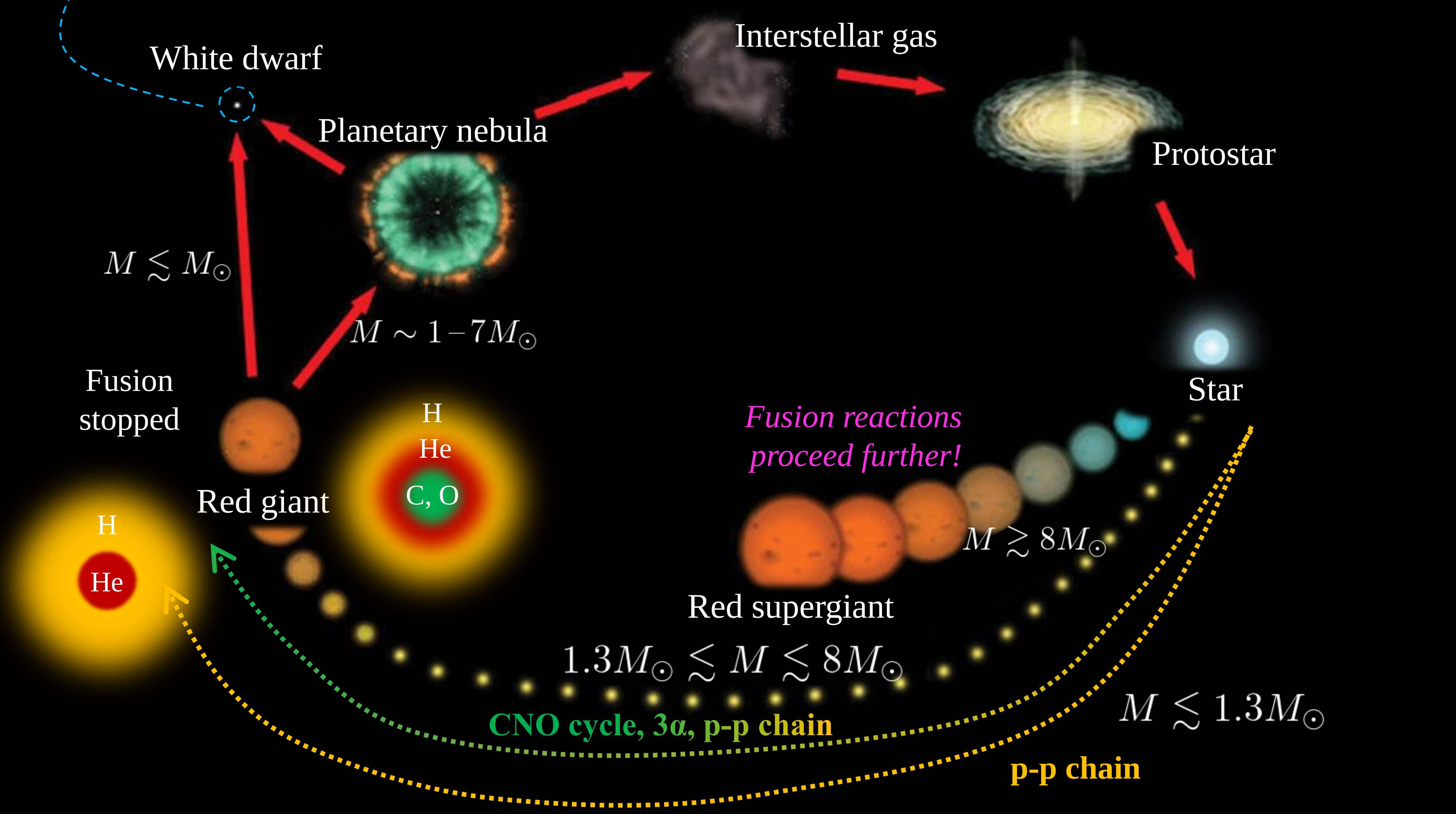
p-p chain

Picture: https://www.astroarts.co.jp/products/stlnav7/spec/plane_list-j.shtml

Life cycle of a **massive** star

The size is like Earth,
but with the solar mass

※The Sun will be a WD
after 5 billion years



White dwarf

Planetary nebula

Interstellar gas

Protostar

Star

$M \lesssim M_\odot$

$M \sim 1-7M_\odot$

Fusion
stopped

Red giant

H

He

C, O

Fusion reactions
proceed further!

$M \gtrsim 8M_\odot$

Red supergiant

$1.3M_\odot \lesssim M \lesssim 8M_\odot$

$M \lesssim 1.3M_\odot$

CNO cycle, 3α, p-p chain

p-p chain

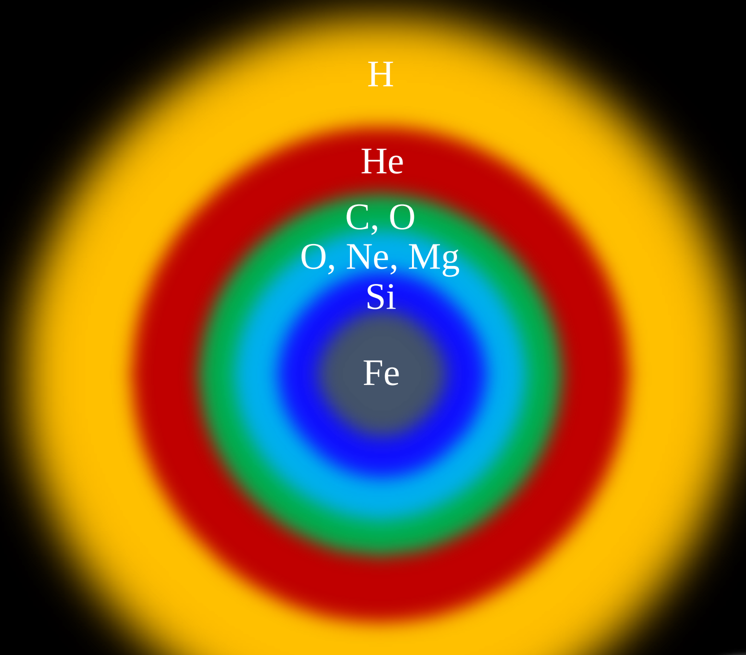
Picture: https://www.astroarts.co.jp/products/stlnav7/spec/plane_list-j.shtml

The fate of a massive star

Nuclear reactions:

$^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C} \rightarrow {}^{16}\mathrm{O} \rightarrow {}^{20}\mathrm{Ne} \rightarrow {}^{24}\mathrm{Mg} \rightarrow {}^{28}\mathrm{Si} \rightarrow \ldots \rightarrow {}^{56}\mathrm{Fe}$

“Onion structure”



After forming the iron core…
→ no more fuel
→ gravitational collapse
→ supernova explosion

The Crab Nebula
Remnant of the SN in 1054

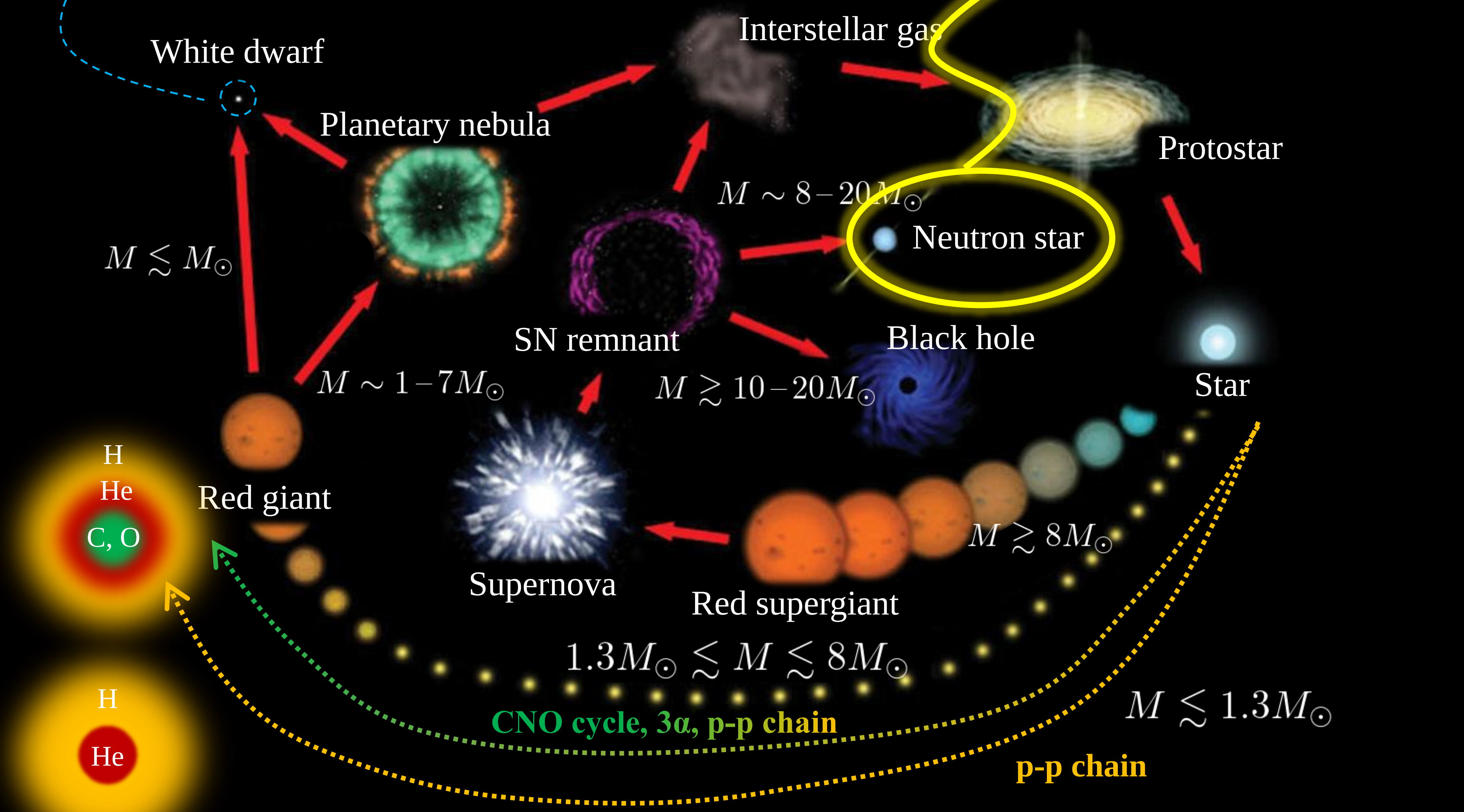
Picture: https://en.wikipedia.org/wiki/Crab_Nebula

Life cyc

An ultra-heavy, super-*N*-rich “nucleus”
→ Paradise for nuclear physicists! :D

The size is like Earth,
but with the solar mass

※The Sun will be a WD
after 5 billion years



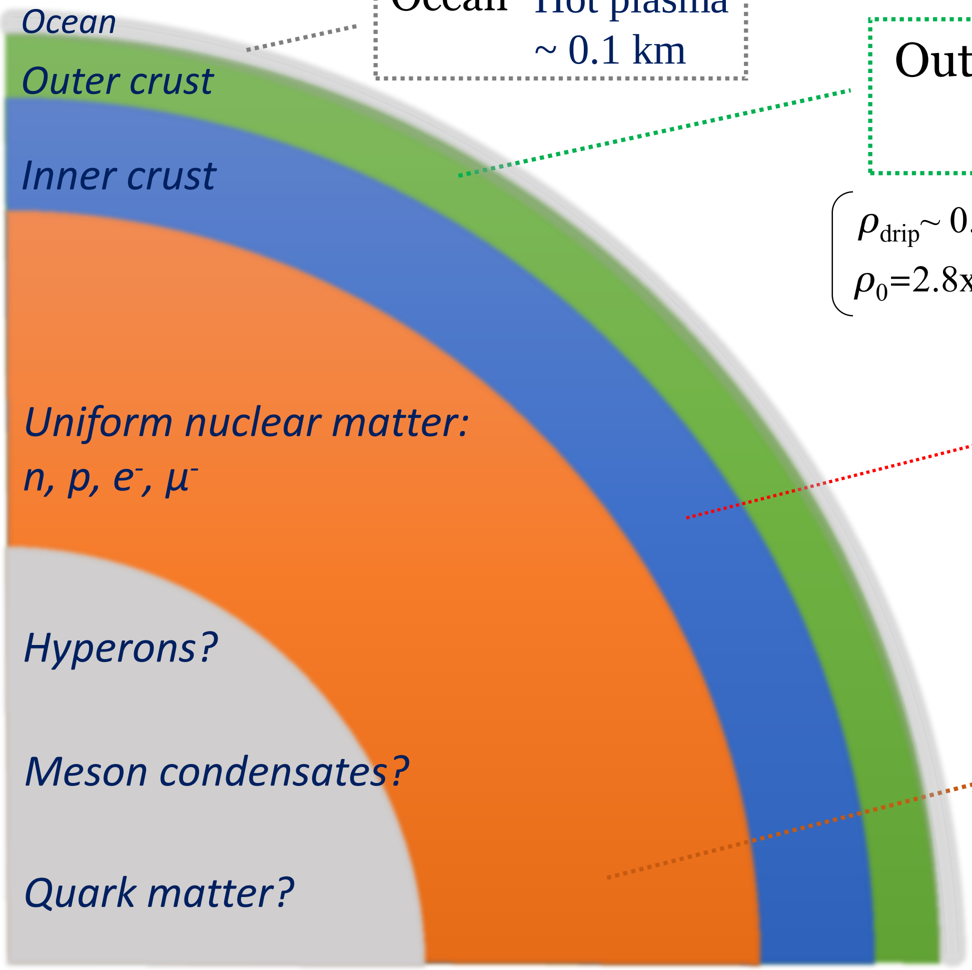
Picture: https://www.astroarts.co.jp/products/stlnav7/spec/plane_list-j.shtml

What’s inside a neutron star?

Structure of a neutron star

Neutron star is a great playground for nuclear physicists

✓ **It offers extreme situations which can not be realized in terrestrial experiments**



Ocean Hot plasma
~ 0.1 km

Outer crust Nuclei (Z=26-42), electrons
0.3-0.6 km $\rho < \rho_{\mathrm{drip}}$

$\rho_{\mathrm{drip}} \sim 0.0014\rho_0$; Above ρ_{drip} unbound neutrons exist outside nuclei
$\rho_0 = 2.8\mathrm{x}10^{14}\ \mathrm{g/cm^3} = 0.16\ \mathrm{fm^{-3}}$; Nuclear saturation density

Inner crust Nuclei, electrons, "dripped" neutrons
0.5-0.8 km $\rho_{\mathrm{drip}} \leq \rho \leq 0.6\rho_0$

Outer/inner core
9-12 km $0.6\rho_0 < \rho \leq 3\text{-}5\rho_0$

In an outer (low-density) region of neutron stars,
nuclear matter is *not* actually *homogeneous*

The nuclear interaction “clusterize” neutrons and protons,
akin to finite nuclei, which form a Coulomb lattice
(*i.e.* a crystal, like a solid)

Let’s see: from the outer crust to the inner crust

Outer crust

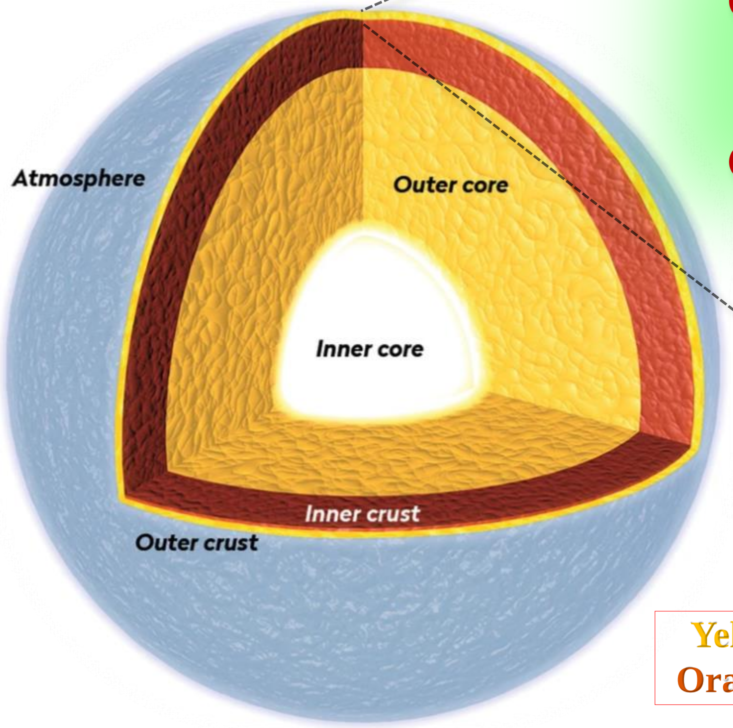
Note:
In the figure showing nuclides composition, the **bold** font indicates known-mass nuclei.

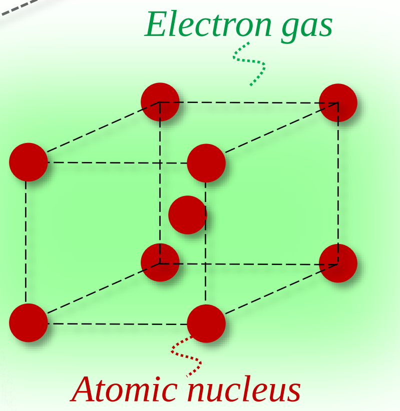
Structure of the outer crust is "similar" to that of a white dwarf

but, nuclei are different and become neutron-rich!

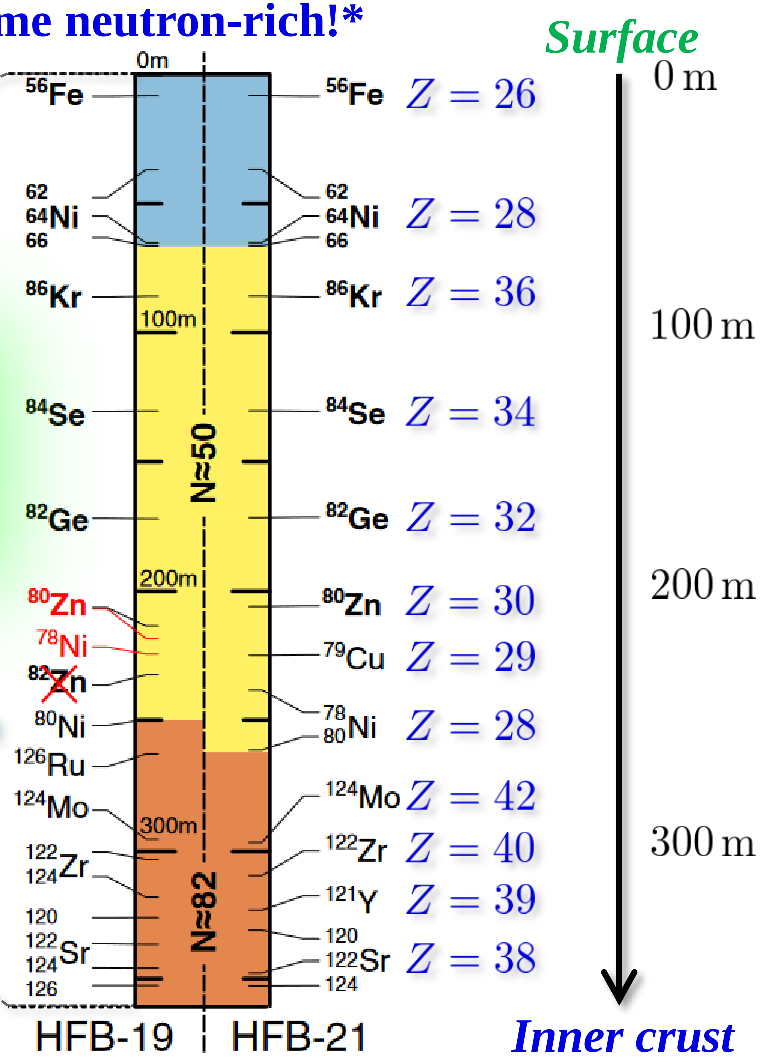
Composition of the outer crust:

✓ Coulomb lattice of nuclei
+ Electron gas





$$p + e^- \rightarrow n + \nu_e$$

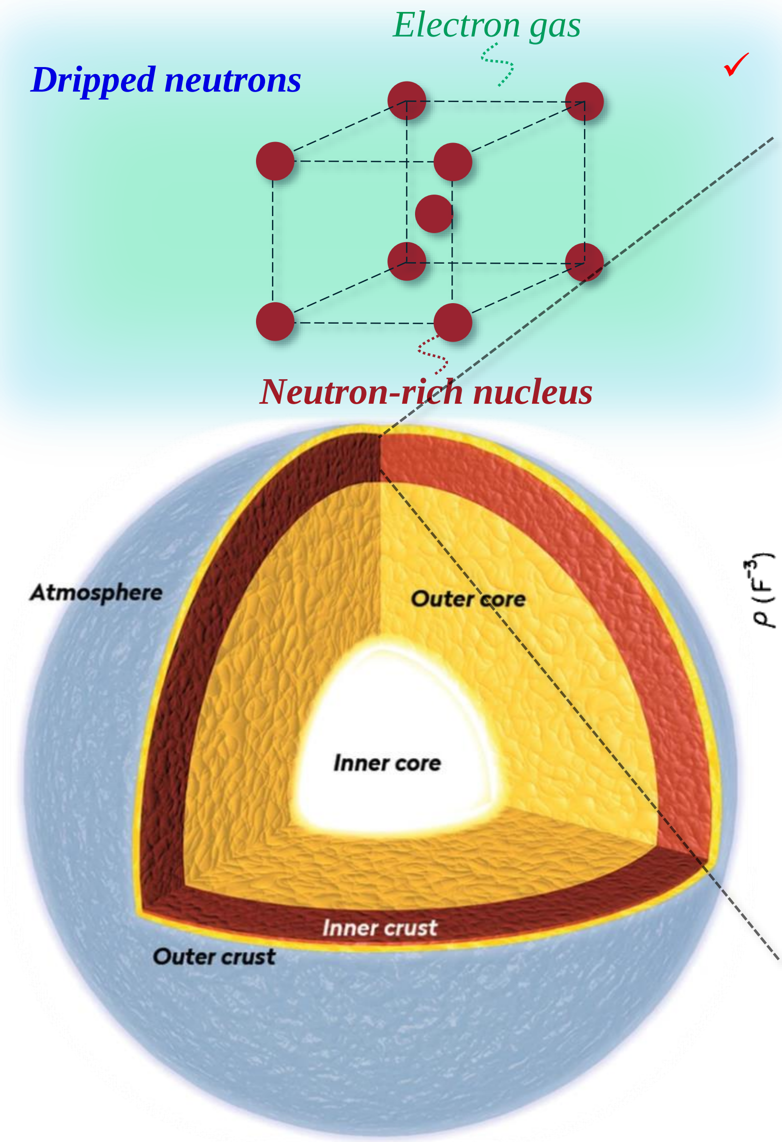


Yellow region: $N \approx 50$
Orange region: $N \approx 82$

Figure: R.N. Wolf et al., Phys. Rev. Lett. **110**, 041101 (2013)
Neutron star: https://www.skyatnightmagazine.com/space-science/neutron-star/

Inner crust

In the inner crust, a sea of "dripped neutrons" permeates the Coulomb lattice



✓ Distance between nuclei decreases with increasing density

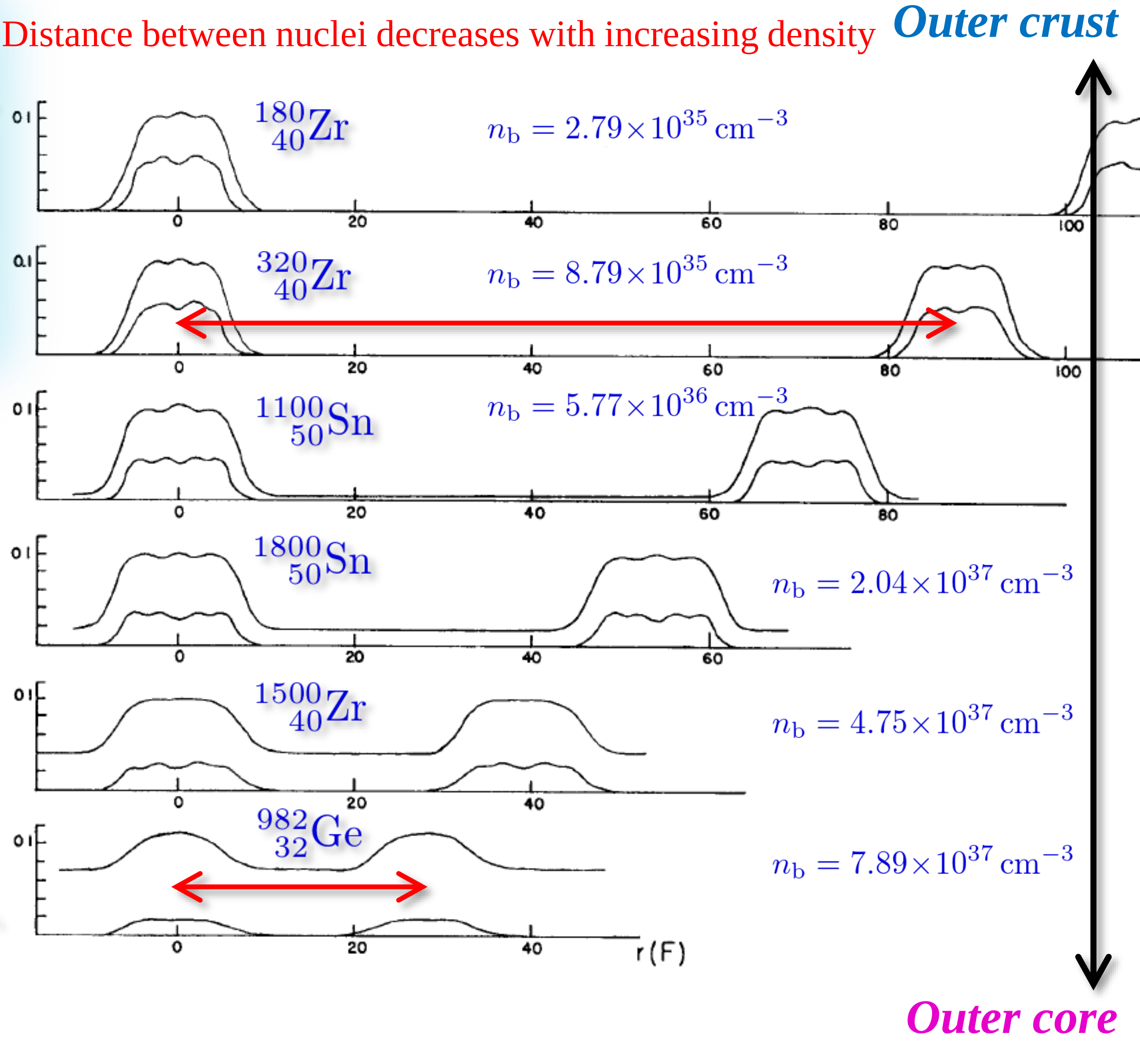


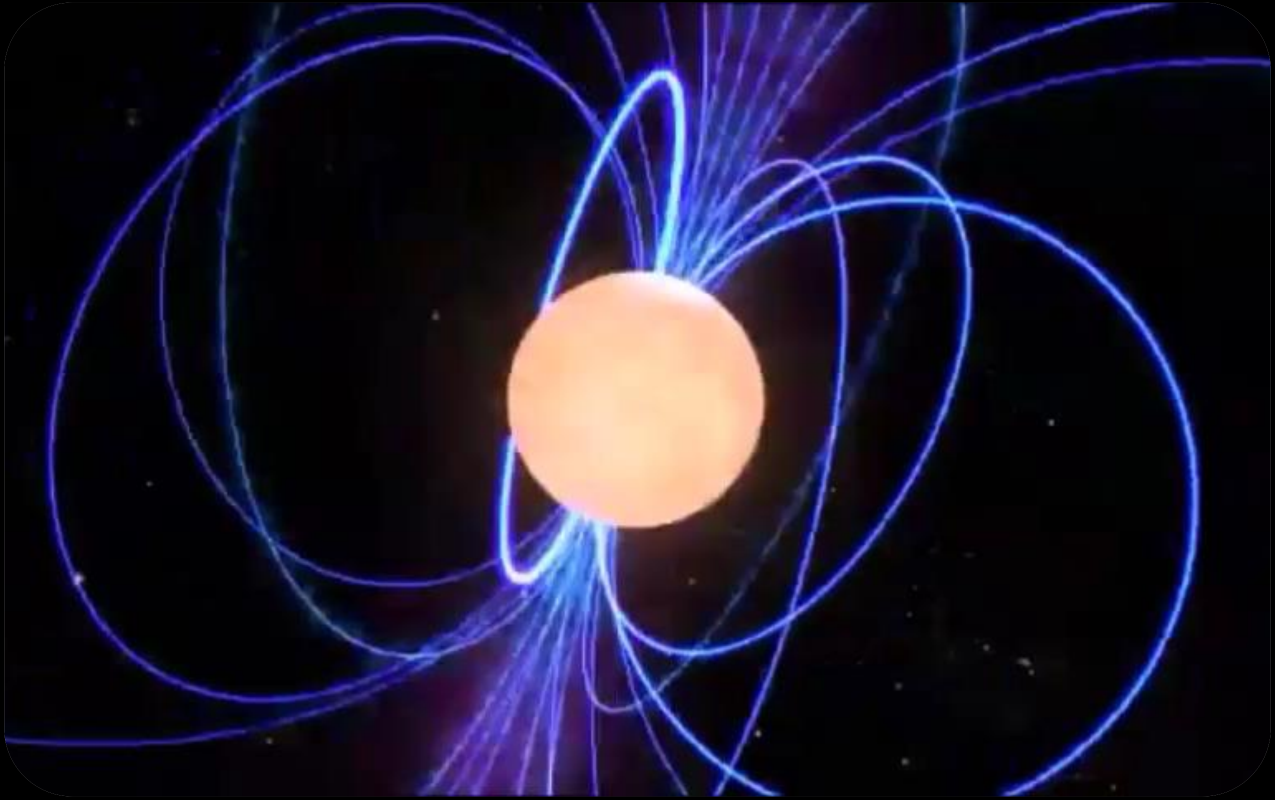
Figure: J.W. Negele and D. Vautherin, Nucl. Phys. **A207**, 298 (1978)
Neutron star: https://www.skyatnightmagazine.com/space-science/neutron-star/

Neutron-star “glitch”

Picture: https://astronomy.com/magazine/ask-astro/2017/12/stellar-magnets

Pulsar - a rotating neutron star

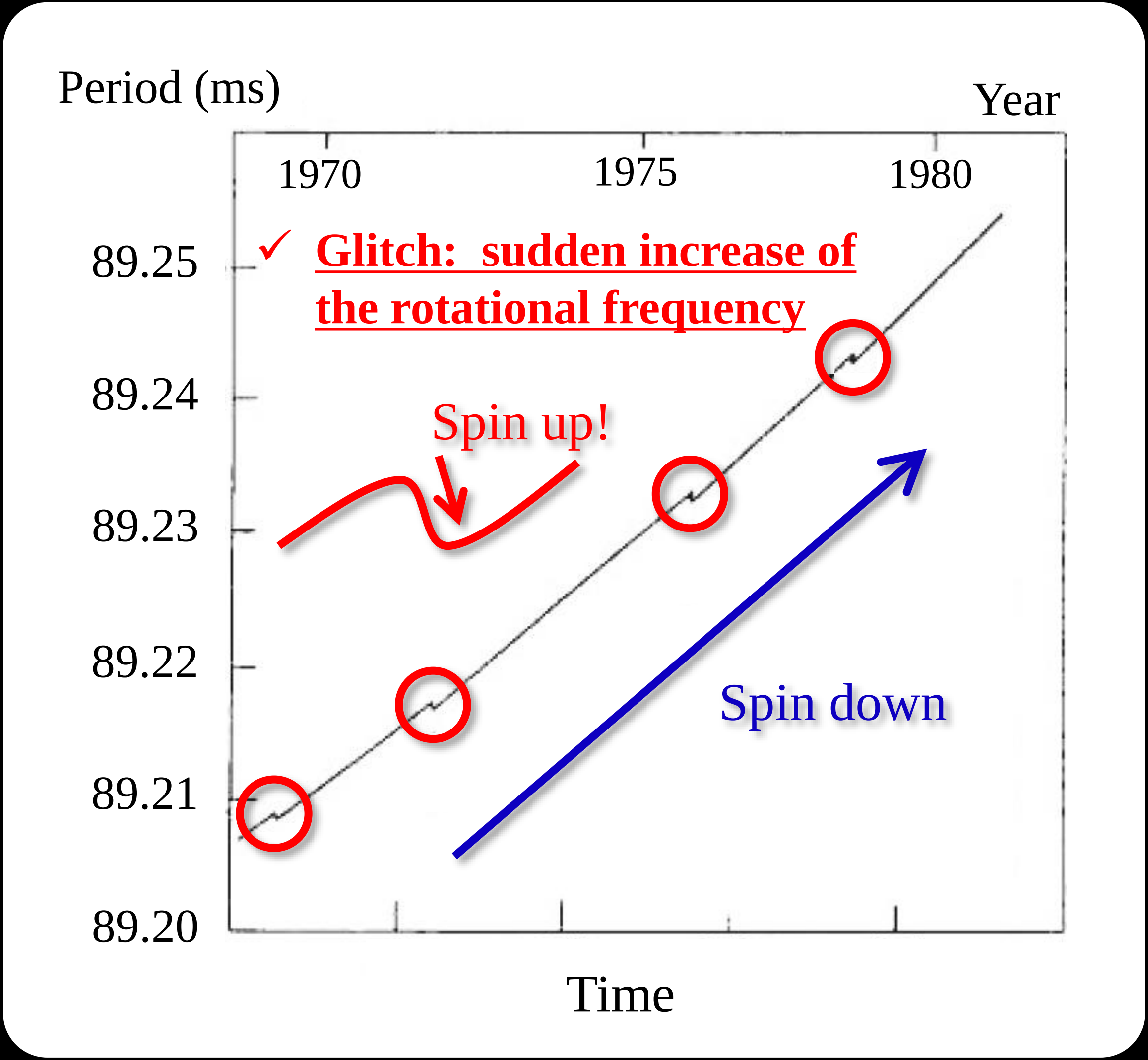
- ✓ First discovery in August 1967 → “Little Green Man” LGM-1 → PSR B1919+21
- ✓ Since then, more than 2650 pulsars have been observed
- ✓ It gradually spins down due to the EM radiation



What is the glitch?

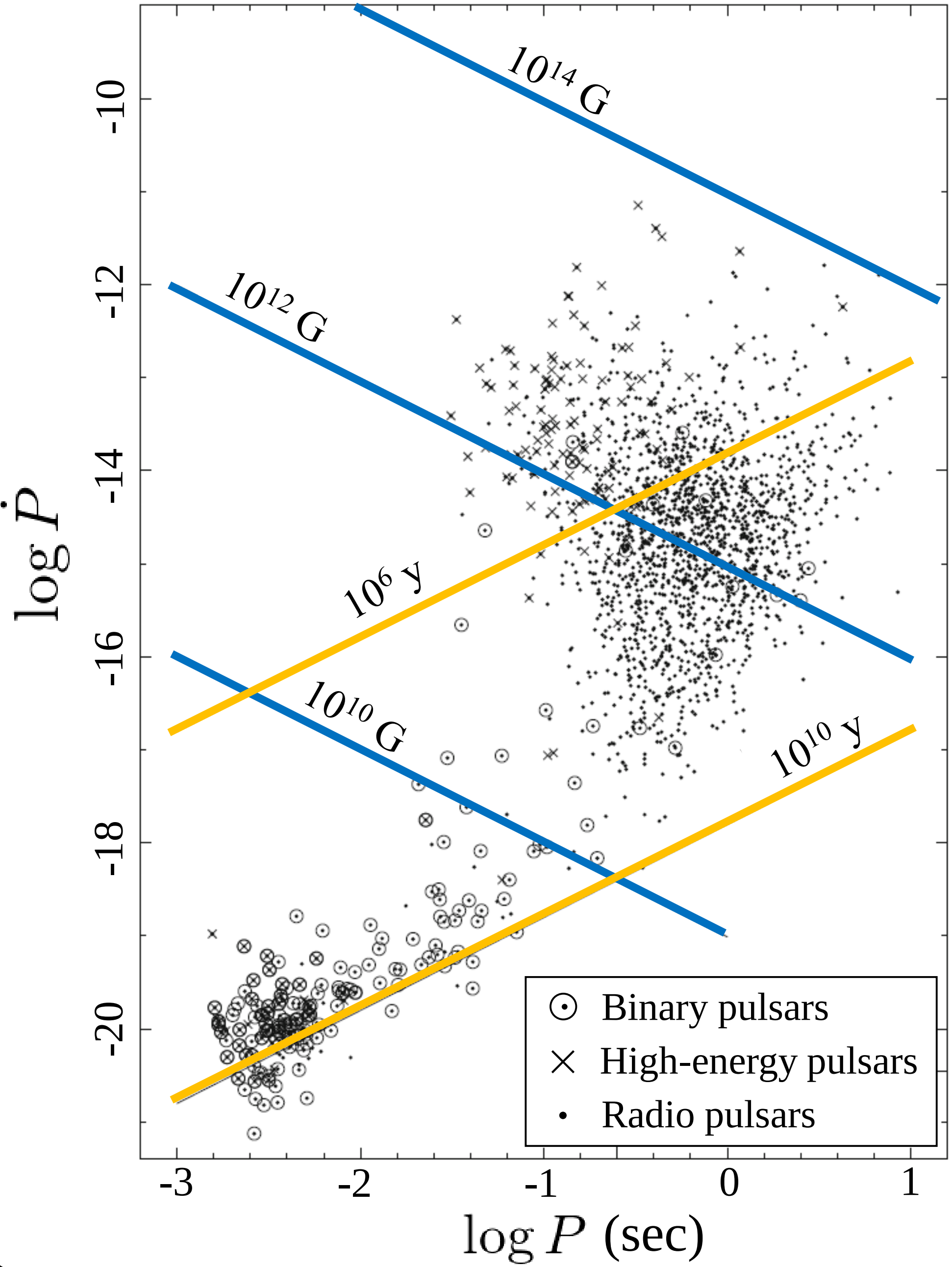
Typical example: the Vela pulsar

- *Irregularity* has been observed from continuous monitoring of the pulsation period



✓ **Glitch: sudden increase of the rotational frequency** cf. Glitch Table and Glitch Catalogue

P-$\dot{P}$ diagram for pulsars (*not necessarily glitchers)



- ✓ Period (P): milliseconds to seconds
- ✓ Gradually spins down ($\dot{P} > 0$) due to the EM radiations
- ✓ Very stable "clock", especially for millisecond pulsars, i.e. $\dot{P} \sim 10^{-20}$

➢ Characteristic age:

$$\tau_c = P/(2\dot{P})$$

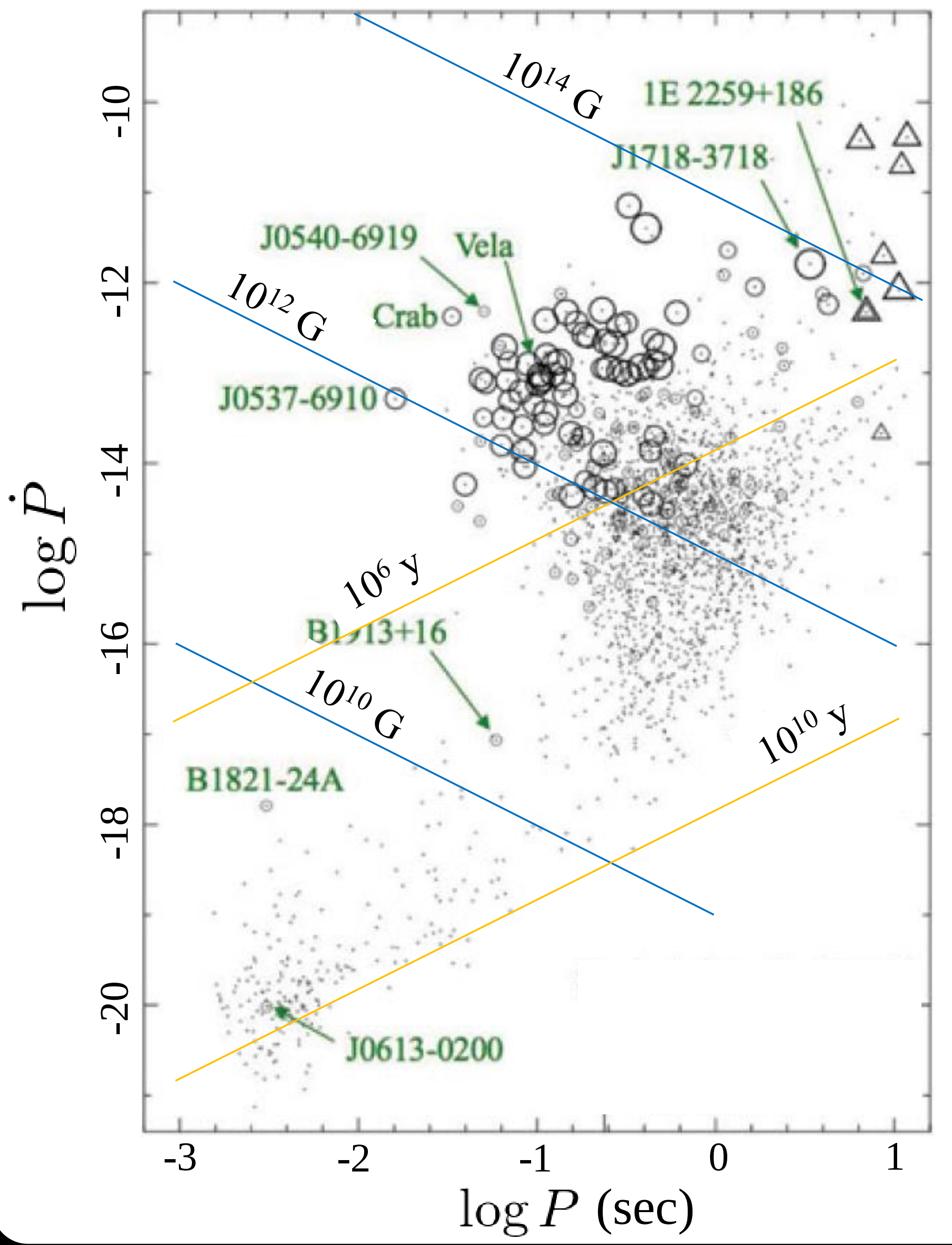
➢ Surface dipole magnetic field strength:

$$B = 3.2 \times 10^{19} (P\dot{P})\ \mathrm{G}$$

Figure taken from:
R.N. Manchester, J. Astrophys. Astr. **38**, 42 (2017)

cf. Glitch Table and Glitch Catalogue

P-$\dot{P}$ diagram for pulsars (*not necessarily glitchers)



- ✓ **More than 548 glitches** have been observed **in more than 180 pulsars**
- ✓ Symbol size: glitch size (typically, log(Δν/ν) ~ 10^{-10}-10^{-5})
- ✓ Young pulsars (including magnetars) exhibit larger glitches than older ones

➢ Characteristic age:

$$\tau_c = P/(2\dot{P})$$

➢ Surface dipole magnetic field strength:

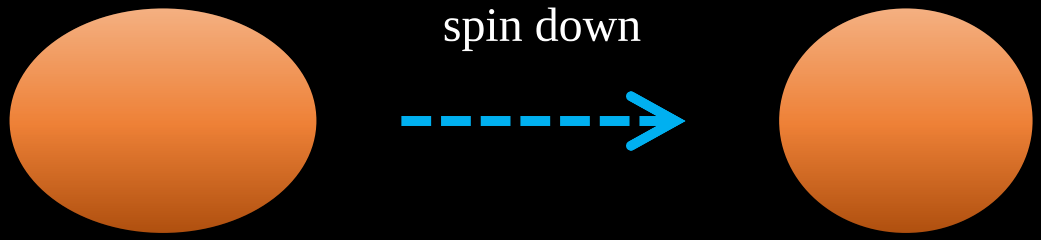
$$B = 3.2 \times 10^{19} (P\dot{P})\ \mathrm{G}$$

Figure taken from:
R.N. Manchester, Proc. IAU Symp. **337**, 197 (2017)

What happened?

Something must happen inside the neutron star!

- ✓ "Starquake" model [G. Baym et al., Nature **224**, 872 (1969)]



oblate

less oblate

spin down

Preferable shape

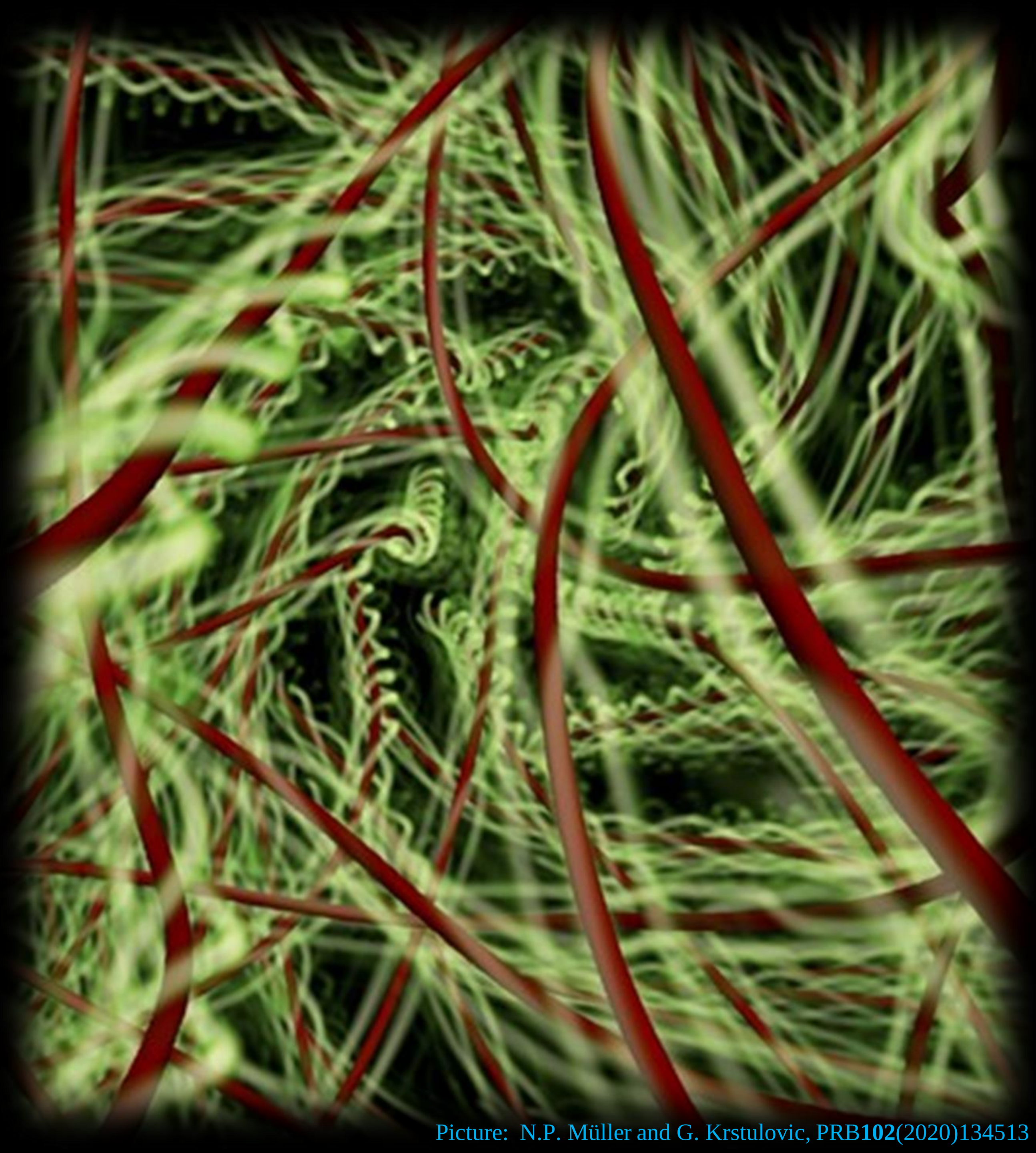
- ✓ Sudden change of moment of inertia → Glitch?

It requires <u>hundreds of years</u> to have a next glitch..

- ✓ Vortex mediated glitch [P.W. Anderson and N. Itoh, Nature **256**, 25 (1975)]

➢ <u>Dynamics of superfluid *"quantized vortices"* play a key role!</u> **!?**

Quantum vortices



Picture: N.P. Müller and G. Krstulovic, PRB**102**(2020)134513

What is a quantum vortex?

In superfluid, vortices are quantized!

Superfluid order parameter:

$$\Delta(\boldsymbol{r},t) = |\Delta(\boldsymbol{r},t)|e^{i\phi(\boldsymbol{r},t)}$$

Superfluid velocity:

$$\boldsymbol{v}_s(\boldsymbol{r},t) = \frac{\hbar}{m}\nabla\phi(\boldsymbol{r},t)$$

Vorticity:

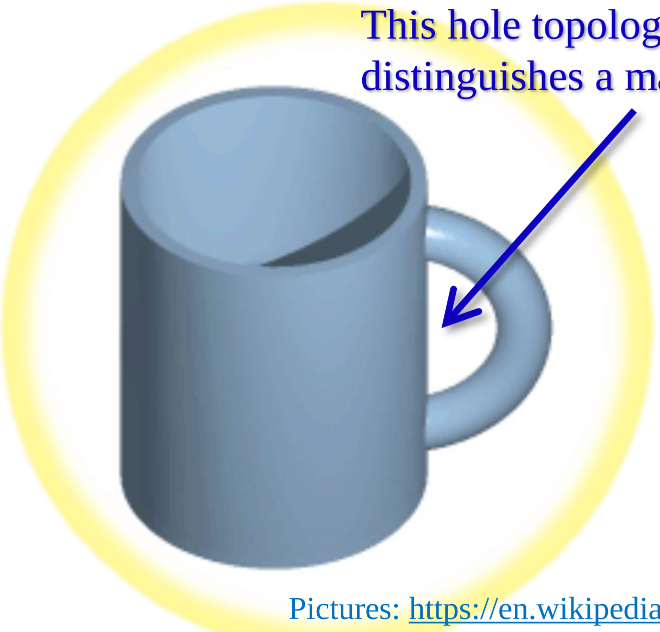
$$\underline{\boldsymbol{\omega} = \nabla \times \boldsymbol{v}_s = 0}$$

superfluid is irrotational

Circulation:

$$\kappa = \int_S (\nabla \times \boldsymbol{v}_s) \cdot d\boldsymbol{S} = 0$$

*Unless, there is no topological defect



This hole topologically distinguishes a mag from a cow

Pictures: https://en.wikipedia.org/wiki/Topology

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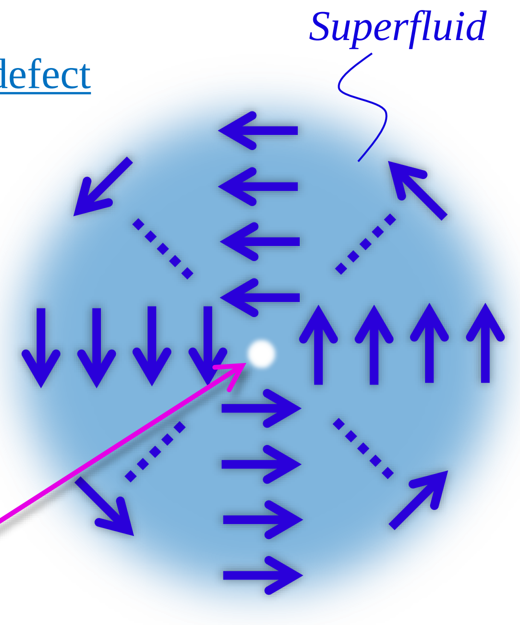
If there is a defect:

$$\kappa = \oint_C \boldsymbol{v}_s \cdot d\boldsymbol{l} = \frac{\hbar}{m}\oint_C \nabla\phi \cdot d\boldsymbol{l} = \frac{\hbar}{m} 2\pi n$$

Quantization of circulation

*the phase $\phi(\boldsymbol{r})$ at the same point must be equivalent!

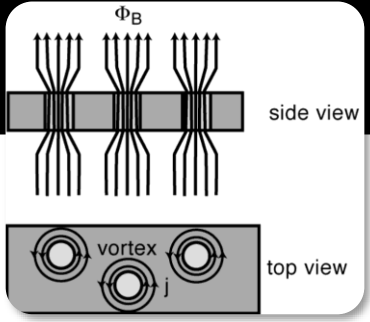
Flow velocity of rotation shall be quantized!



A hole at which superfluidity is lost

Quantum vortex

What is a quantum vortex?



In superconductor, magnetic flux is quantized!

Magnetic flux:

Meissner effect

$$\Phi = \int_S \boldsymbol{B} \cdot d\boldsymbol{S} = \int_S (\nabla \times \boldsymbol{A}) \cdot d\boldsymbol{S} \overset{\downarrow}{=} 0$$

$$\boldsymbol{j}_s = -\frac{n_s e_s^2}{m_s}\boldsymbol{A} + \frac{n_s e_s \hbar}{m_s}\nabla\phi$$: the London equation

If there is a defect:

$$\Phi = \oint_C \boldsymbol{A} \cdot d\boldsymbol{l} \approx \frac{\hbar}{e_s} \oint_C \nabla\phi \cdot d\boldsymbol{l} = \frac{\hbar}{e_s} 2\pi n$$

n_s, m_s, e_s: density, mass, and charge of a carrier (Cooper pair)

Quantization of magnetic flux (fluxtube, fluxoid, or fluxon)

Circulation:

$$\kappa = \int_S (\nabla \times \boldsymbol{v}_s) \cdot d\boldsymbol{S} = 0$$

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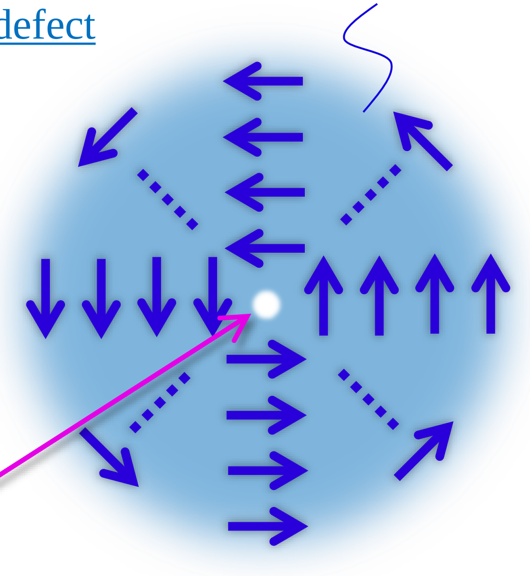
Superfluid

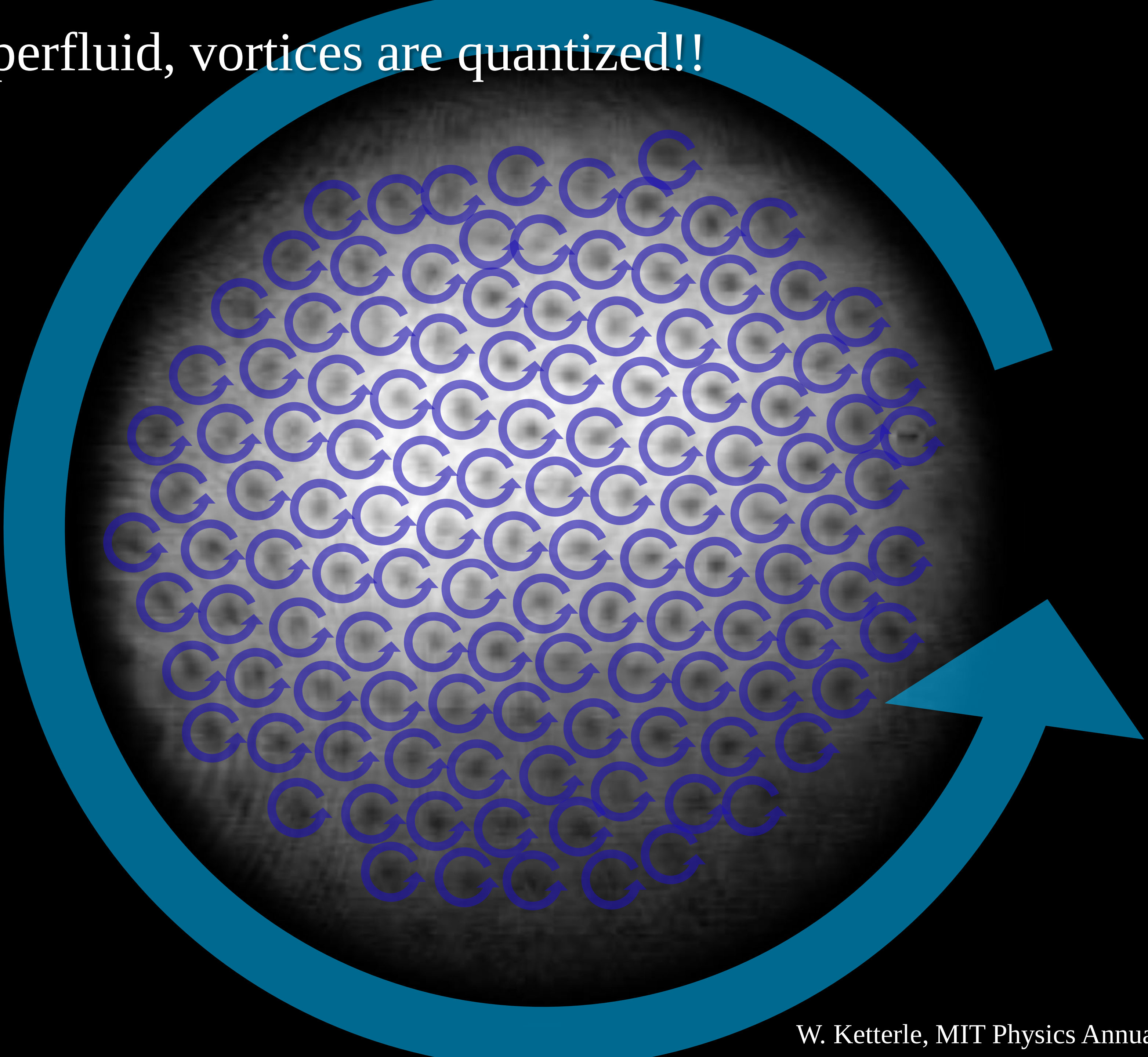


A hole at which superfluidity is lost

Quantum vortex

In daily life, a vortex is continuous..

In superfluid, vortices are quantized!!



W. Ketterle, MIT Physics Annual. 2001

G.P. Bewley, D.P. Lathrop, and K.R. Sreenivasan, Nature **441**, 588 (2006)

A movie from a talk by W. Guo (available from https://youtu.be/P2ckefSAN20) at
INT Program 19-1a “Quantum Turbulence: Cold Atoms, Heavy Ions, and Neutron Stars”
March 18 - April 19, 2019

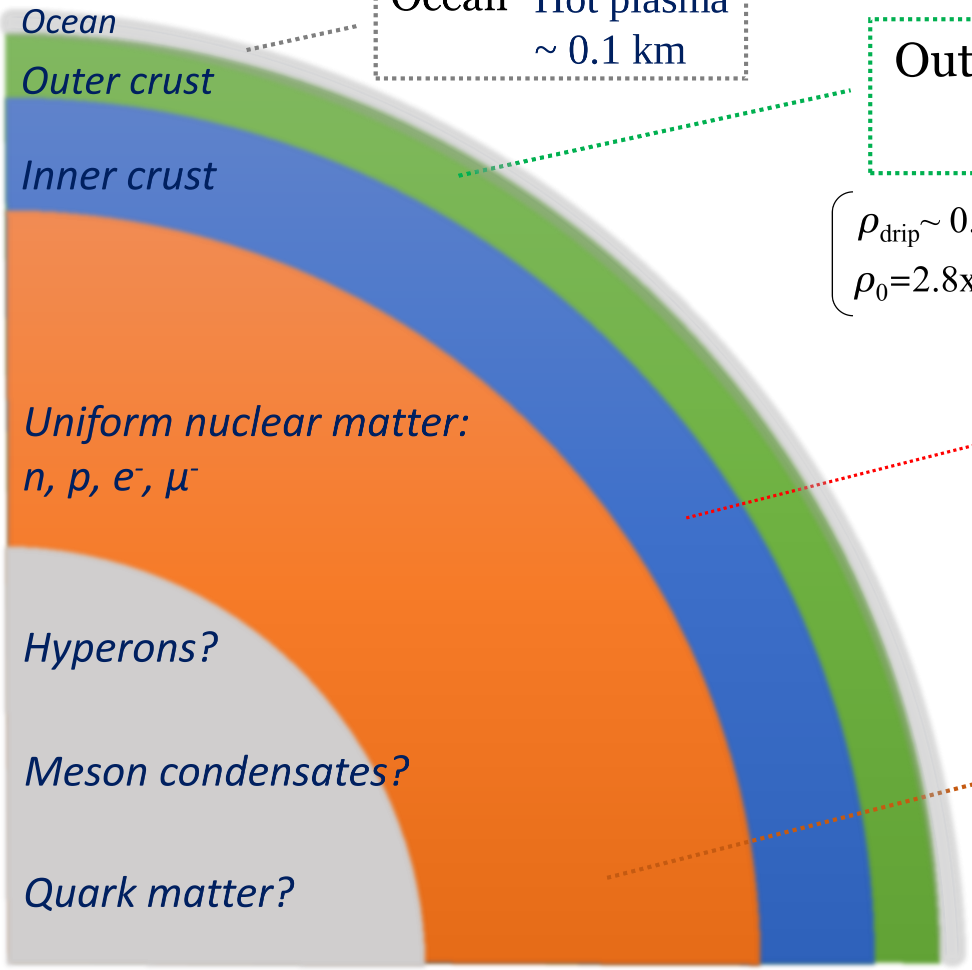
Direct visualization of quantized vortices

Hydrogen particles were trapped in the vortex core, then worked as a tracer

Structure of a neutron star

Neutron star is a great playground for nuclear physicists

✓ **It offers extreme situations which can not be realized in terrestrial experiments**



Ocean Hot plasma
~ 0.1 km

Outer crust Nuclei (Z=26-42), electrons
0.3-0.6 km $\rho < \rho_{\mathrm{drip}}$

$\rho_{\mathrm{drip}} \sim 0.0014\rho_0$; Above ρ_{drip} unbound neutrons exist outside nuclei
$\rho_0 = 2.8\mathrm{x}10^{14}\ \mathrm{g/cm^3} = 0.16\ \mathrm{fm^{-3}}$; Nuclear saturation density

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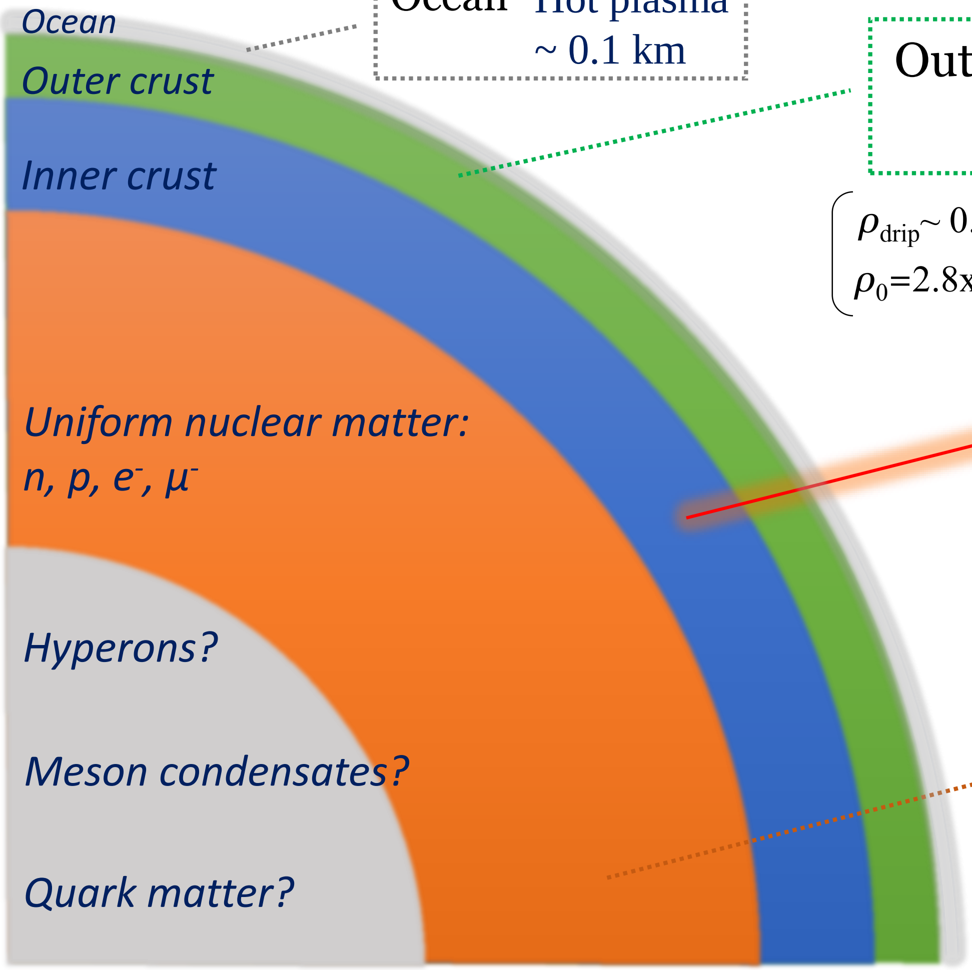

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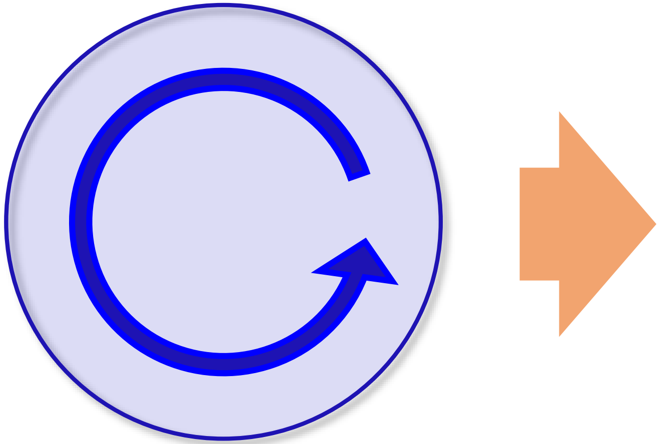
Quantum vortices in a neutron star

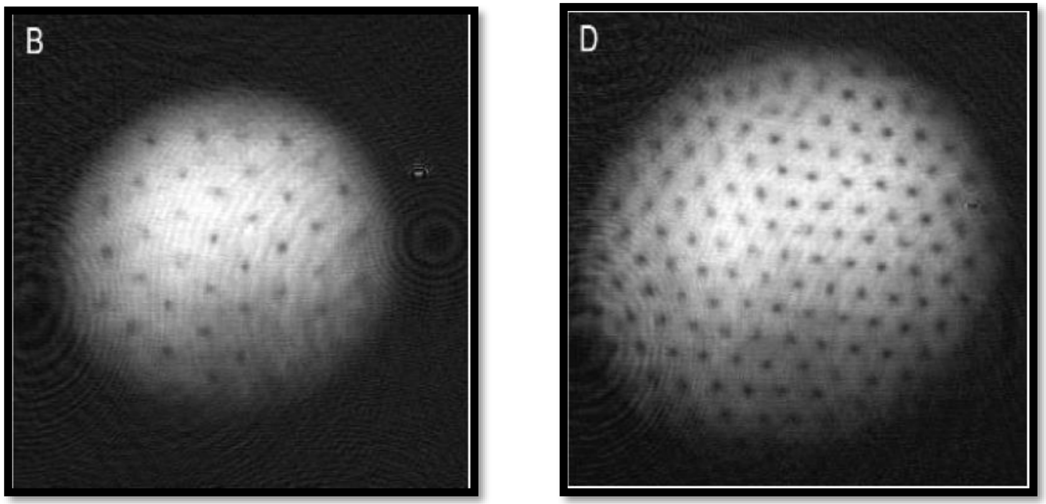
In rotating superfluid, an array of quantum vortices is generated

- Observation in ultra-cold atomic gases



Vortex-number density $\propto$ Rotation speed

Low ⟷ High



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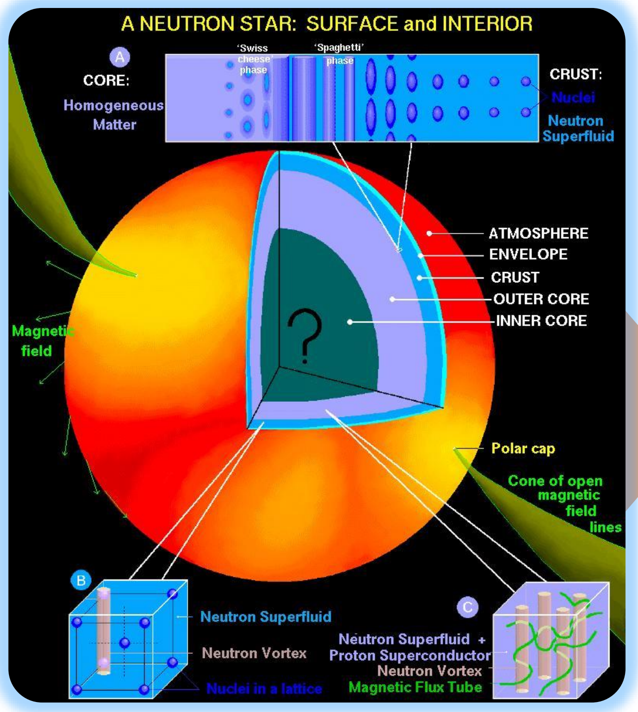

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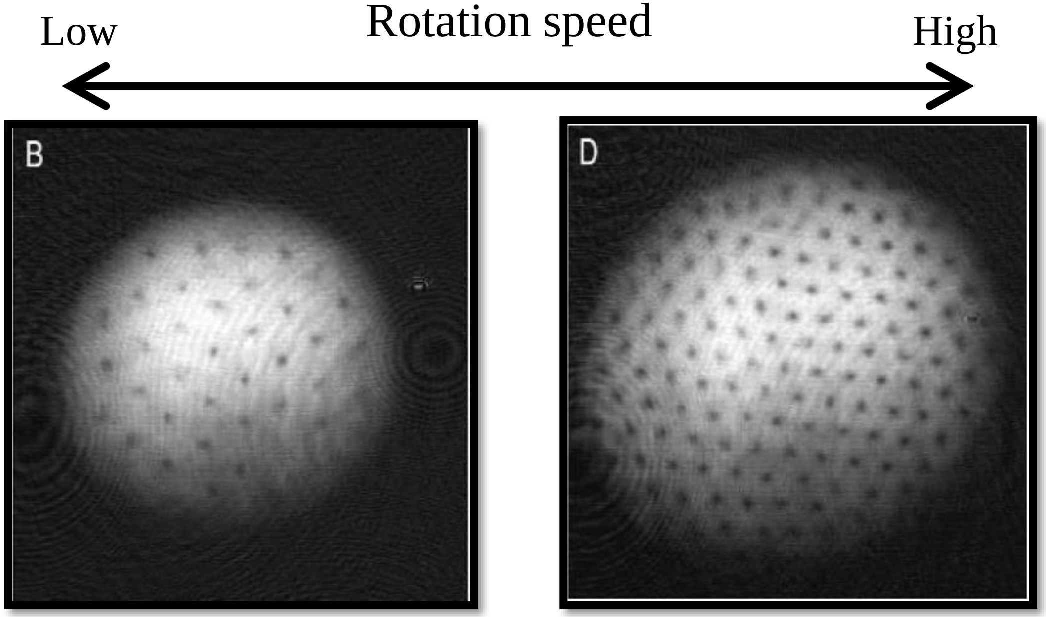
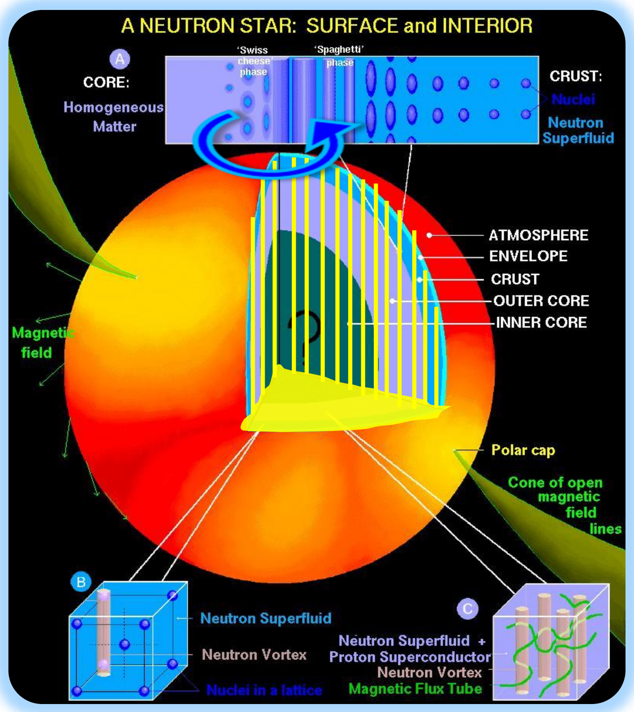


W. Ketterle, MIT Physics Annual. 2001

Quantum vortices in a neutron star

There must be a huge number ($\sim 10^{18}$) of vortices inside a neutron star!!

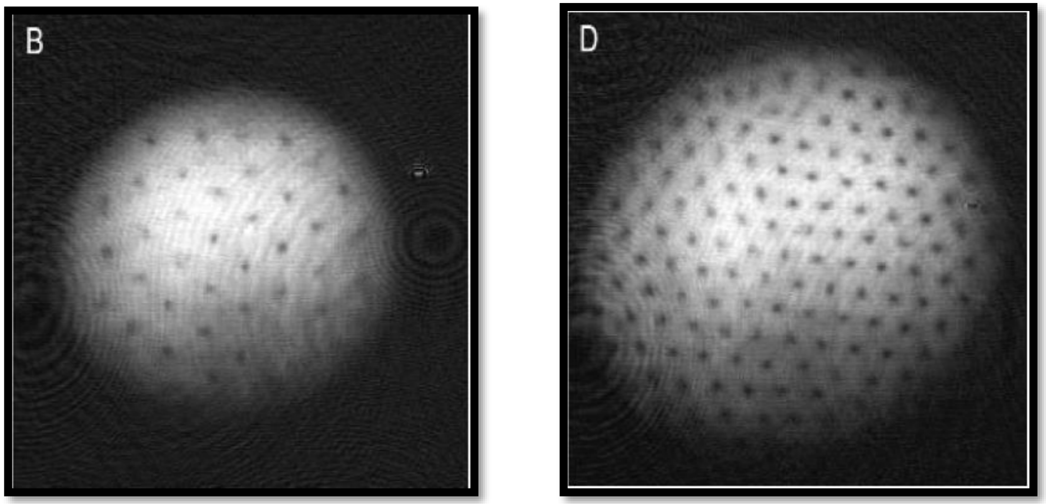


Vortex-number density

$\propto$

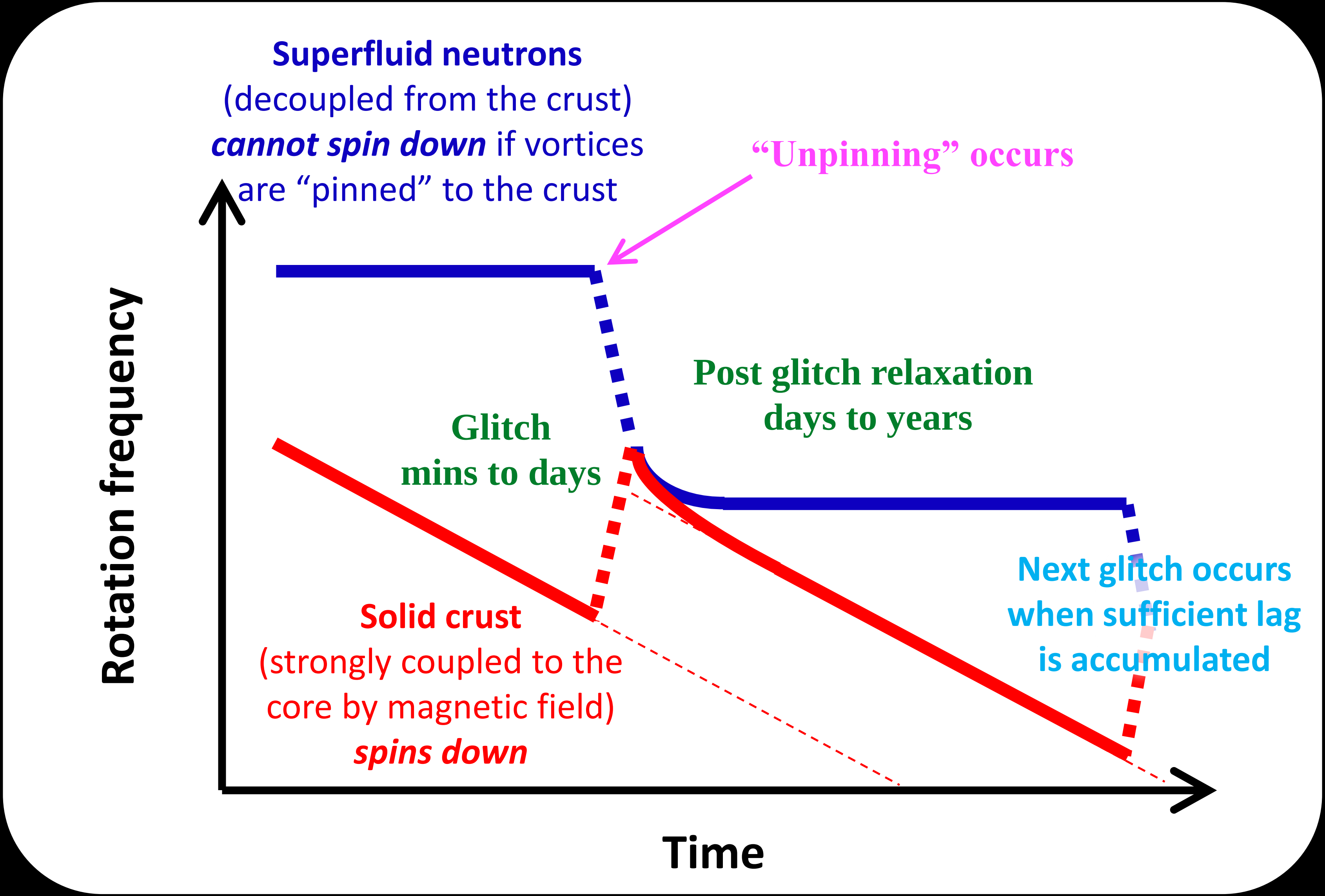
Rotation speed

Low ⟷ High



W. Ketterle, MIT Physics Annual. 2001

The vortex mediated glitch: Naive picture



Superfluid neutrons
(decoupled from the crust)
cannot spin down if vortices
are "pinned" to the crust

"Unpinning" occurs

Glitch
mins to days

Post glitch relaxation
days to years

Next glitch occurs
when sufficient lag
is accumulated

Solid crust
(strongly coupled to the
core by magnetic field)
spins down

To fully understand the glitches, we need to clarify:

Glitch dynamics

How do vortices move?

and, of course,
details of NS matter..

Pinning mechanism

How are vortices pinned?

Trigger mechanism

How are vortices unpinned?

We attacked this problem using
the state-of-the-art microscopic nuclear theory

We attack this problem with HPC on GPU supercomputers
with TDDFT for superfluid systems, TDSLDA!

TDSLDA (Time-Dependent Superfluid Local Density Approximation)

TDSLDA: TDDFT with local treatment of pairing

Kohn-Sham scheme is extended for non-interacting quasiparticles

- TDSLDA equations (formally equivalent to TDHFB or TD-BdG equations)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow\uparrow}(\boldsymbol{r},t) & h_{\uparrow\downarrow}(\boldsymbol{r},t) & 0 & \Delta(\boldsymbol{r},t) \\ h_{\downarrow\uparrow}(\boldsymbol{r},t) & h_{\downarrow\downarrow}(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & -h^*_{\uparrow\uparrow}(\boldsymbol{r},t) & -h^*_{\uparrow\downarrow}(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & 0 & -h^*_{\downarrow\uparrow}(\boldsymbol{r},t) & -h^*_{\downarrow\downarrow}(\boldsymbol{r},t) \end{pmatrix}\begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix}$$

$$h_\sigma = \frac{\delta E}{\delta n_\sigma}$$: s.p. Hamiltonian

$$\Delta = -\frac{\delta E}{\delta \nu^*}$$: pairing field

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_k<E_c} |v_{k,\sigma}(\boldsymbol{r},t)|^2$$: number density

$$\nu(\boldsymbol{r},t) = \sum_{E_k<E_c} u_{k,\uparrow}(\boldsymbol{r},t)v^*_{k,\downarrow}(\boldsymbol{r},t)$$: anomalous density

$$\boldsymbol{j}_\sigma(\boldsymbol{r},t) = \hbar\sum_{E_k<E_c} \mathrm{Im}[v^*_{k,\sigma}(\boldsymbol{r},t)\boldsymbol{\nabla}v_{k,\sigma}(\boldsymbol{r},t)]$$: current

A large number (10^4-10^6) of 3D coupled non-linear PDEs have to be solved!!

# of qp orbitals ~ # of grid points

TDSLDA (Time-Dependent Superfluid Local Density Approximation)

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Supercomputing!!

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$\Delta = -\frac{\delta E}{\delta \nu^*}$: pairing field

$n_\sigma(\boldsymbol{r},t) = \sum_{E_k<E_c} |v_{k,\sigma}(\boldsymbol{r},t)|^2$: number density

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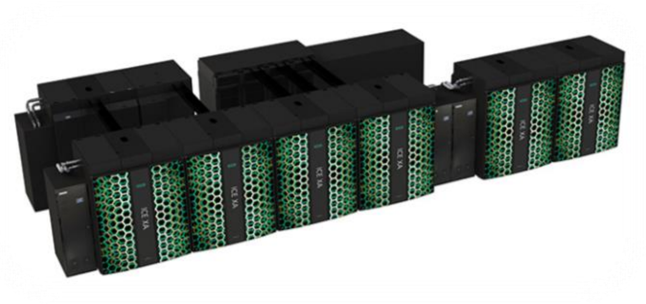
# of qp orbitals ~ # of grid points

*The number indicates the rank according to the [TOP500 list (June 2022)]


Piz Daint, CSCS, Switzerland (No. 23)


TITAN, ORNL, USA


TSUBAME3.0, Japan (No. 64)


Summit, ORNL, USA (No. 4)
GPU, 200 PFlops/s

GPU machines within Nos. 1-20

No.4: Summit, ORNL, USA
No.5: Sierra, LLNL, USA
No.7: Perlmutter, NERSC, USA
No.8: Selene, NVIDIA Co., USA
No.11: JUWELS Booster Module, FZJ, Germany
No.12: HPC5, Eni S. p. A., Italy
No.13: Voyager-EUS2, Azure East US 2, USA
No.14: Polaris, ANL, USA
No.15: SSC-21, Samsung Electronics, South Korea
No.18: Damman-7, Saudi Aramco, Saudi Arabia
No.19: ABCI 2.0, AIST, Japan

Certainly, GPU is competing with CPU machines!!

Applications of TDSLDA

TDSLDA has been successfully applied for both UFG and nuclear systems

Unitary Fermi Gas (UFG)

$$k_{\mathrm{F}}a \to \infty,\ k_{\mathrm{F}}r_{\mathrm{eff}} \to 0$$

a : s-wave scattering length
r_{eff} : effective range

A. Bulgac and S. Yoon, PRL**102**(2009)085302.	Large-amplitude pairing field dynamics
A. Bulgac *et al.*, Science **332**(2011)1288.	Dynamics of quantum vortices
A. Bulgac *et al.*, PRL**108**(2012)150401.	Quantum shock waves and domain walls
A. Bulgac *et al.*, PRL**112**(2014)025301.	Dynamics of vortex rings
G. Wlazłowski *et al.*, PRA**91**(2015)031602(R).	Dynamics of vortices and quantum turbulence
G. Wlazłowski *et al.*, PRL**120**(2018)253002.	Solitonic cascades in spin-polarized UFG
P. Magierski et al., PRA**100**(2019)033613.	Spin-polarized droplet in UFG

Nuclear systems

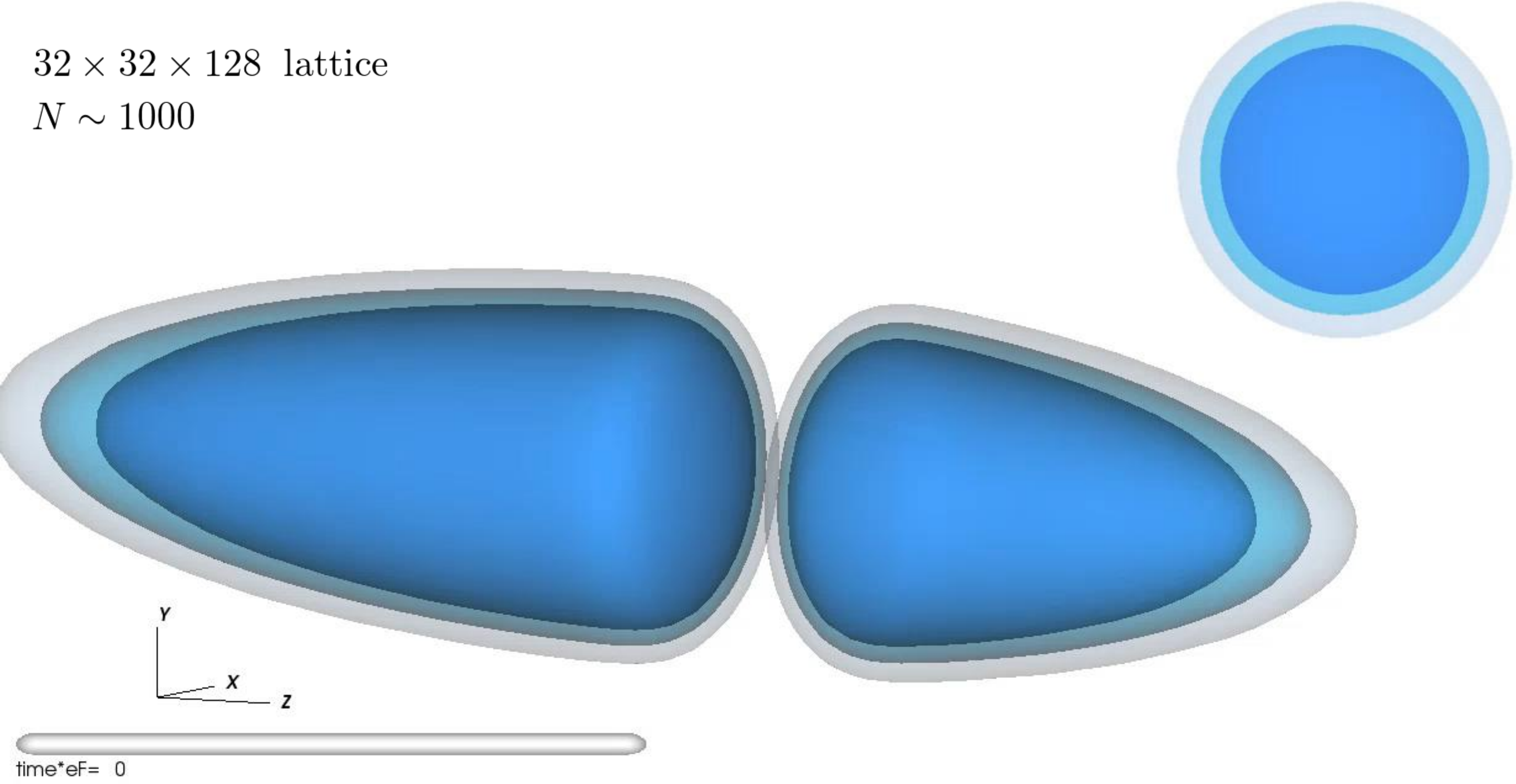
I. Stetcu *et al.*, PRC**84**(2011)051309(R).	Isovector giant dipole resonance (IVGDR)
I. Stetcu *et al.*, PRL**114**(2015)012701.	Relativistic coulomb excitation
A. Bulgac *et al.*, PRL**116**(2016)122504; PRC**100**(2019)034615.	Induced fission of ^{240}Pu
G. Wlazłowski *et al.*, PRL**117**(2016)232701.	Vortex-nucleus interaction

Heavy soliton?: Nature **499**(2013)426
Vortex ring?: PRL**112**(2014)025301

Result of TDSLDA simulation:

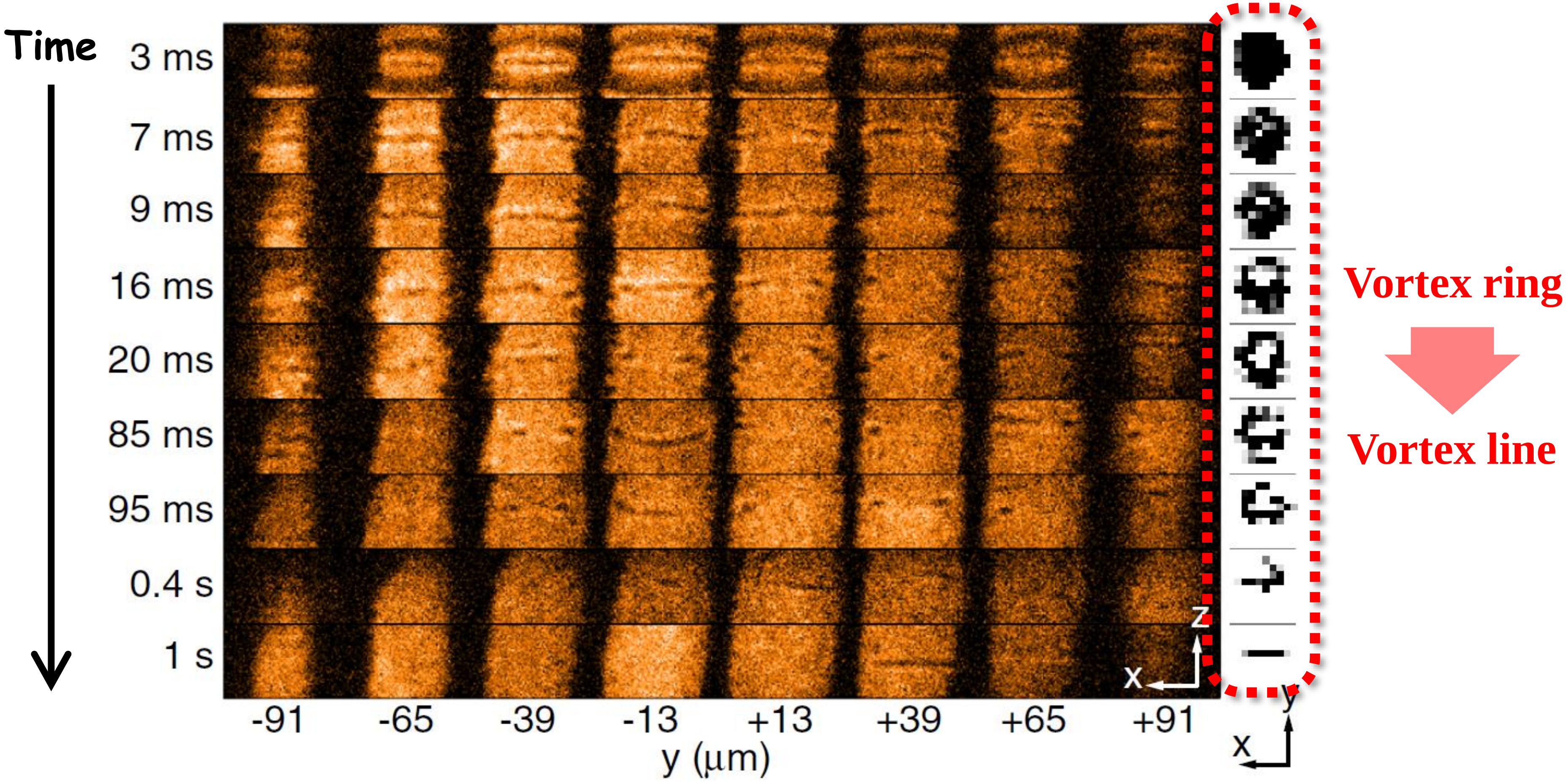
Phase discontinuity creates a vortex ring which decays into a vortex line

$32 \times 32 \times 128$ lattice
$N \sim 1000$



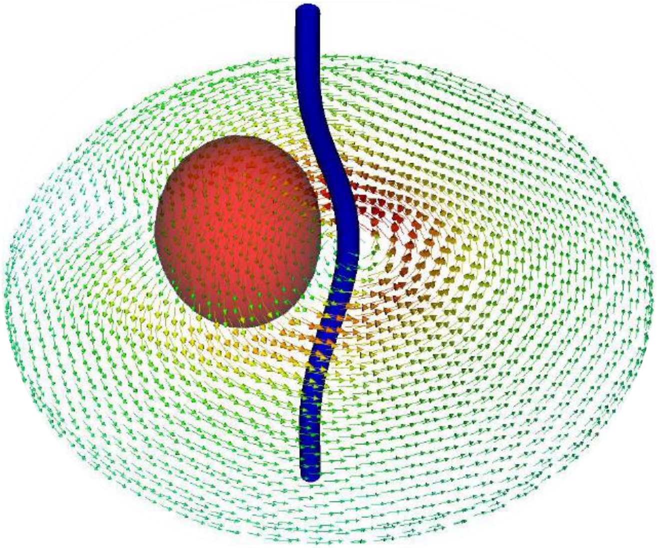
G. Wlazłowski, A. Bulgac, M.M. Forbes, and K.J. Roche, Phys. Rev. A **91**, 031602(R) (2015)

The cascades of solitonic excitations have been identified experimentally



M.J.H. Ku, B. Mukherjee, T. Yefsah, and M.W. Zwierlein, Phys. Rev. Lett. **116**, 045304 (2016)

Vortex-nucleus dynamics within TDSLDA



G. Wlazłowski, K. Sekizawa, P. Magierski, A. Bulgac, and M.M. Forbes,
Phys. Rev. Lett. **117**, 232701 (2016)

A key to understand the glitches is:

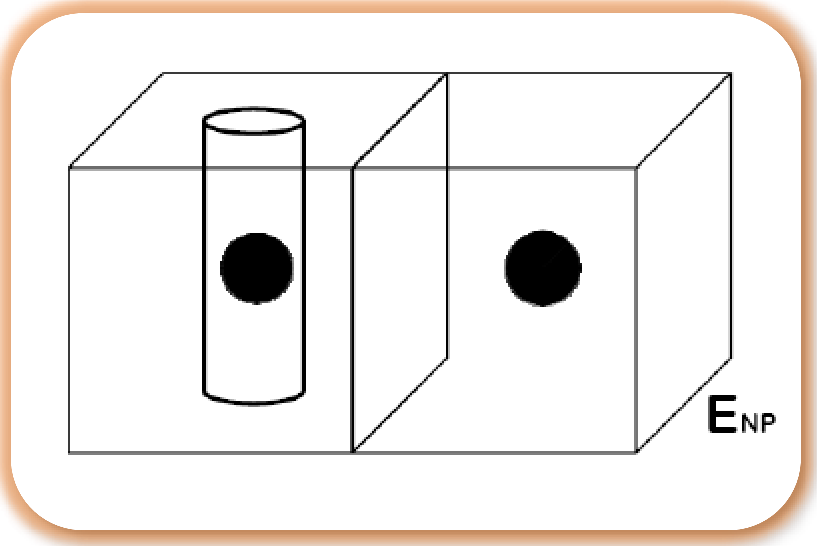
Vortex pinning mechanism in the inner crust of neutron stars

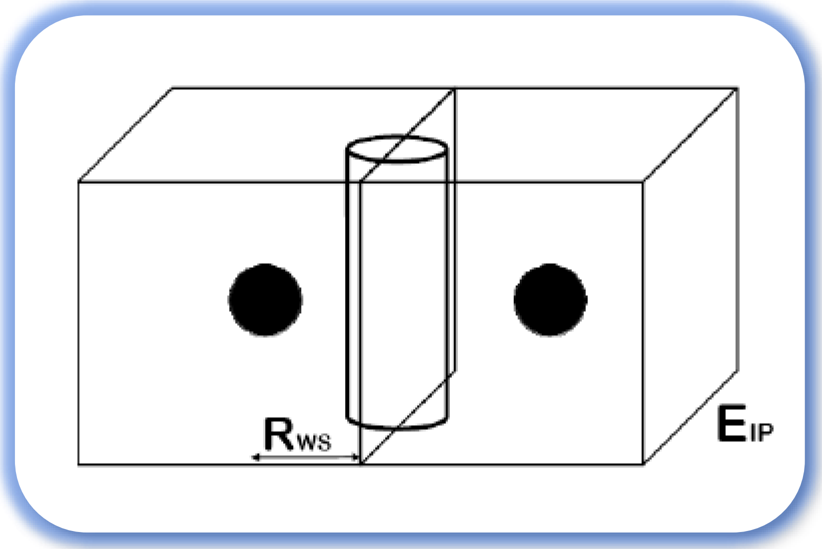
Q. Is the vortex-nucleus interaction

Attractive?

or

Repulsive?

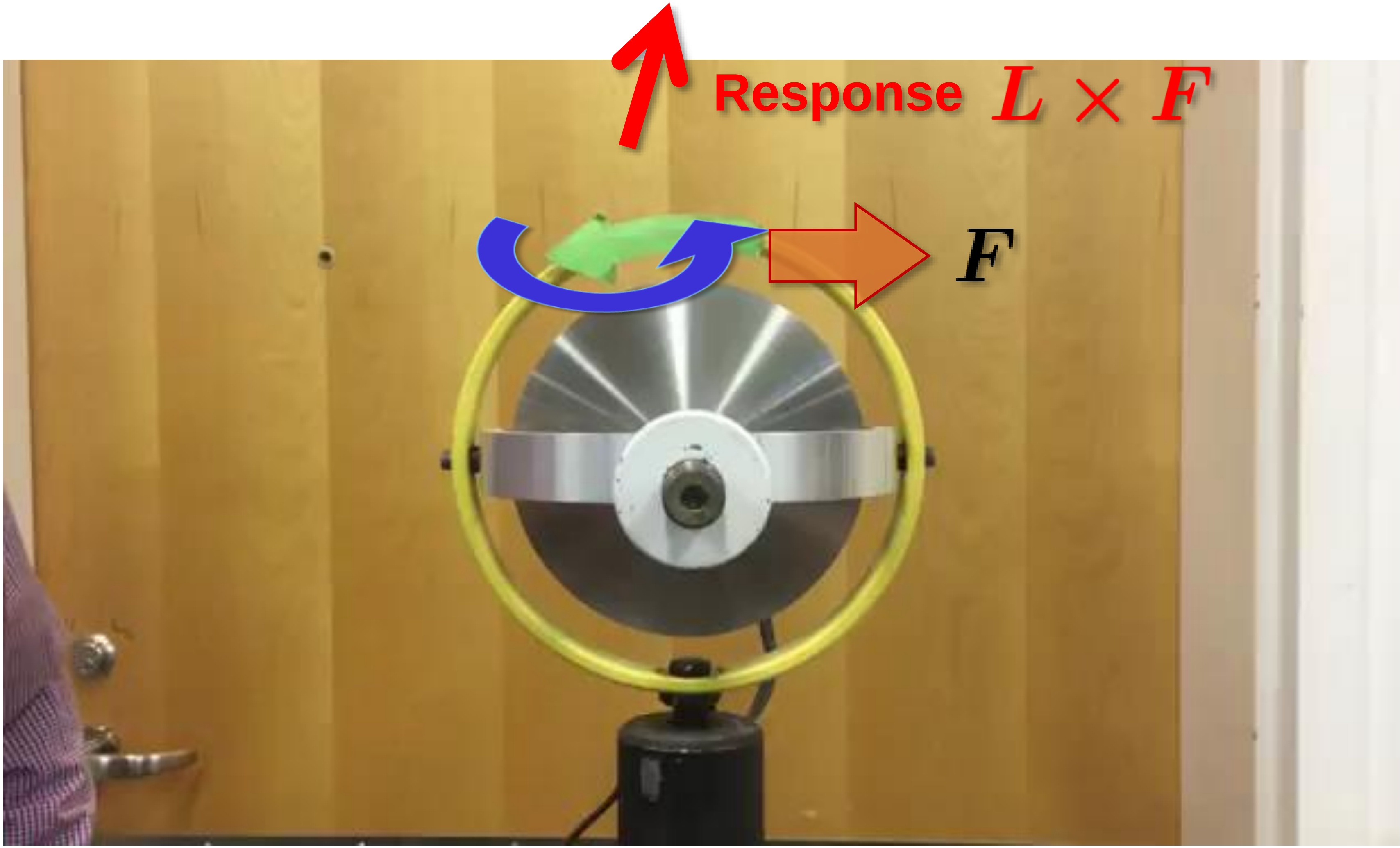




“Nuclear pinning”

“Interstitial pinning”

Response of a spinning gyroscope when pushed



Method - TDSLDA

We performed 3D, dynamical simulations by TDDFT with superfluidity

☐ TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}u_i(\mathbf{r})\\ v_i(\mathbf{r})\end{pmatrix}=\begin{pmatrix}h(\mathbf{r}) & \Delta(\mathbf{r})\\ \Delta^*(\mathbf{r}) & -h(\mathbf{r})\end{pmatrix}\begin{pmatrix}u_i(\mathbf{r})\\ v_i(\mathbf{r})\end{pmatrix}$$

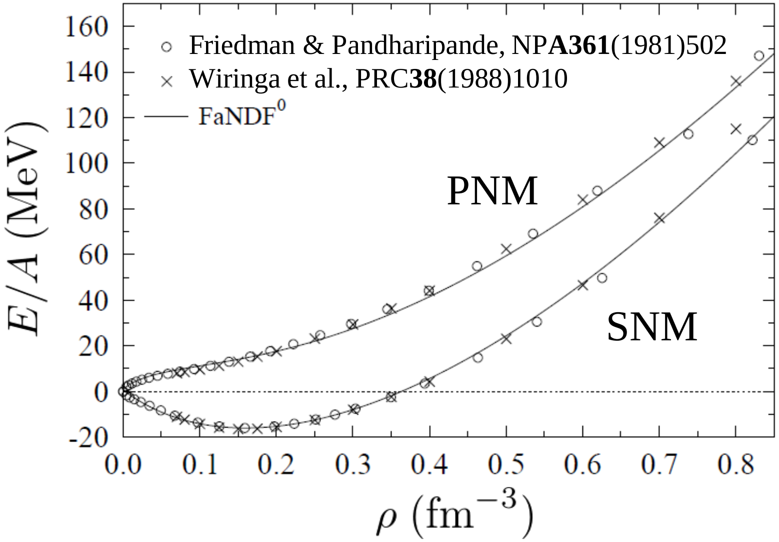
☐ Energy density functional (EDF)

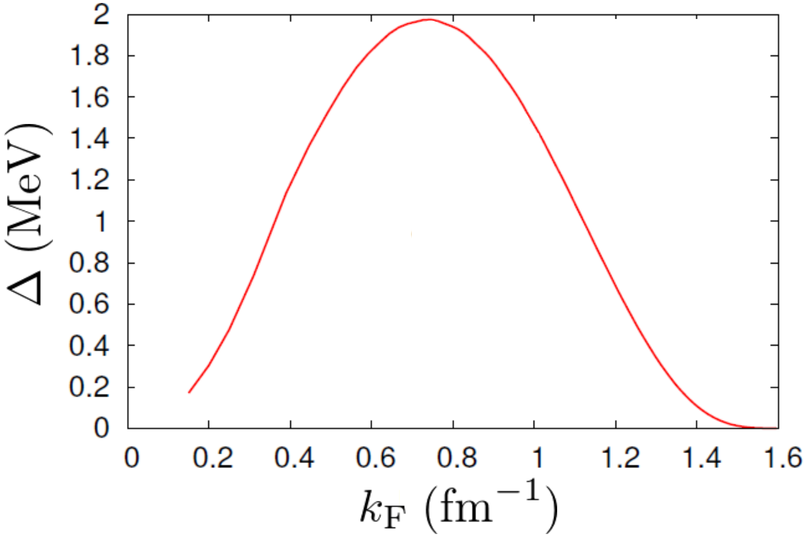
$$\mathcal{E}(\mathbf{r})=\mathcal{E}_0(\mathbf{r})+\mathcal{E}_{\mathrm{pair}}(\mathbf{r})$$

$\mathcal{E}_0(\mathbf{r})$: Fayans EDF (FaNDF^0) w/o LS

$$\mathcal{E}(\boldsymbol{r})=\sum_{q=n,p} g[\rho_q(\boldsymbol{r})]|\nu_q(\boldsymbol{r})|^2$$

S.A. Fayans, JETP Lett. **68**, 169 (1998)





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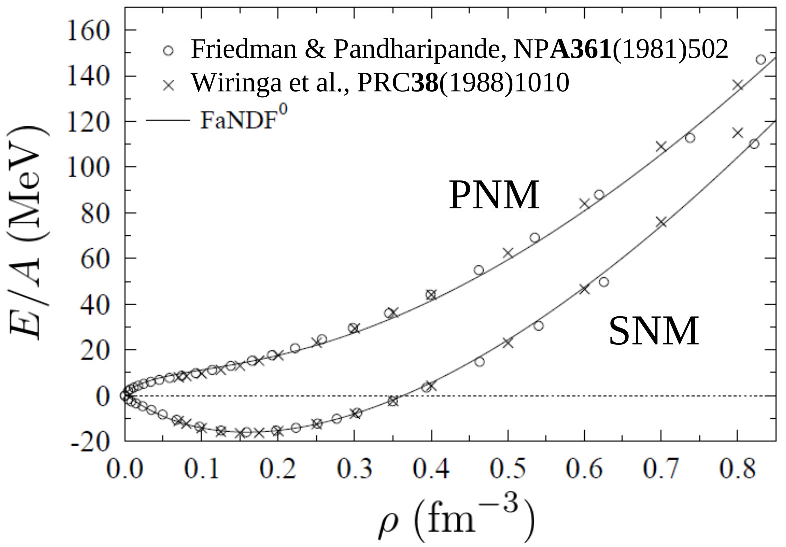

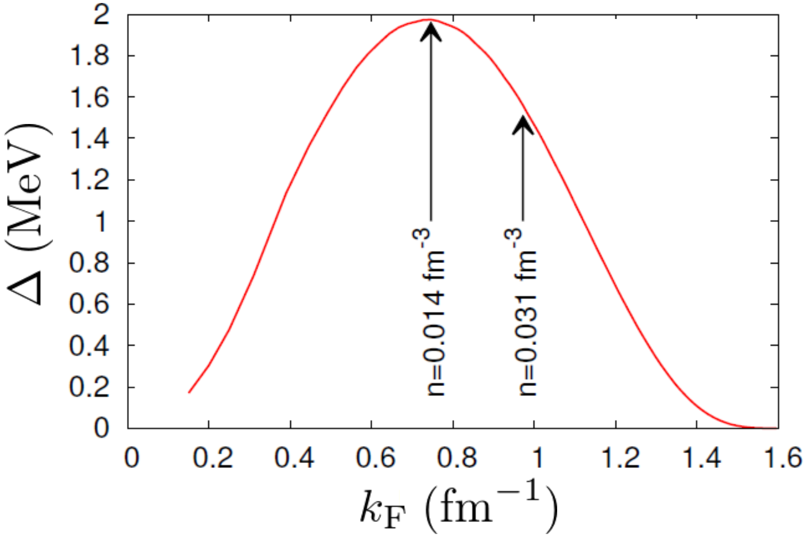

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☐ Computational details

$75\ \mathrm{fm} \times 75\ \mathrm{fm} \times 60\ \mathrm{fm}$

$(50 \times 50 \times 40,\ \Delta x = 1.5\ \mathrm{fm})$

$k_{\mathrm{c}} = \pi/\Delta x > k_{\mathrm{F}} \qquad k_{\mathrm{F}} = (3\pi^2\rho_n)^{1/3}$

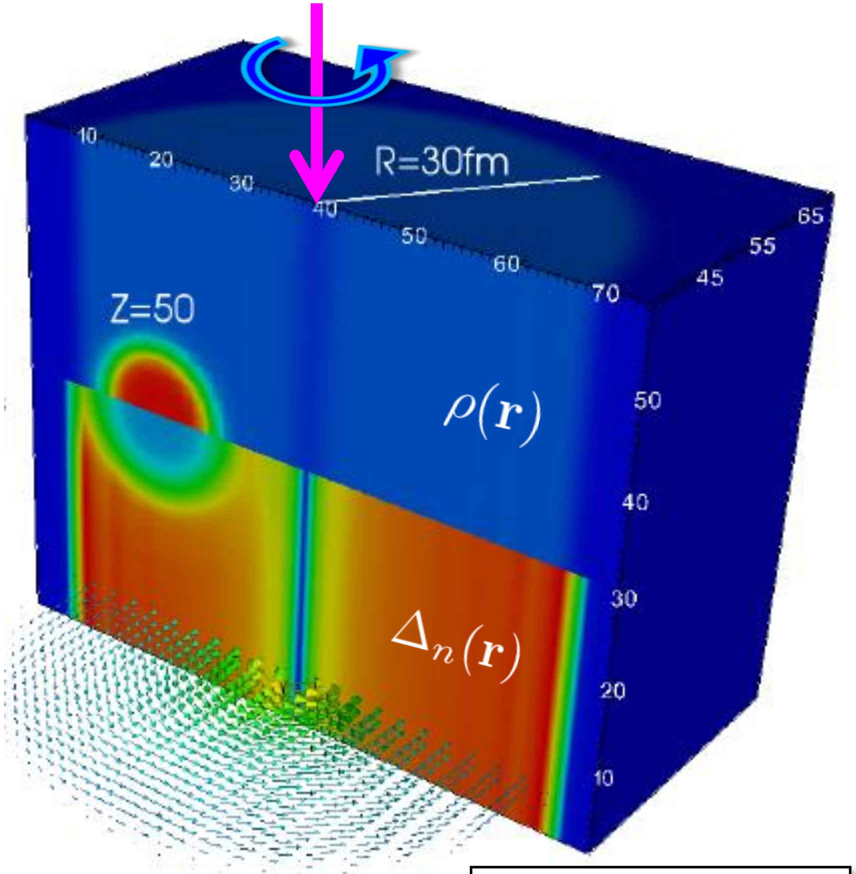
Nuclear impurity: $Z = 50$

$\rho_n \simeq 0.014\ \mathrm{fm}^{-3}\ (N \simeq 2{,}530)$

$\rho_n \simeq 0.031\ \mathrm{fm}^{-3}\ (N \simeq 5{,}714)$

# of quasi-particle w.f. $\approx 100{,}000$

a vortex line exists here



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75 fm × 75 fm × 60 fm

$(50\times50\times40,\ \Delta x=1.5\ \mathrm{fm})$

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$\rho_n\simeq0.031\ \mathrm{fm}^{-3}\ (N\simeq5{,}714)$

# of quasi-particle w.f. $\approx 100,000$

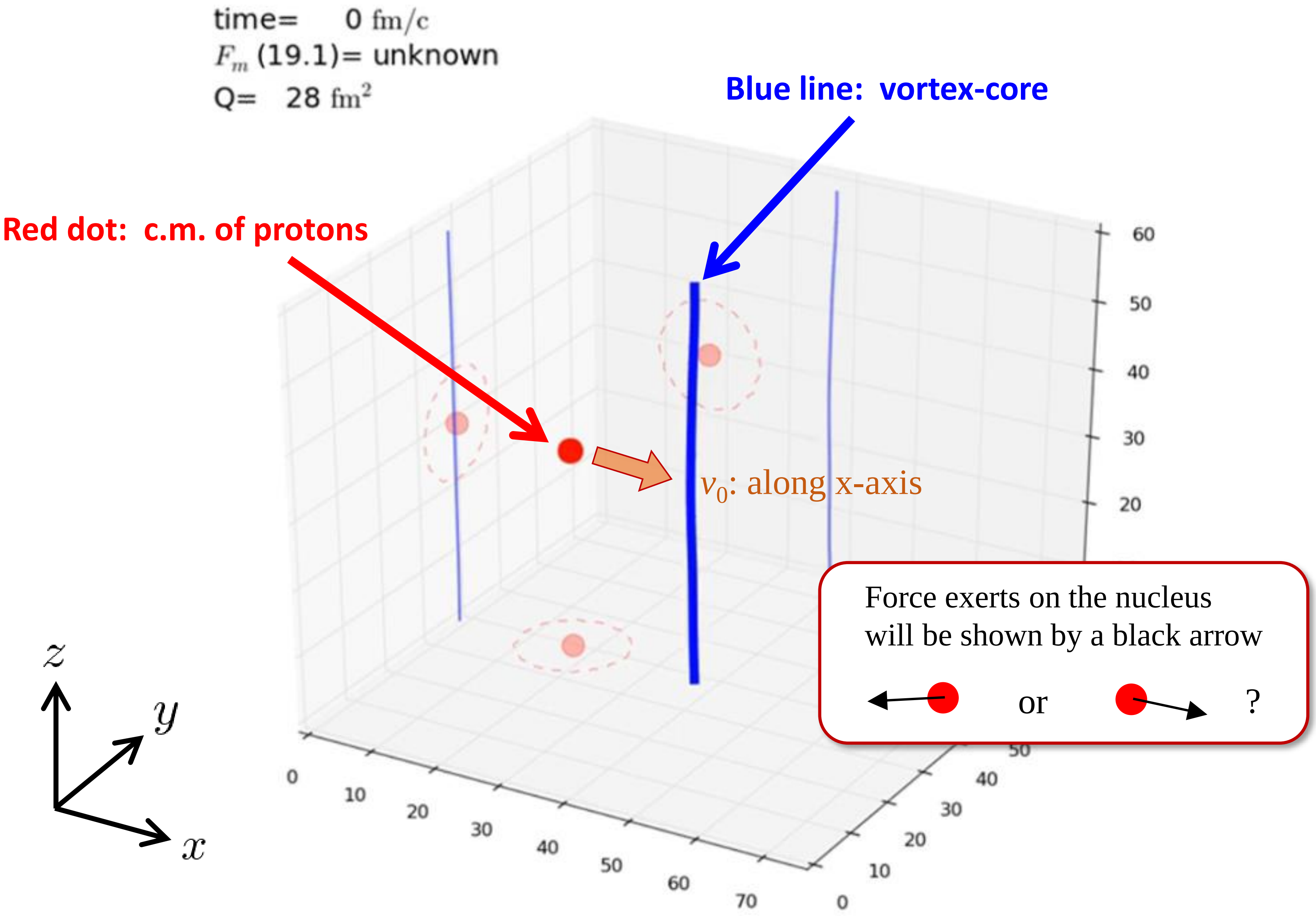
MPI+GPU
→ 48h w/ 200GPUs
for 10,000 fm/c

TITAN, Oak Ridge

NERSC Edison, Berkeley

HA-PACS, Tsukuba

Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$



Blue line: vortex-core

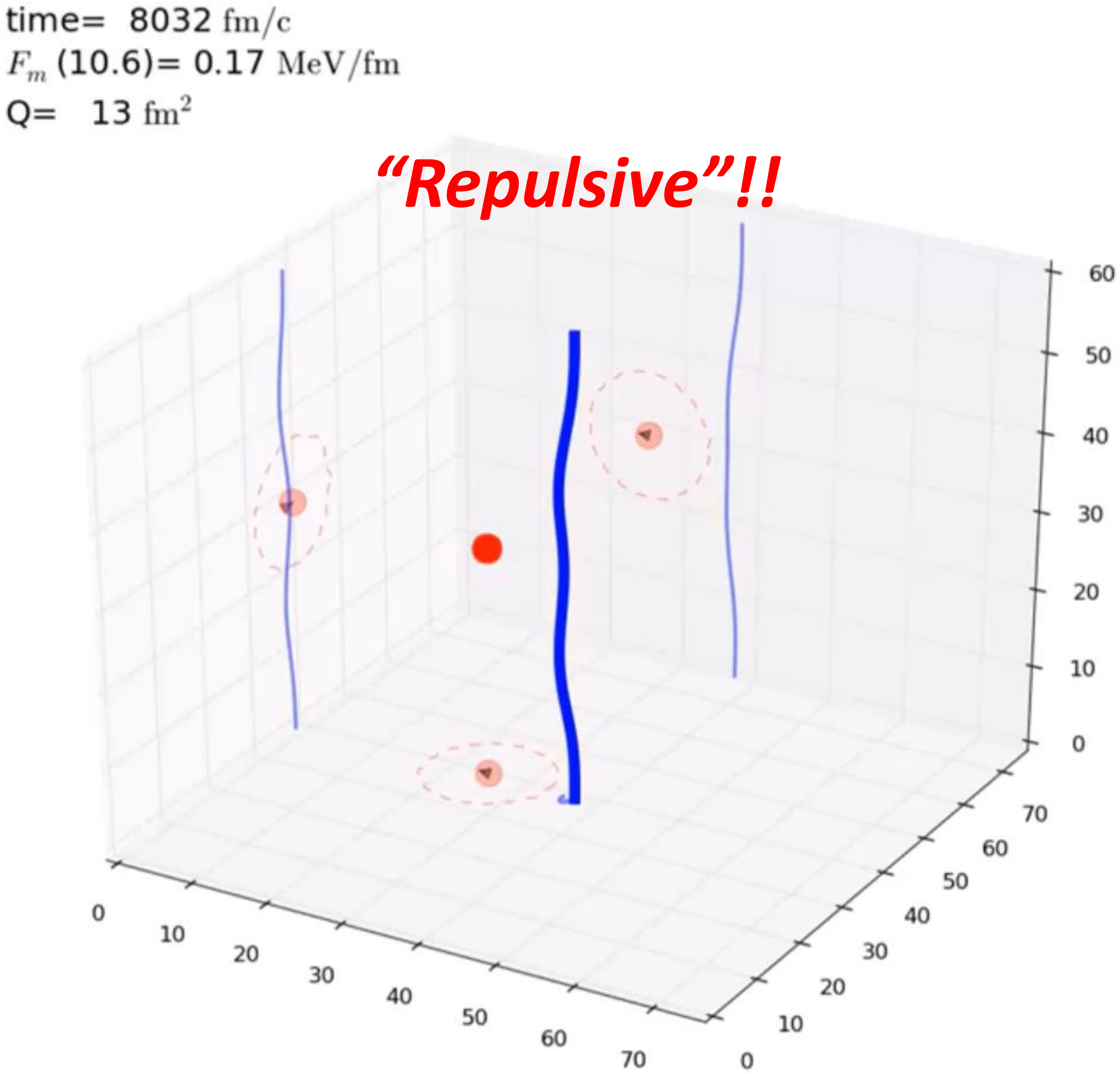
Red dot: c.m. of protons

v_0: along x-axis

Force exerts on the nucleus will be shown by a black arrow

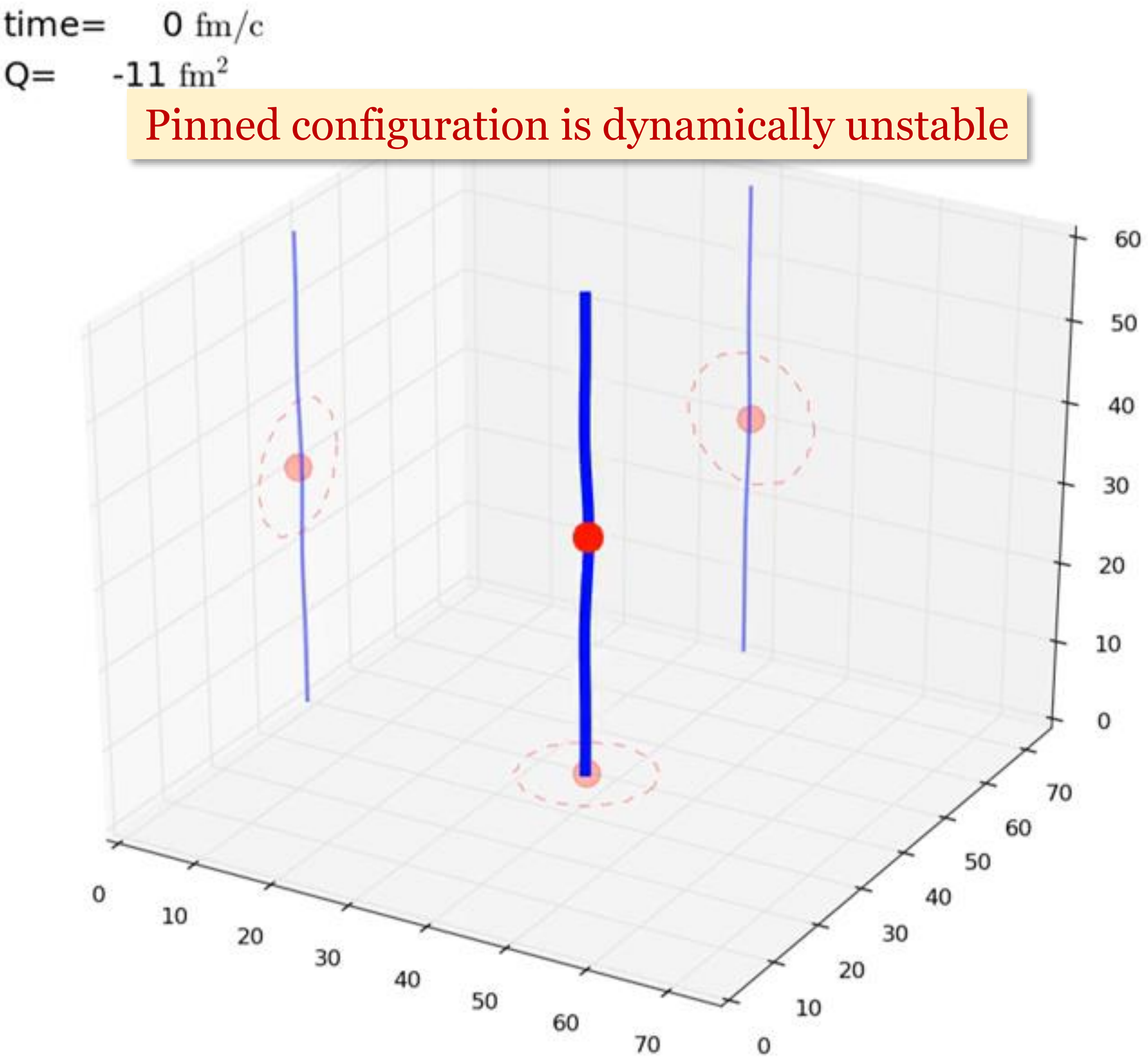
or ?

Results of TDSLDA calculation: $\rho_n \simeq 0.014\,\mathrm{fm}^{-3}$



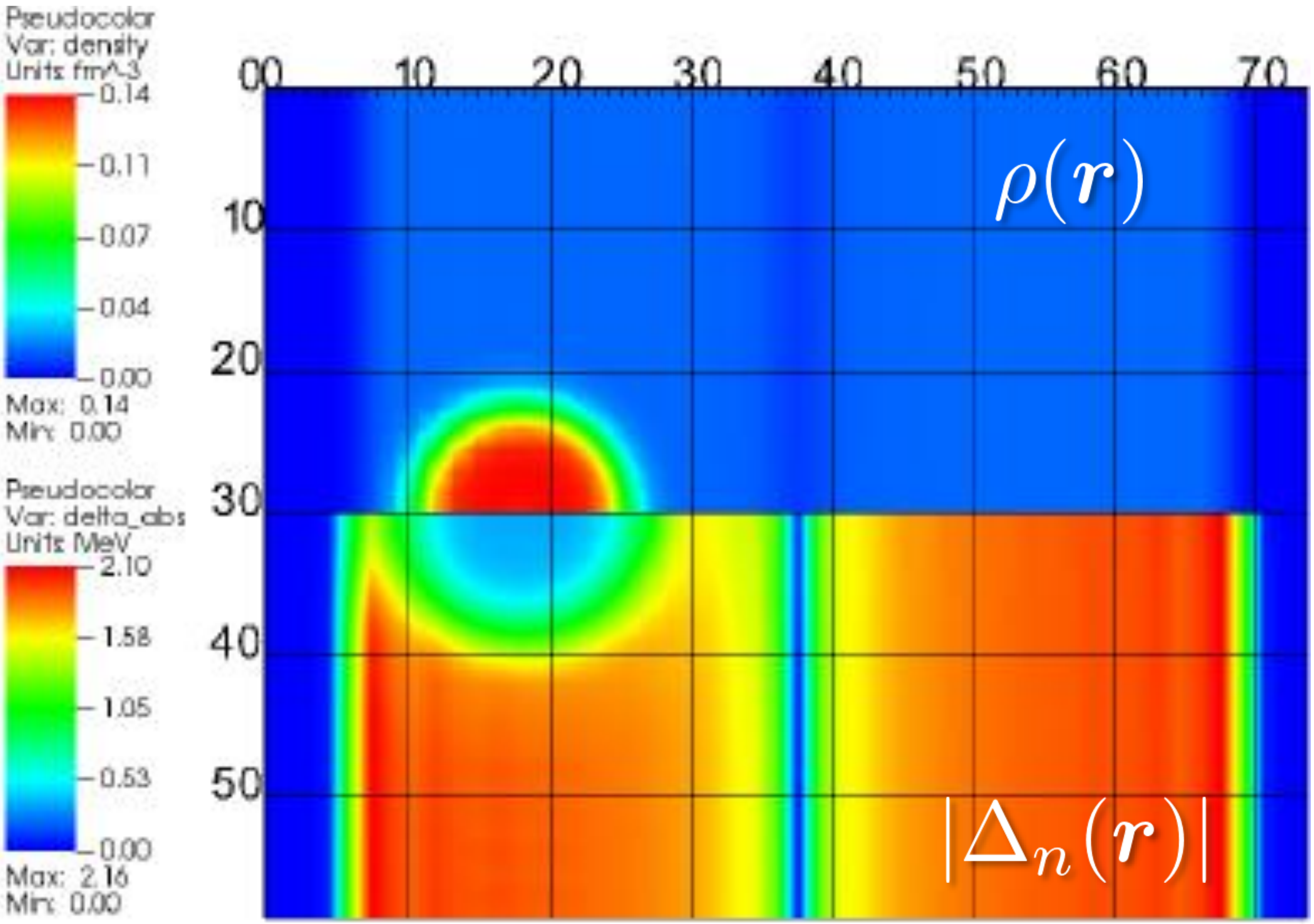
"Repulsive"!!

Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$

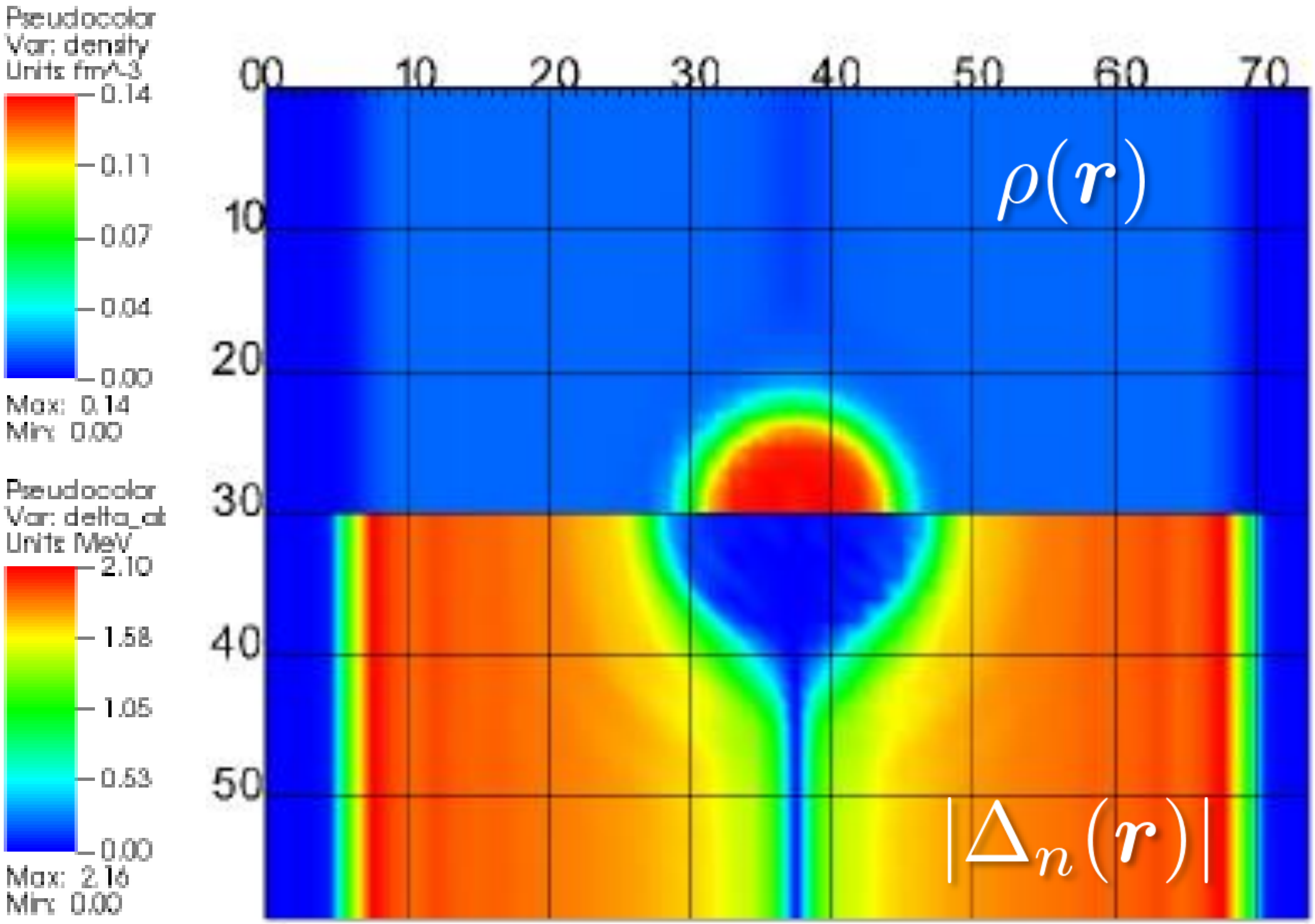


Pinned configuration is dynamically unstable

"Unpinned configuration"

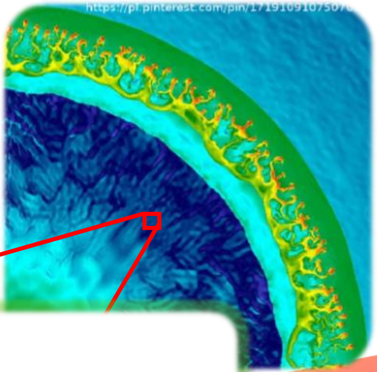


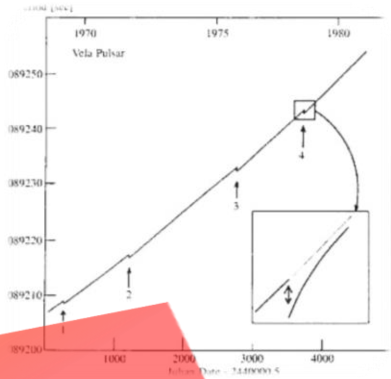
"Pinned configuration"



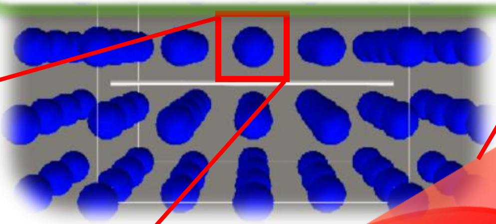
Our goal and strategy

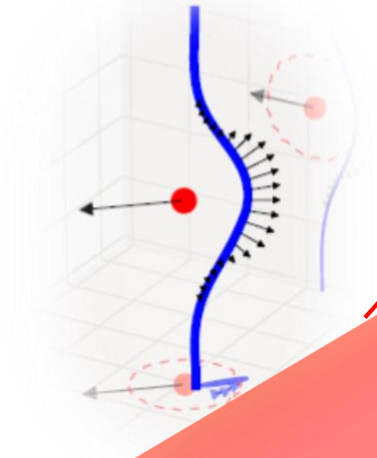
Goal: Unveil the mechanism of glitches





New collaboration started:
Nicolaus Copernicus Astronomical Centre
B. Haskell et al.





10⁴m

Macroscopic

- observations
- hydrodynamics

~10^{-10}m

Mesoscopic

- dynamics of *vortices* in a lattice of *nuclei*
  (e.g. filament model)

Provide model ingredients

10^{-15}-10^{-13}m

Microscopic

Nuclear Physics!!

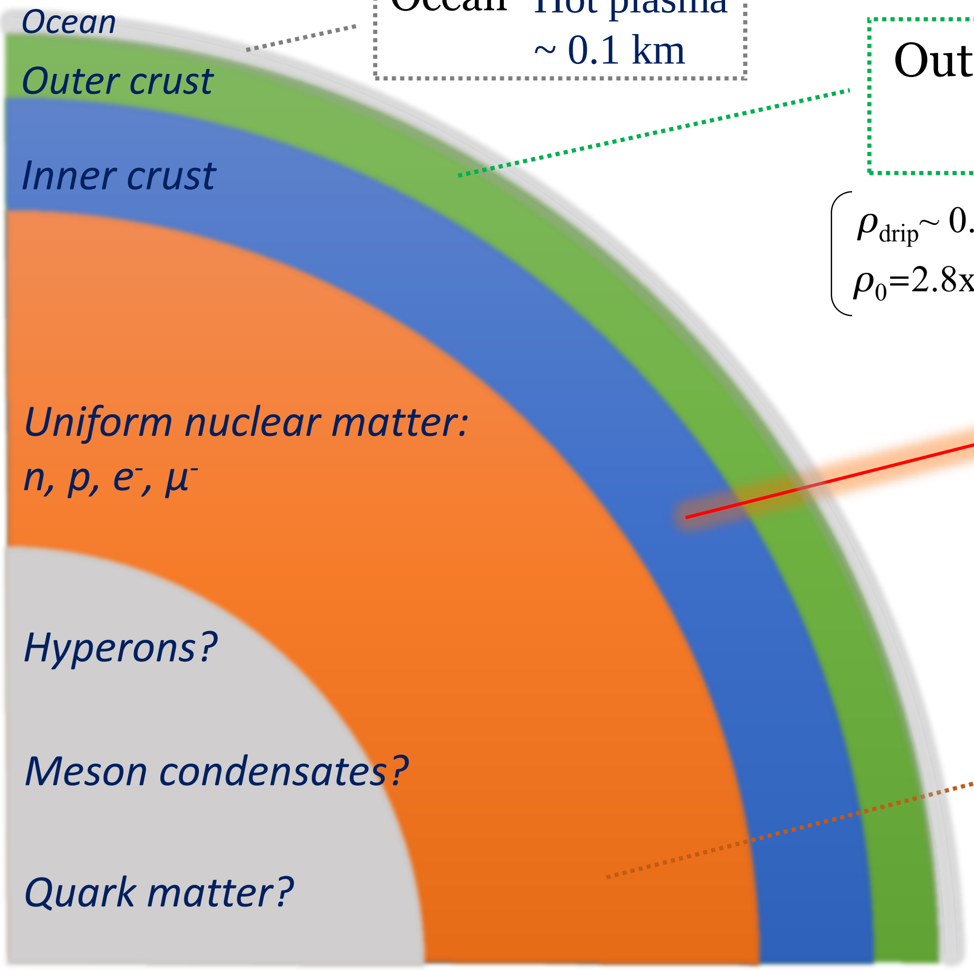
- vortex-nucleus dynamics from *neutrons and protons*

Time-Dependent Band Theory for the Inner Crust of Neutron Stars

https://www.newsweek.com/nuclear-pasta-neutron-star-strongest-material-universe-1127491

Structure of a neutron star

Neutron star is a great playground for nuclear physicists



Ocean Hot plasma
~ 0.1 km

Outer crust Nuclei (Z=26-42), electrons
0.3-0.6 km $\rho < \rho_{\mathrm{drip}}$

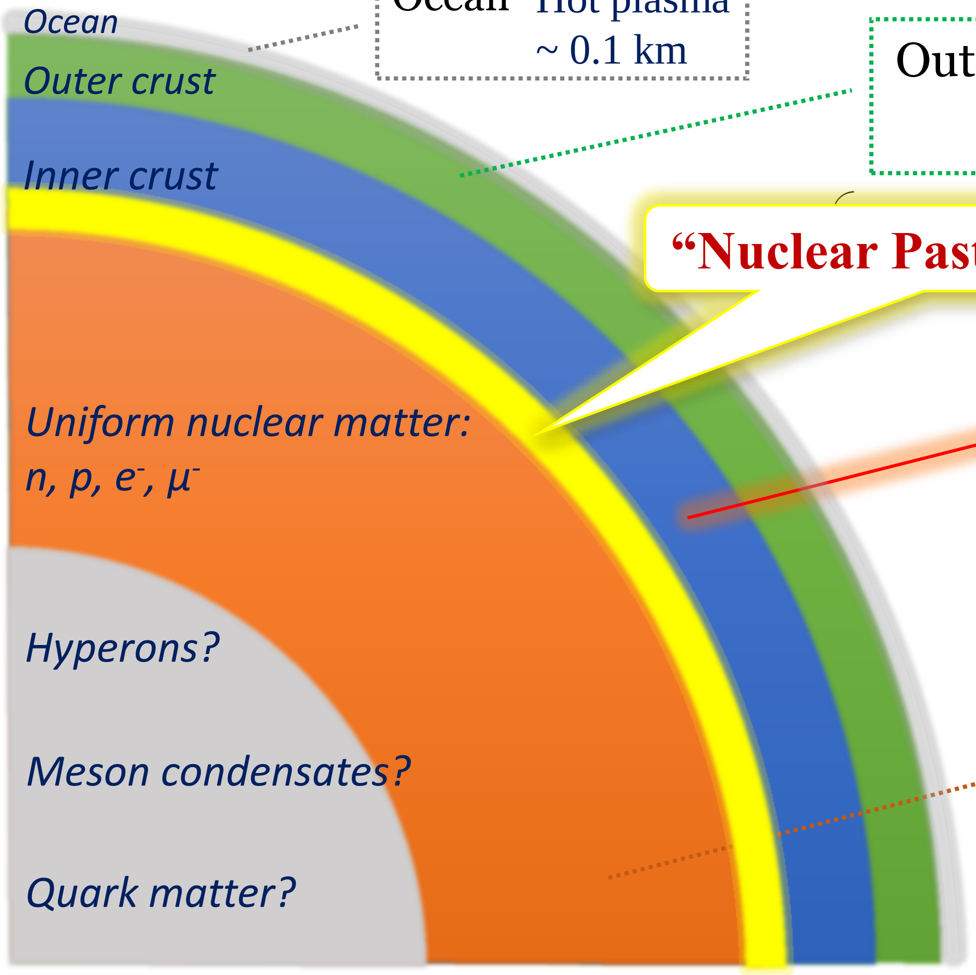
$\rho_{\mathrm{drip}} \sim 0.0014\rho_0$; Above ρ_{drip} unbound neutrons exist outside nuclei
$\rho_0 = 2.8\mathrm{x}10^{14}\ \mathrm{g/cm^3} = 0.16\ \mathrm{fm^{-3}}$; Nuclear saturation density

Inner crust Nuclei, electrons,
"dripped" neutrons
0.5-0.8 km $\rho_{\mathrm{drip}} \leq \rho \leq 0.6\rho_0$

Outer/inner core
9-12 km $0.6\rho_0 < \rho \leq 3\text{-}5\rho_0$

Structure of a neutron star

Neutron star is a great playground for nuclear physicists



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~ 0.1 km

Outer crust Nuclei (Z=26-42), electrons
0.3-0.6 km $\rho < \rho_{\text{drip}}$

ound neutrons exist outside nuclei
; Nuclear saturation density

"Nuclear Pasta" may exist!

Inner crust Nuclei, electrons, "dripped" neutrons
0.5-0.8 km $\rho_{\text{drip}} \le \rho \le 0.6\rho_0$

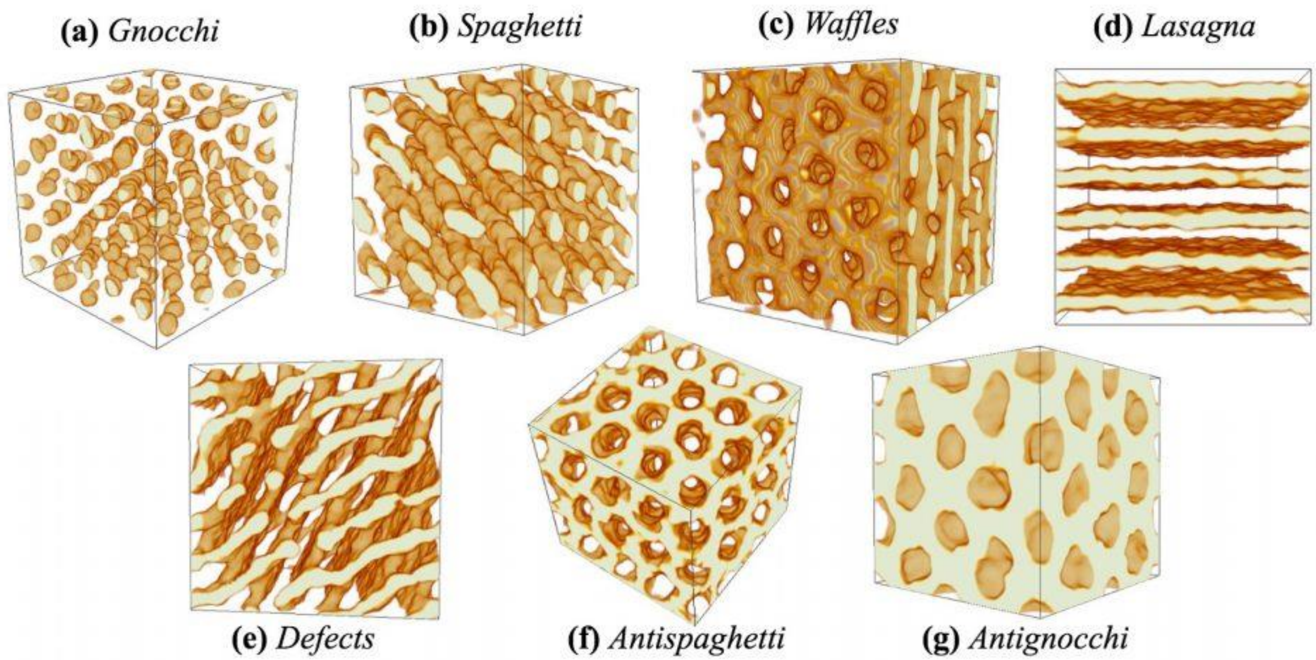
Outer/inner core
9-12 km $0.6\rho_0 < \rho \le 3\text{-}5\rho_0$

What is Nuclear Pasta? 😋

Gnocchi

Lasagna

Spaghetti



M. E. Caplan and C. J. Horowitz, Rev. Mod. Phys. **89**, 041002 (2017)

What is the “band structure” in solids?

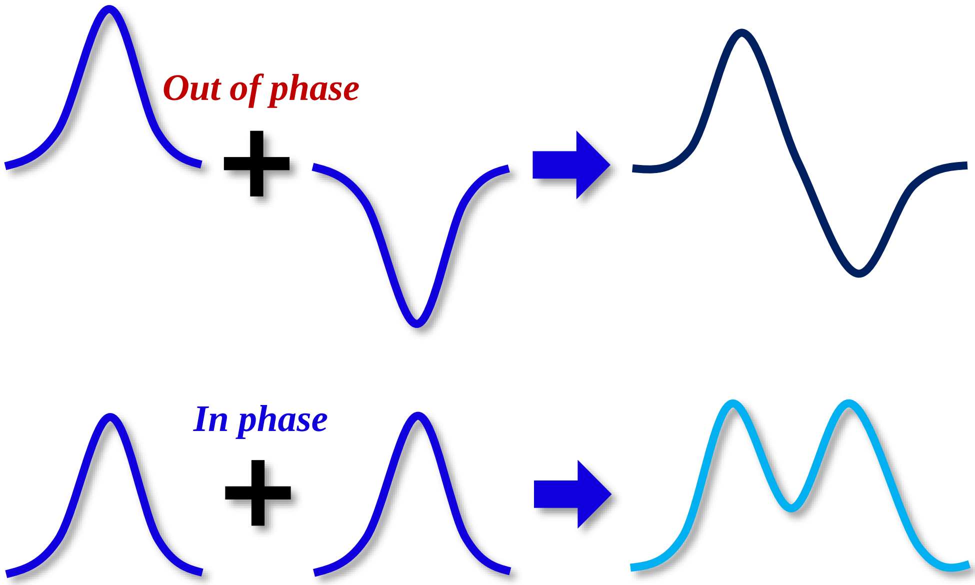
An energy band: a bunch of shifted energy levels of atomic orbitals

- ✓ **Energy levels must be the same for each atom when those atoms are *completely separated***
- ✓ As they come closer, energy levels are spread out (cf. bonding/anti-bonding molecular orbital)
- ✓ The energy splitting depend on the inter-atomic distance (an optimum value is realized)

One atom

Two atoms

cf. Bonding & anti-bonding molecular orbitals



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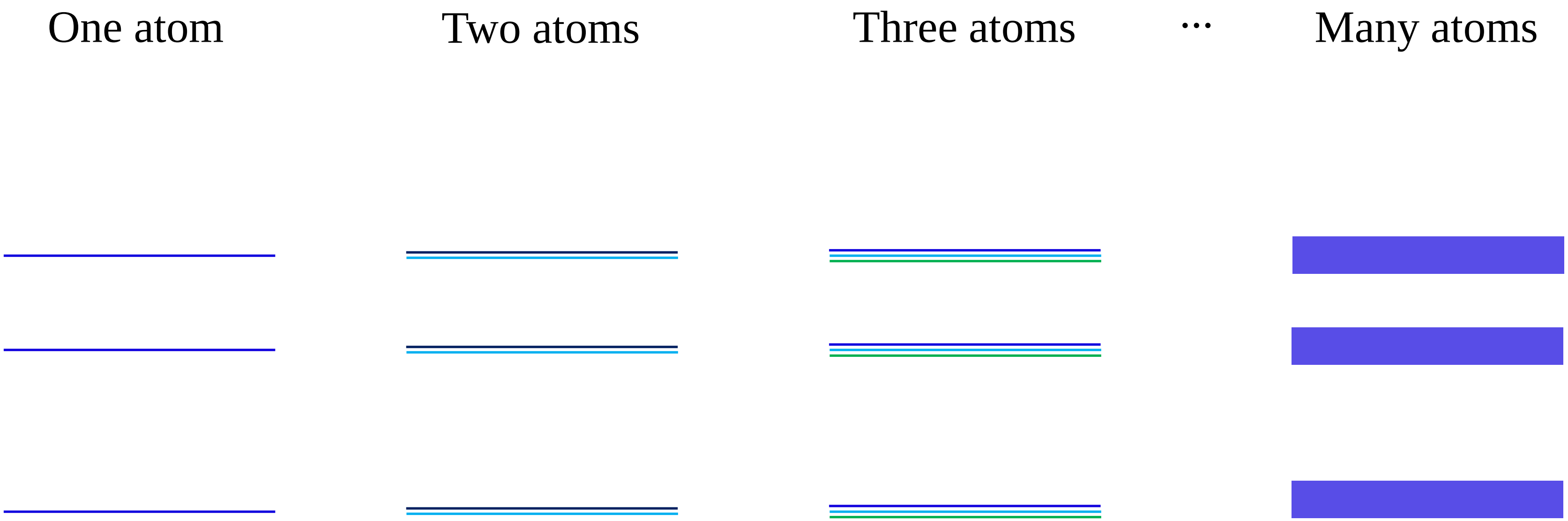

One atom　Two atoms　Three atoms　…　Many atoms

What is the "band structure" in solids?

An energy band: a bunch of shifted energy levels of atomic orbitals

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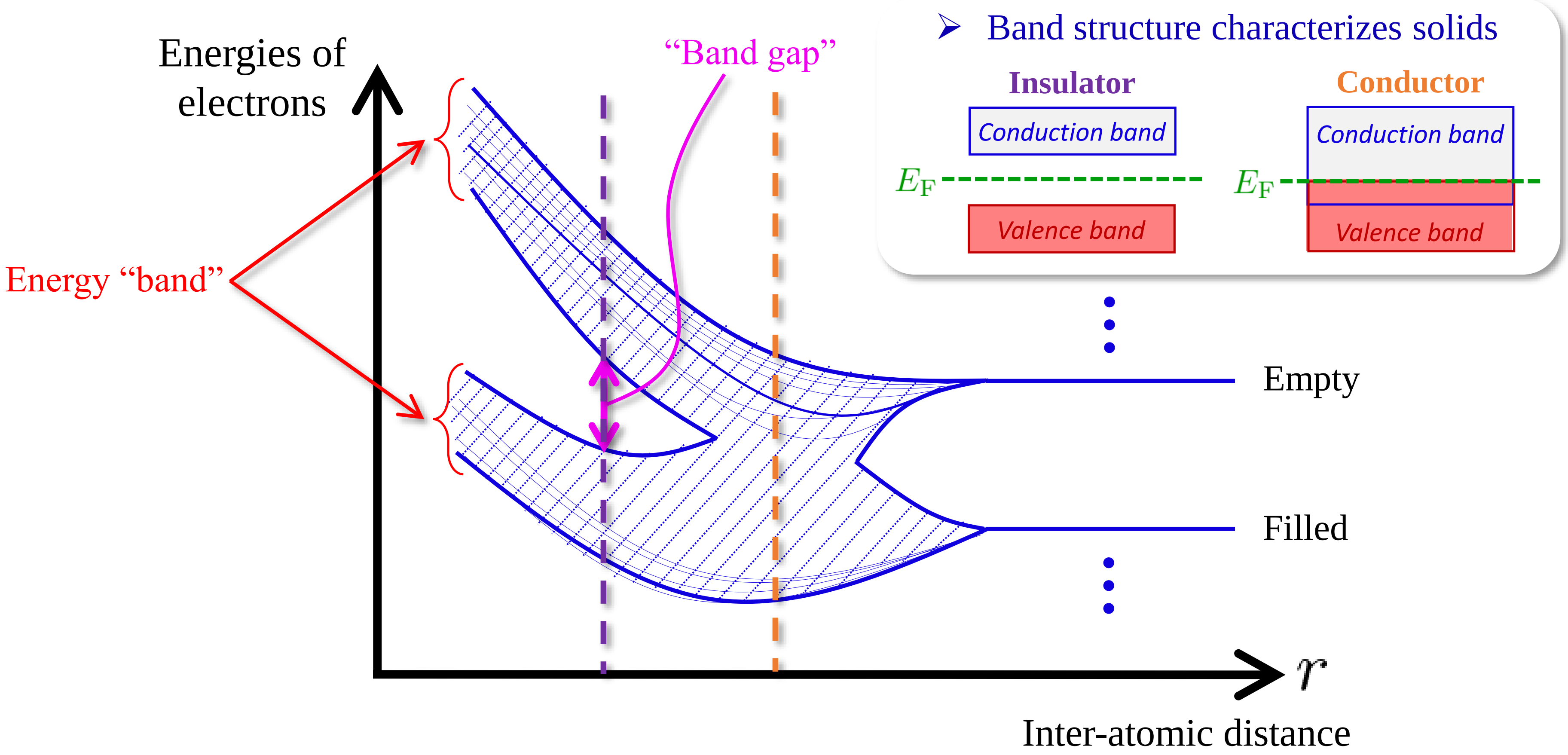


➢ Band structure characterizes solids

The rest of the talk is based on one of my most recent publications:

PHYSICAL REVIEW C **105**, 045807 (2022)

Time-dependent extension of the self-consistent band theory for neutron star matter: Anti-entrainment effects in the slab phase

Kazuyuki Sekizawa,[1,2,*] Sorataka Kobayashi,[3] and Masayuki Matsuo[4,†]

[1] *Center for Transdisciplinary Research, Institute for Research Promotion, Niigata University, Niigata 950-2181, Japan*
[2] *Nuclear Physics Division, Center for Computational Sciences, University of Tsukuba, Ibaraki 305-8577, Japan*
[3] *Graduate School of Science and Technology, Niigata University, Niigata 950-2181, Japan*
[4] *Department of Physics, Faculty of Science, Niigata University, Niigata 950-2181, Japan*

(Received 28 December 2021; accepted 4 April 2022; published 25 April 2022)

in collaboration with

Sorataka Kobayashi
(Finished MSc in Mar. 2019)

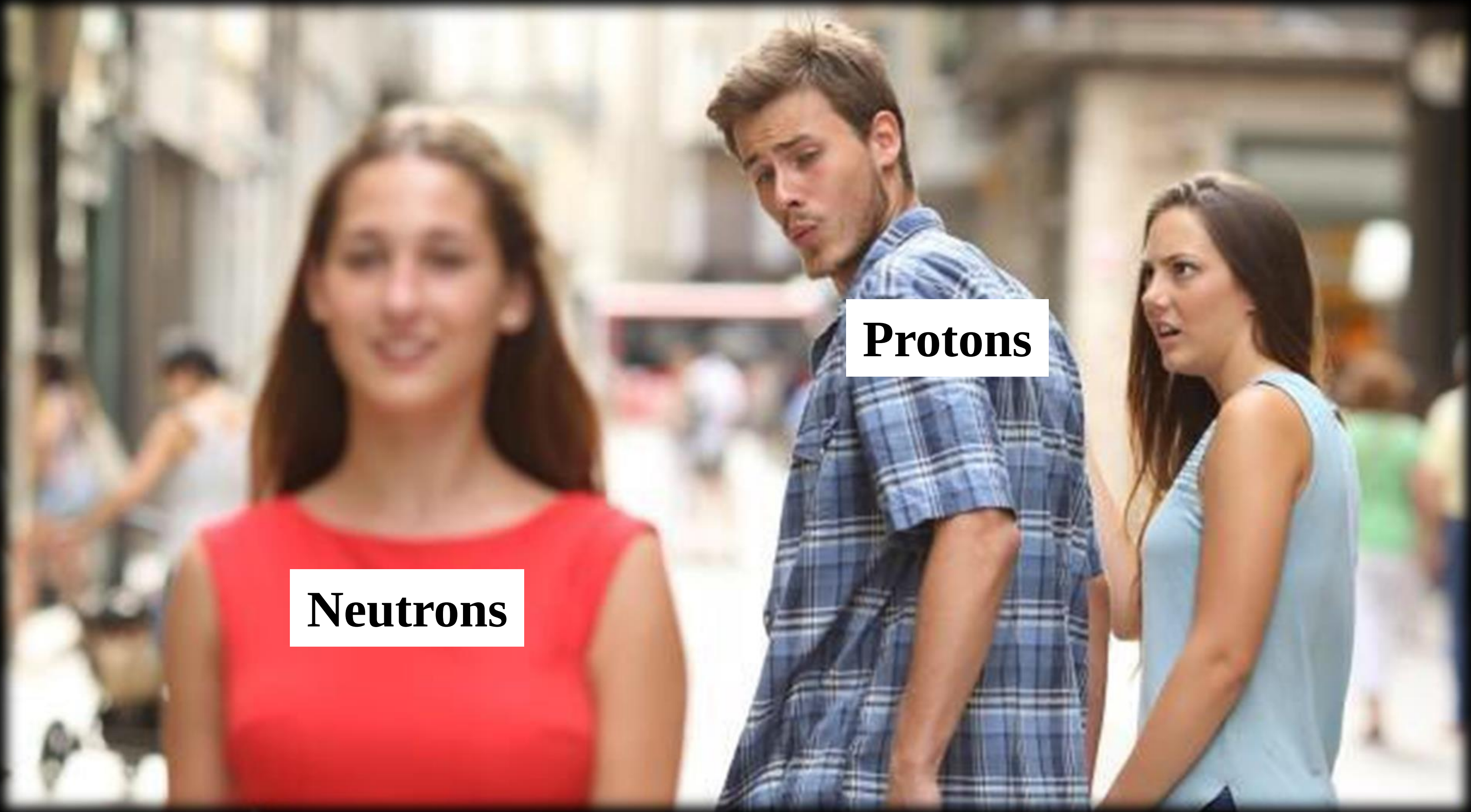
Masayuki Matsuo

Kenta Yoshimura (M1)

真の強さを学ぶ。
新潟大學
NIIGATA UNIVERSITY

東京工業大学
Tokyo Institute of Technology

"Entrainment" is a phenomenon between two species (particles, gases, fluids, etc.), where a motion of one component attracts the other.



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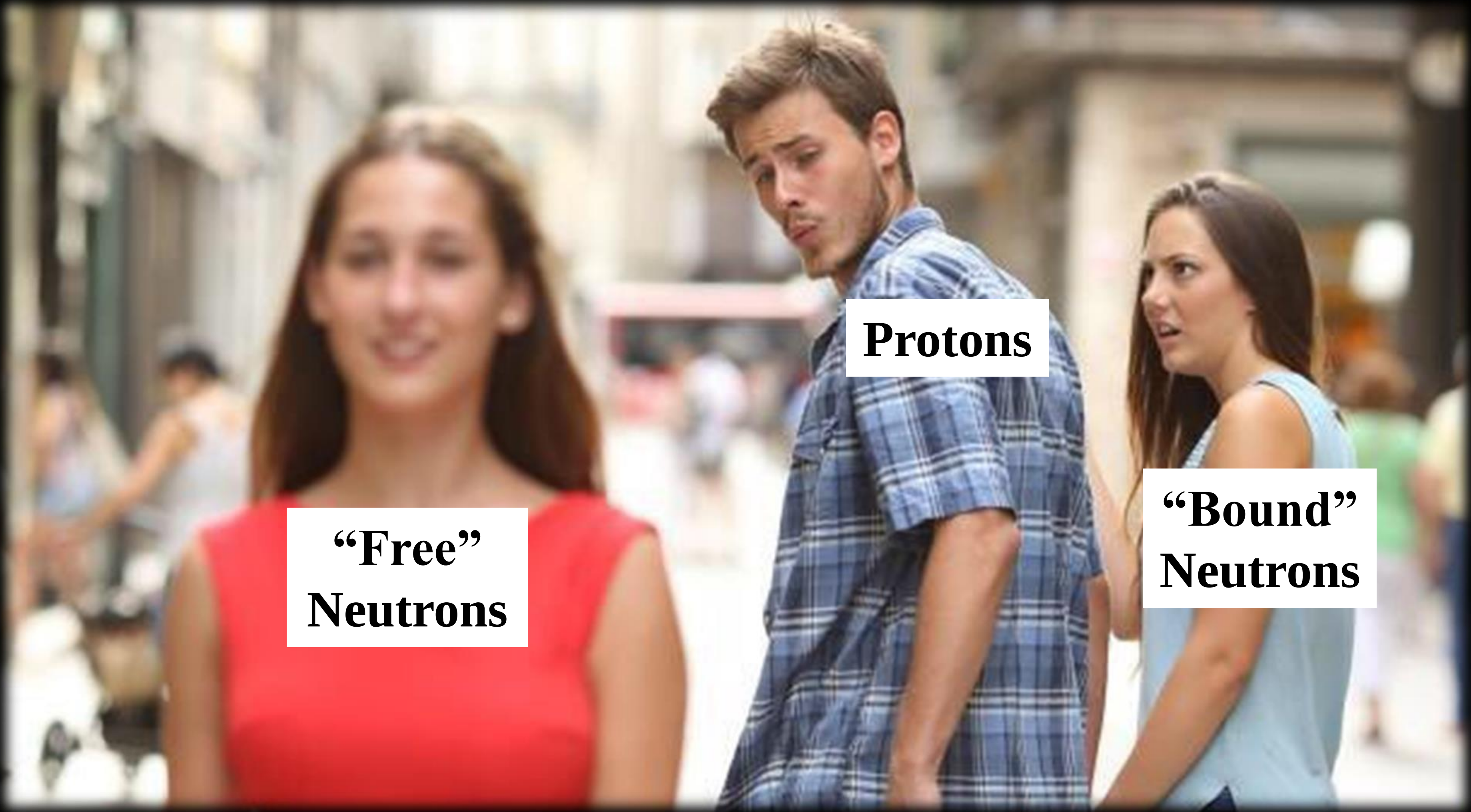


“Entrainment” in the inner crust

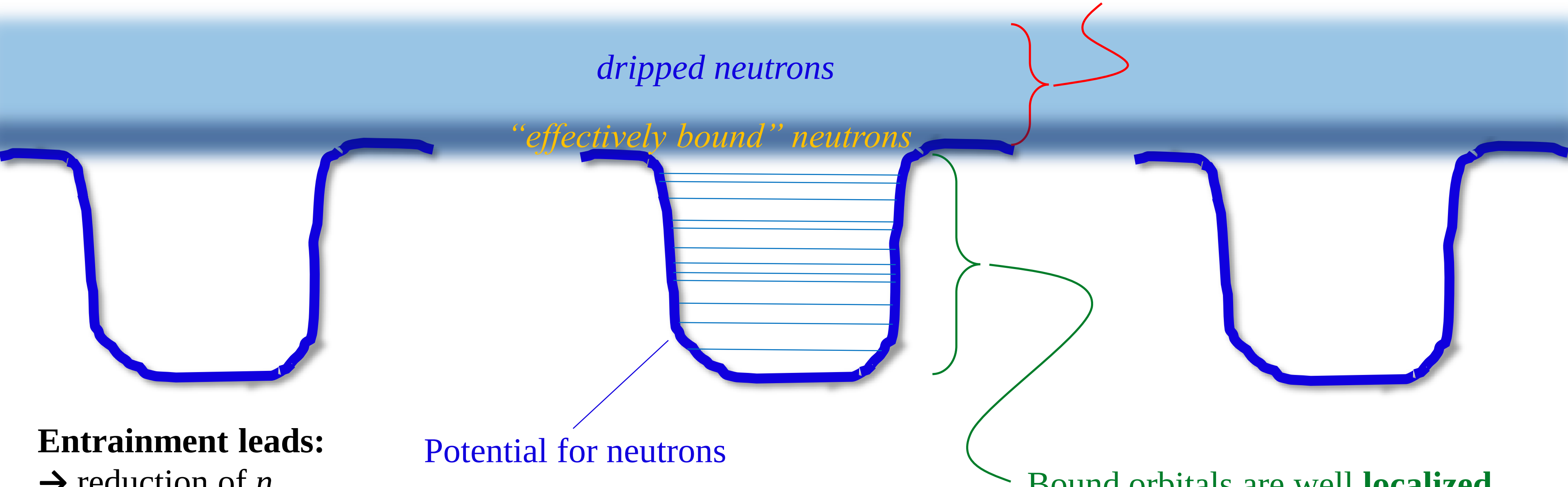
- **Part of dripped neutrons are “effectively bound” (immobilized) by the periodic structure (due to Bragg scatterings), resulting in a larger effective mass**

$$m_{\mathrm{n}} n_{\mathrm{n}}^{\mathrm{f}} = m_{\mathrm{n}}^{\star} n_{\mathrm{n}}^{\mathrm{c}}$$

$n_{\mathrm{n}}^{\mathrm{c}}$: Conduction neutron number density (neutrons that can actually flow)

$m_{\mathrm{n}}^{\star}$: (Macroscopic) Effective mass

Dripped neutrons extend spatially
→ Affected by the lattice, and a band structure is formed!



dripped neutrons

“effectively bound” neutrons

Potential for neutrons

Bound orbitals are well **localized**
→ Not affected by the lattice

Entrainment leads:
→ reduction of n_c
→ enhancement of m^*

Band calculations for the inner crust

The "entrainment effect" is still a debatable problem

- The first consideration for 1D, square-well potential

K. Oyamatsu and Y. Yamada, NPA**578**(1994)184

- Band calculations for slab (1D) and rod (2D) phases

B. Carter, N. Chamel, and P. Haensel, NPA**748**(2005)675

  - Entrainment effects are **weak** for the slab & rod phases:

$$\frac{m^\star}{m} \sim \begin{cases} 1.02-1.03 & \text{for the slab phase} \\ 1.11-1.40 & \text{for the rod phase} \end{cases}$$

- Band calculations for cubic-lattice (3D) phases

N. Chamel, NPA**747**(2005)109 (2005); NPA**773**(2006)263; PRC**85**(2012)035801; J. Low Temp. Phys. **189**, 328 (2017)

  - **Significant** entrainment effects were found in a low-density region:

$$\frac{m^\star}{m} \gtrsim 10 \text{ or more!} \quad \text{for the cubic lattice}$$

- The first *self-consistent* band calculation for the slab (1D) phase (based on DFT with a BCPM EDF)
  - ***"Reduction"*** of the effective mass was observed for the slab phase:

$$\frac{m^\star}{m} \sim 0.65-0.75 \quad \text{for the slab phase}$$

Yu Kashiwaba and T. Nakatsukasa, PRC**100**(2019)035804

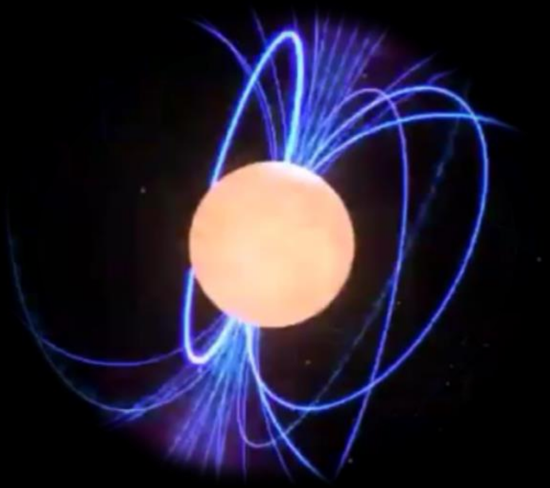
New!

- **Time-dependent extension of the self-consistent band theory (based on TDDFT with a Skyrme EDF)**
  - ***"Reduction"*** was observed, consistent with the Tsukuba group.

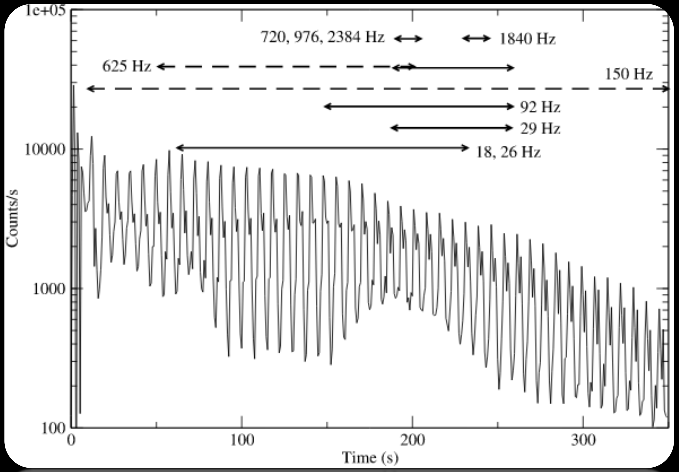
K. Sekizawa, S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

It may affect interpretation of various phenomena, e.g.:

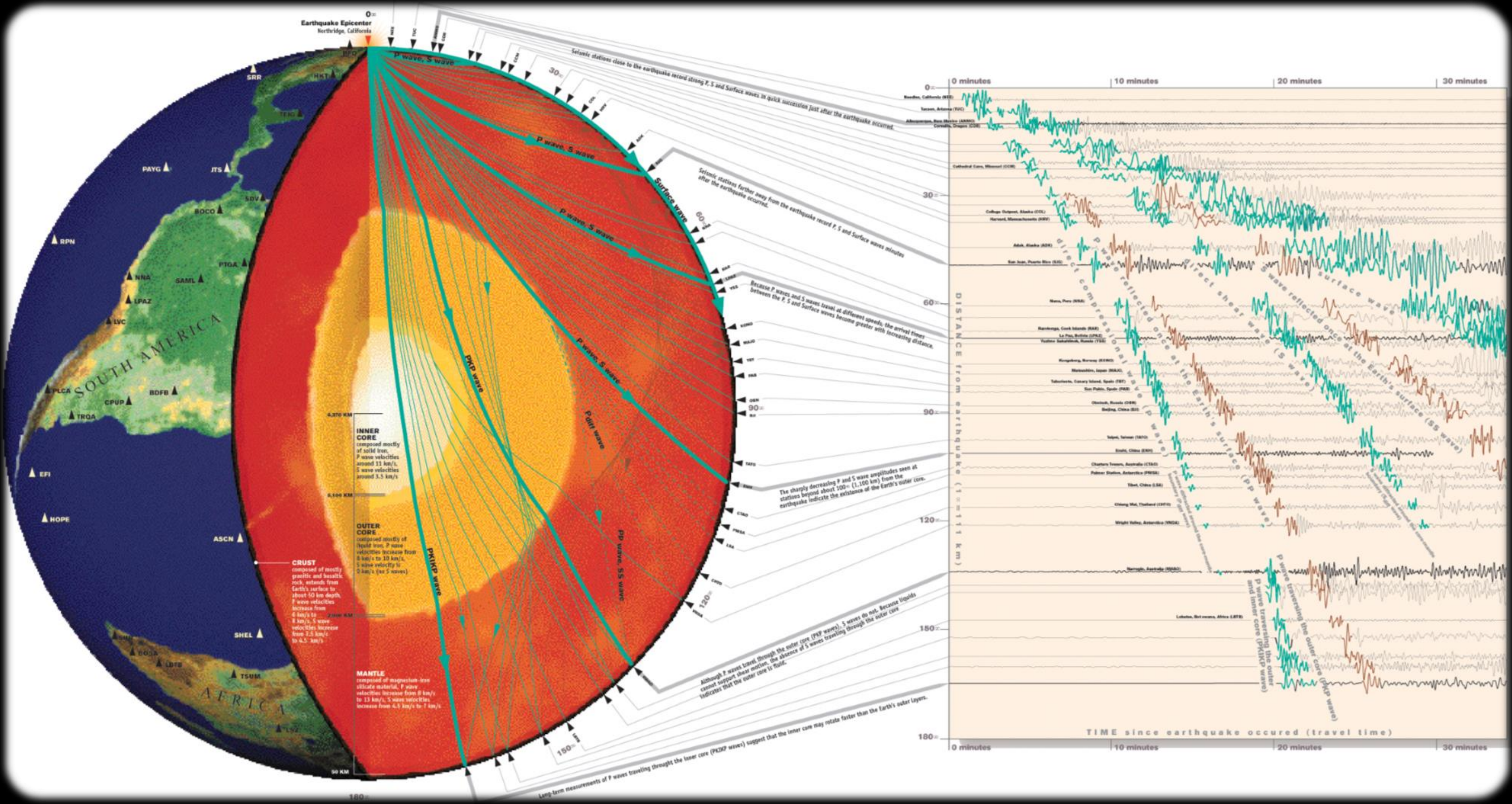
Neutron-star glitch



Quasi-periodic oscillation

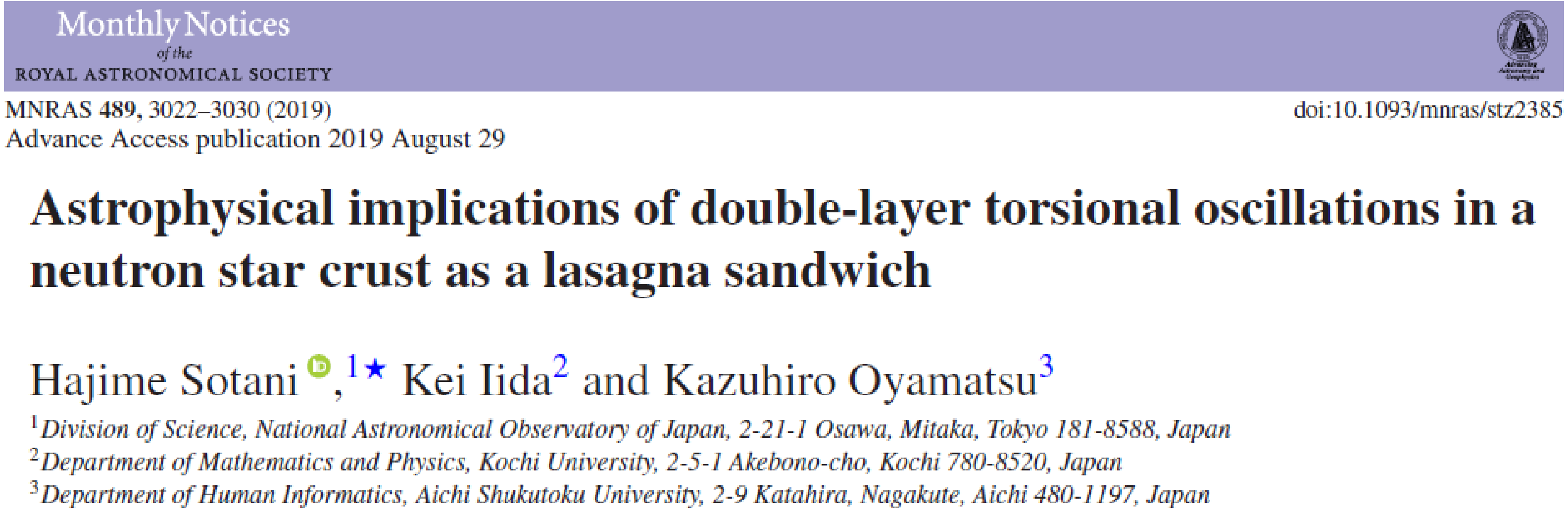


Seismology (地震学): Studying inside of the Earth from earthquakes and their propagation



Picture taken from AusPass by Australian National University

QPOs as "asteroseismology" ("星振学")

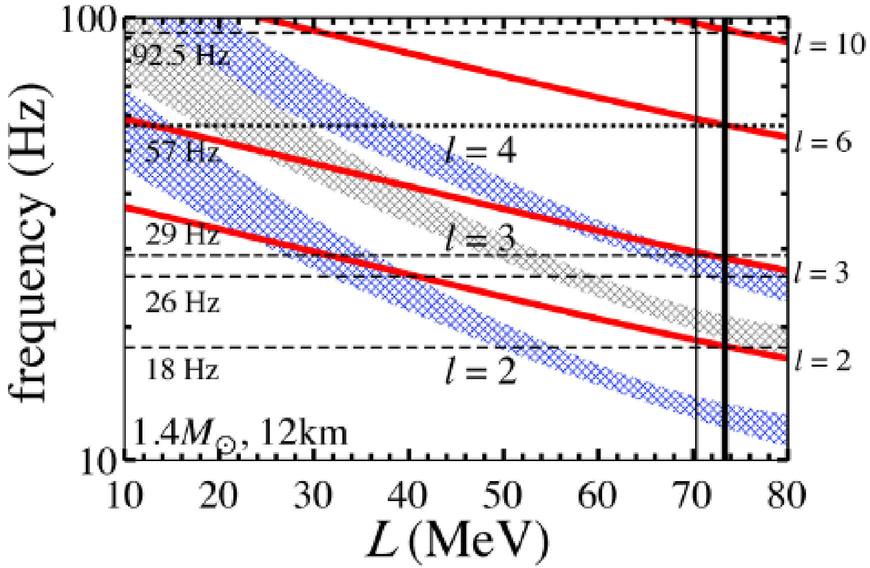


Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS **489,** 3022–3030 (2019)
Advance Access publication 2019 August 29
doi:10.1093/mnras/stz2385

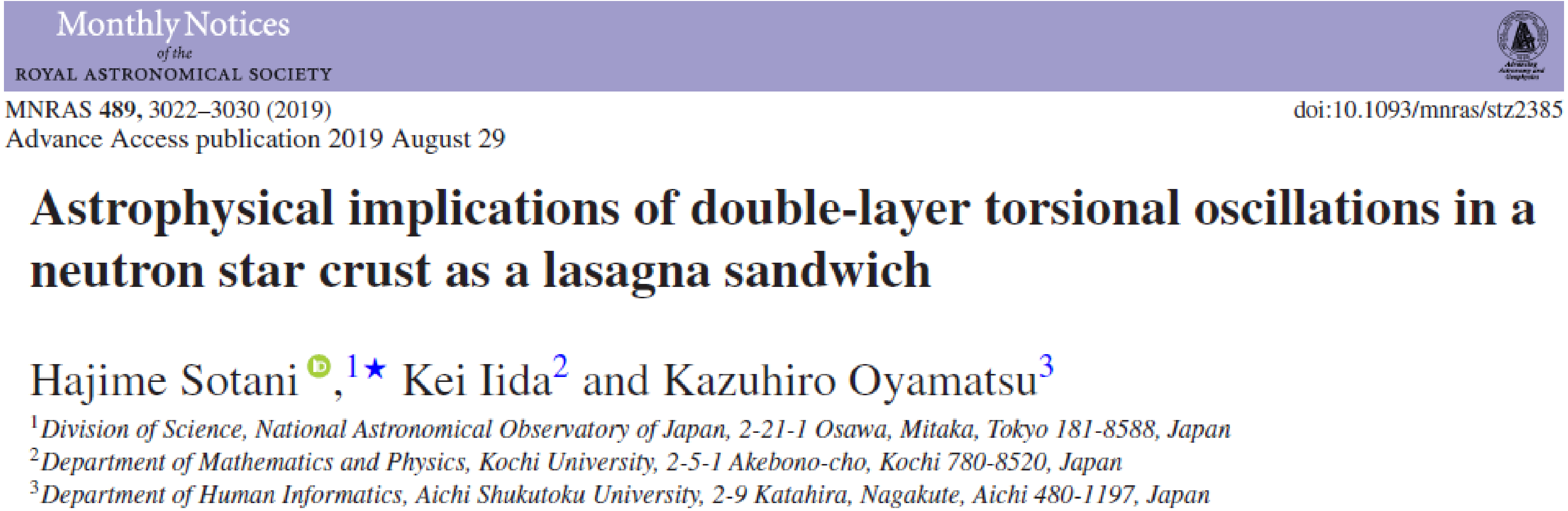
Astrophysical implications of double-layer torsional oscillations in a neutron star crust as a lasagna sandwich

Hajime Sotani[1★] Kei Iida[2] and Kazuhiro Oyamatsu[3]
[1] *Division of Science, National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan*
[2] *Department of Mathematics and Physics, Kochi University, 2-5-1 Akebono-cho, Kochi 780-8520, Japan*
[3] *Department of Human Informatics, Aichi Shukutoku University, 2-9 Katahira, Nagakute, Aichi 480-1197, Japan*



- Many (~30) observed QPO frequencies, and prediction by a Bayesian analysis, have been nicely explained by torsional oscillations of tube–bubble or sphere–cylinder layer

QPOs as "asteroseismology" ("星振学")



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doi:10.1093/mnras/stz2385

Astrophysical implications of double-layer torsional oscillations in a neutron star crust as a lasagna sandwich

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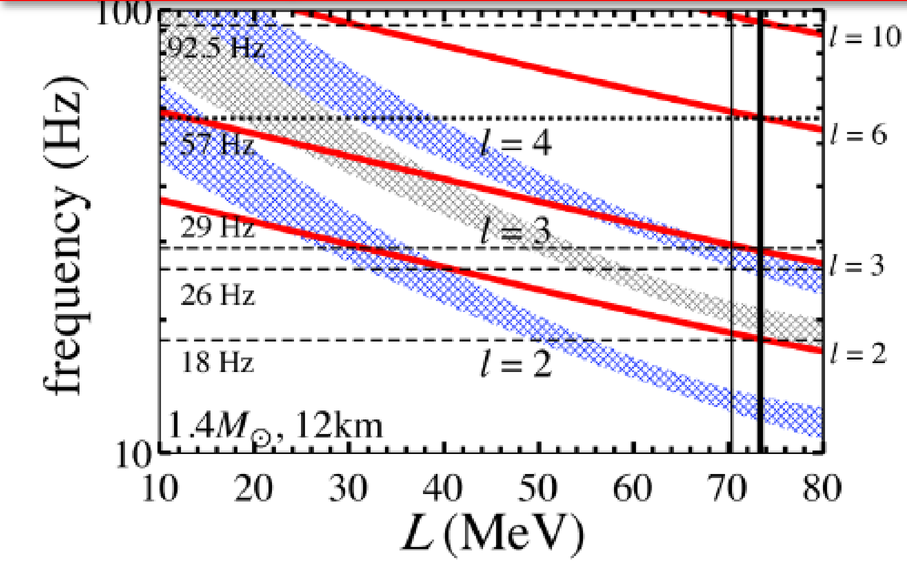
[2]*Department of Mathematics and Physics, Kochi University, 2-5-1 Akebono-cho, Kochi 780-8520, Japan*
[3]*Department of Human Informatics, Aichi Shukutoku University, 2-9 Katahira, Nagakute, Aichi 480-1197, Japan*

The interpretation could be affected by the entrainment effects!



- Many (~30) observed QPO frequencies, and prediction by a Bayesian analysis, have been nicely explained by torsional oscillations of tube–bubble or sphere–cylinder layer

Formulation: Band theory for the slab phase

<u>K.S.</u>, S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

We employ the Skyrme-Kohn-Sham DFT with <u>the Bloch boundary condition</u>

✓ The Bloch boundary condition for single-particle orbitals

$$\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) = \frac{1}{\sqrt{V}} u^{(q)}_{\alpha\boldsymbol{k}}(z) e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \qquad \underline{u^{(q)}_{\alpha\boldsymbol{k}}(z+na) = u^{(q)}_{\alpha\boldsymbol{k}}(z)}$$

Periodicity of the slabs

α: Band index $\boldsymbol{k}$: Bloch wave vector q: Isospin (n or p) a: Period of the slabs

✓ Skyrme EDF

$$\frac{E}{A} = \frac{1}{N_\mathrm{b}} \int_0^a \left(\frac{\hbar^2}{2m}\tau(z) + \sum_{t=0,1} \left[C^{\rho}_t[n] n^2_t(z) + C^{\Delta\rho}_t n_t(z) \partial^2_z n_t(z) + C^{\tau}_t \left(n_t(z)\tau_t(z) - \boldsymbol{j}^2_t(z) \right) \right] + \mathcal{E}^{(p)}_{\mathrm{Coul}}(z) \right) dz$$

Number density:

$$n_q(z) = 2 \sum^{\mathrm{occ.}}_{\alpha,\boldsymbol{k}} \left| \psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) \right|^2$$

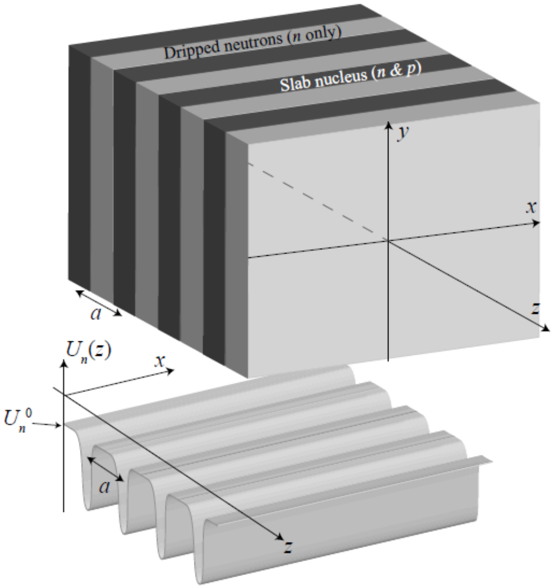
Kinetic density:

$$\tau_q(z) = 2 \sum^{\mathrm{occ.}}_{\alpha,\boldsymbol{k}} \left| \nabla\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) \right|^2$$

Current (momentum) density:

$$\boldsymbol{j}_q(z) = 2 \sum^{\mathrm{occ.}}_{\alpha,\boldsymbol{k}} \mathrm{Im}\left[\psi^{(q)*}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) \nabla \psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) \right]$$

*Uniform background electrons are assumed for the charge neutrality condition: $n_e = \bar{n}_p$



Picture from PRC**100**(2019)035804

✓ Skyrme-Kohn-Sham equations

Note: While we deal with 3D slabs, the equations to be solved are 1D!

$$\hat{h}^{(q)}(z)\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) = \varepsilon^{(q)}_{\alpha\boldsymbol{k}}\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) \quad \Rightarrow \quad \left(\hat{h}^{(q)}(z) + \hat{h}^{(q)}_{\boldsymbol{k}}(z) \right) u^{(q)}_{\alpha\boldsymbol{k}}(z) = \varepsilon^{(q)}_{\alpha\boldsymbol{k}} u^{(q)}_{\alpha\boldsymbol{k}}(z)$$

Ordinary single-particle Hamiltonian:

$$\hat{h}^{(q)}(z) = -\nabla\cdot\frac{\hbar^2}{2m^{\oplus}_q(z)}\nabla + U^{(q)}(z) + \frac{1}{2i}\left[\nabla\cdot\boldsymbol{I}^{(q)}(z) + \boldsymbol{I}^{(q)}(z)\cdot\nabla \right]$$

Additional (k-dependent) term:

$$\hat{h}^{(q)}_{\boldsymbol{k}}(z) = \frac{\hbar^2\boldsymbol{k}^2}{2m^{\oplus}_q(z)} + \hbar\boldsymbol{k}\cdot\underline{\hat{\boldsymbol{v}}^{(q)}(z)}$$

Velocity operator:

$$\hat{\boldsymbol{v}}^{(q)}(z) \equiv \frac{1}{i\hbar}\left[\boldsymbol{r}, \hat{h}^{(q)}(z) \right]$$

Results: Band structure ($Y_p = 0.4$)

K.S., S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

Proton fraction:

$$Y_\mathrm{p} = \frac{\bar{n}_\mathrm{p}}{\bar{n}_\mathrm{n} + \bar{n}_\mathrm{p}}$$

Average nucleon density:

$$\bar{n}_q = \frac{1}{a}\int_0^a n_q(z)dz$$

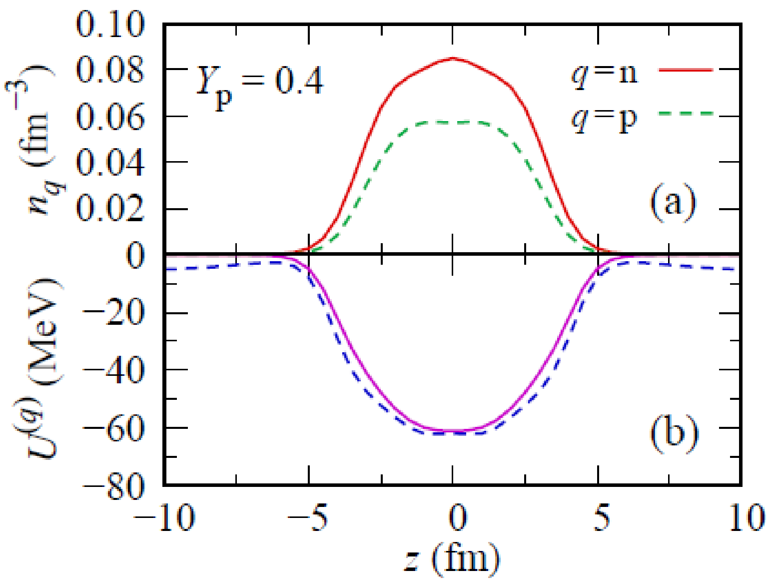
Single-particle energy:

$$\varepsilon^{(q)}_{\alpha\boldsymbol{k}} = \underbrace{e^{(q)}_{\alpha\boldsymbol{k}}}_{z\text{-component}} + \underline{\varepsilon^{(q)}_{\text{kin-}xy,\alpha\boldsymbol{k}}} \approx \frac{\hbar^2 k_\parallel^2}{2m} \qquad k_\parallel = \sqrt{k_x^2 + k_y^2}$$

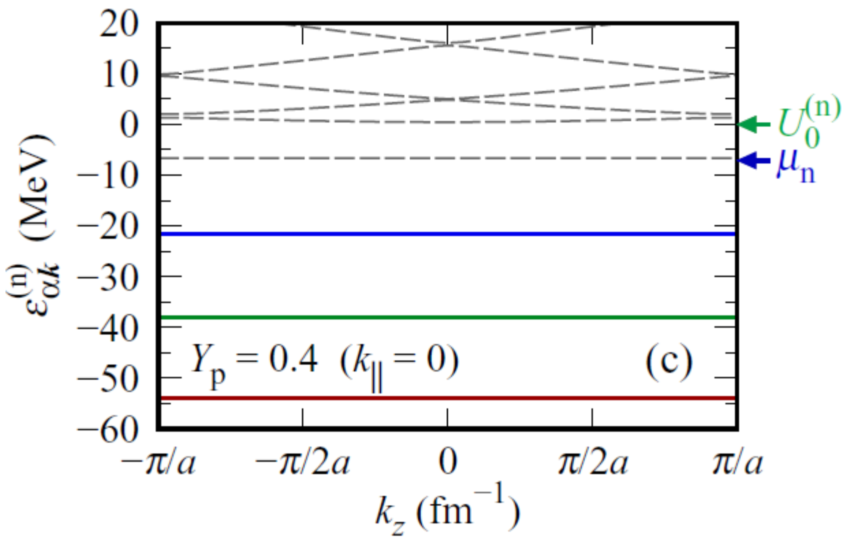
✓ Bound orbitals do not show band structure (k_z dependence)

Y_p = 0.4, n_B = 0.4 fm^{-3}: Isolated slab (no neutron drip)

Density and potential



Neutron single-particle energies



Results: *Band structure* ($Y_p = 0.1$)

<u>K.S.</u>, S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

Proton fraction:

$$Y_{\rm p} = \frac{\bar{n}_{\rm p}}{\bar{n}_{\rm n} + \bar{n}_{\rm p}}$$

Average nucleon density:

$$\bar{n}_q = \frac{1}{a}\int_0^a n_q(z)dz$$

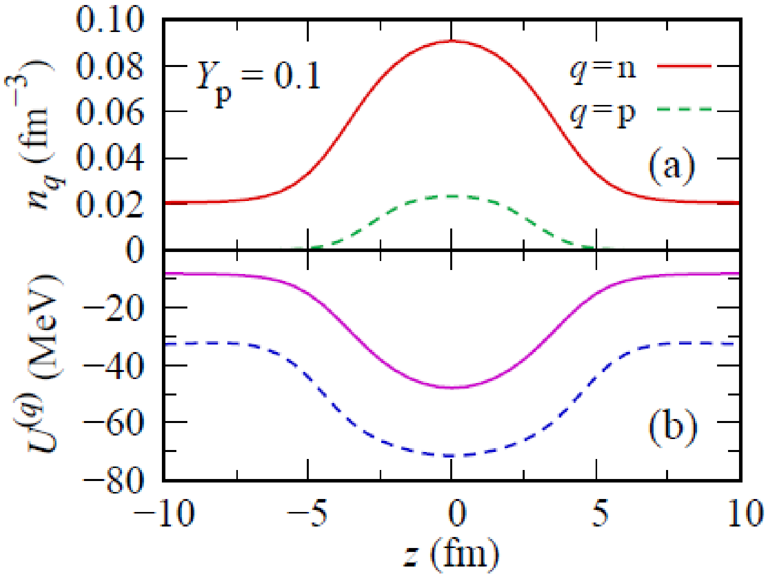
Single-particle energy:

$$\varepsilon^{(q)}_{\alpha\boldsymbol{k}} = \underset{z\text{-component}}{\underline{e^{(q)}_{\alpha\boldsymbol{k}}}} + \underline{\varepsilon^{(q)}_{\text{kin-}xy,\alpha\boldsymbol{k}}} \approx \frac{\hbar^2 k_\parallel^2}{2m} \qquad k_\parallel = \sqrt{k_x^2 + k_y^2}$$

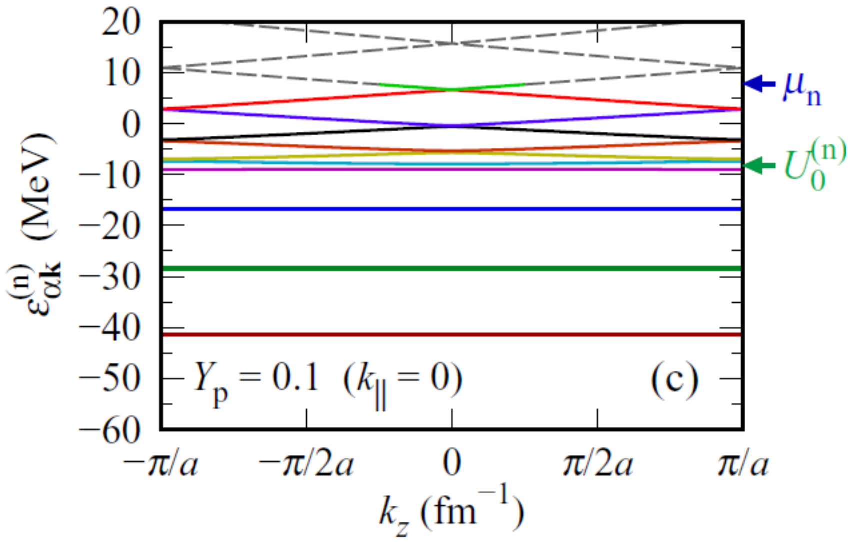
✓ <u>Dripped neutrons</u> show band structure (k_z dependence)

$Y_{\rm p}$ = 0.1, $n_{\rm B}$ = 0.4 fm^{-3}: Neutron-dripped slab

Density and potential



Neutron single-particle energies



Results: Band structure ($Y_p = 0.1$)

K.S., S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

Proton fraction:

$$Y_{\rm p} = \frac{\bar{n}_{\rm p}}{\bar{n}_{\rm n} + \bar{n}_{\rm p}}$$

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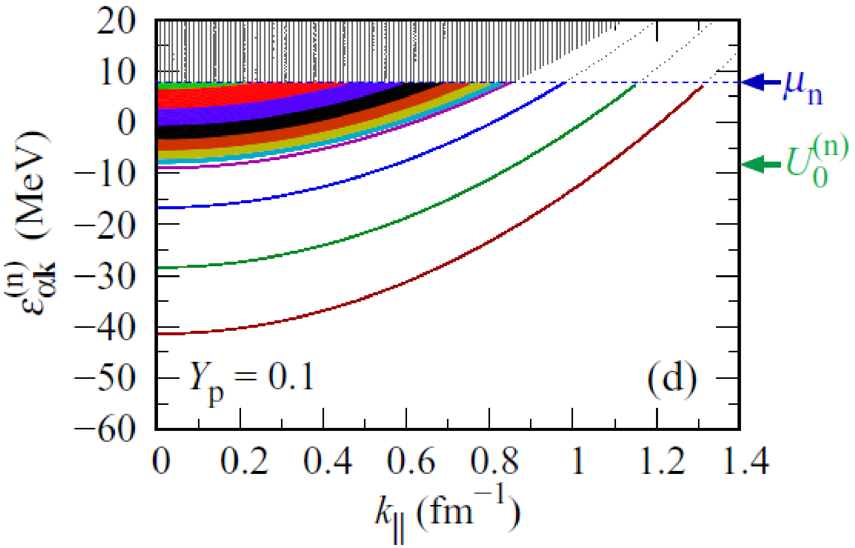
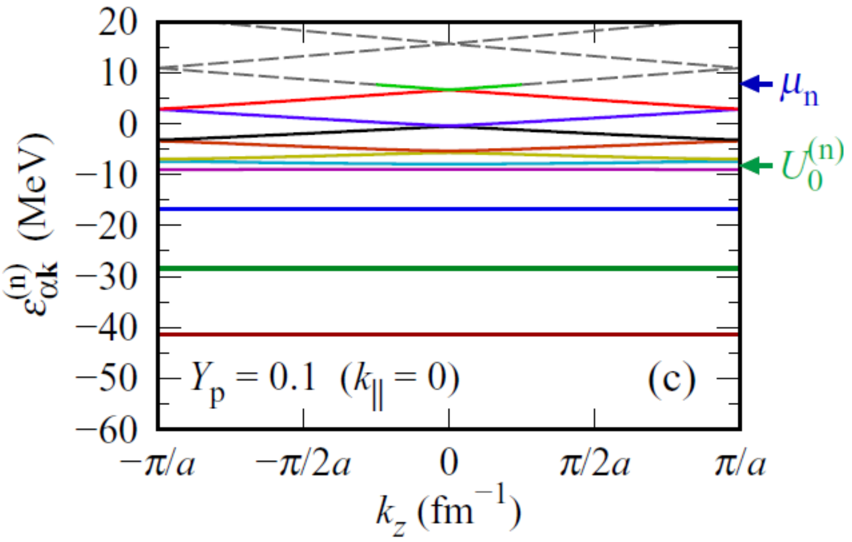
✓ Dripped neutrons show band structure (k_z dependence)

$Y_{\rm p}$ = 0.1, $n_{\rm B}$ = 0.4 fm^{-3}: Neutron-dripped slab

$k_\parallel$ dependence of s.p. energies:



Neutron single-particle energies



Static approach for conduction neutrons

✓ In the static approach, **conduction neutrons** are analyzed

In the **static** approach, the *conduction neutron number density* is defined by

$$n_{\mathrm{n}}^{\mathrm{c}} \equiv m_{\mathrm{n,bg}}^{\oplus} \mathcal{K}_{zz}^{(\mathrm{n})}$$

where $\mathcal{K}_{zz}^{(\mathrm{n})}$ is the so-called *mobility coefficient*:

$$\mathcal{K}_{zz}^{(\mathrm{n})} = \frac{1}{\pi L} \sum_{\alpha, k_z} \int k_{\parallel} \left(m_{\mathrm{n},\alpha\boldsymbol{k}}^{\star -1} \right)_{zz} \theta(\mu_{\mathrm{n}} - \varepsilon_{\alpha\boldsymbol{k}}^{(\mathrm{n})})\, \mathrm{d}k_{\parallel}$$

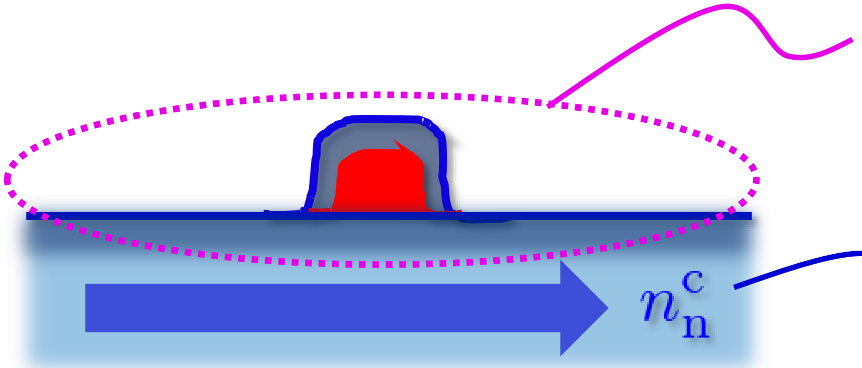
Inverse of the "macroscopic" effective mass tensor

$$\left(m_{\mathrm{n},\alpha\boldsymbol{k}}^{\star -1} \right)_{\mu\nu} = \frac{1}{\hbar^2} \frac{\partial^2 \varepsilon_{\alpha\boldsymbol{k}}^{(\mathrm{n})}}{\partial k_\mu \partial k_\nu}$$

For bound orbitals, there is no k_z dependence ⇒ $1/m \to 0$, i.e., $m \to \infty$ (can not conduct).

⇒ The mobility coefficient quantifies dripped neutrons that can actually conduct.

Let’s look at the same phenomenon from a different side



Instead, we may quantify the rest: the effectively bound neutrons that can not actually conduct.

The static approach quantifies conduction neutrons through the analysis of band structure

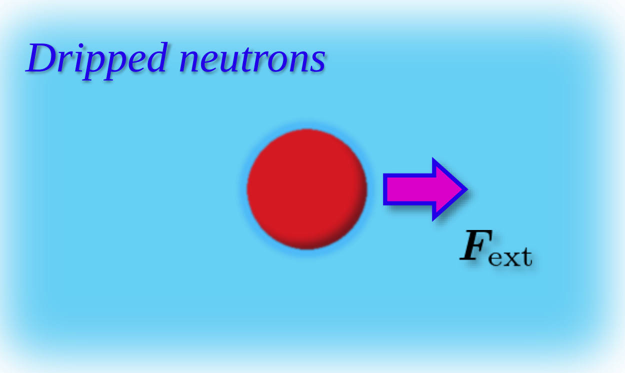
Figure was taken from: https://matome.eternalcollegest.com/post-2134590520376671801

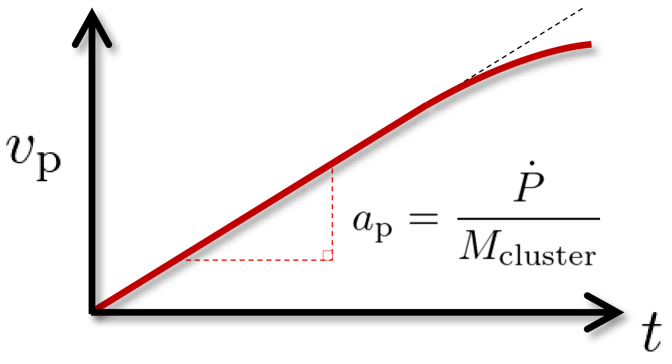
Real-time method for the collective mass

K.S., S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

✓ The collective mass is extracted from **acceleration motion under constant force**

The real-time method: Idea





How to introduce spatially-uniform electric field

✓ TDKS equation in a "velocity gauge"

Spatially-uniform Vector potential

$$i\hbar\frac{\partial\tilde{u}^{(q)}_{\alpha\boldsymbol{k}}(z,t)}{\partial t} = \left(\hat{h}^{(q)}(z,t)+\hat{h}^{(q)}_{\boldsymbol{k}(t)}(z,t)\right)\tilde{u}^{(q)}_{\alpha\boldsymbol{k}}(z,t) \qquad \boldsymbol{k}(t) = \boldsymbol{k} + \frac{e}{\hbar c}A_z(t)\hat{\boldsymbol{e}}_z$$

Gauge transformation for the Bloch orbitals:

$$\tilde{u}^{(q)}_{\alpha\boldsymbol{k}}(z,t) = \exp\left[-\frac{ie}{\hbar c}A_z(t)z\right]u^{(q)}_{\alpha\boldsymbol{k}}(z,t)$$

Electric field:

$$E_z(t) = -\frac{1}{c}\frac{dA_z}{dt}$$

k-dependent term:

$$\hat{h}^{(q)}_{\boldsymbol{k}}(z) = \frac{\hbar^2\boldsymbol{k}^2}{2m^{\oplus}_q(z)} + \hbar\boldsymbol{k}\cdot\hat{\boldsymbol{v}}^{(q)}(z)$$

Velocity operator:

$$\hat{\boldsymbol{v}}^{(q)}(z) \equiv \frac{1}{i\hbar}\left[\boldsymbol{r},\hat{h}^{(q)}(z)\right]$$

cf. K. Yabana and G.F. Bertsch, Phys. Rev. B **54**, 4484 (1996); G.F. Bertch *et al.*, Phys. Rev. B **62**, 7998 (2000)

Results: The collective mass

K.S., S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

Acceleration:

$$a_{\rm p} = \frac{d^2Z}{dt^2}$$

C.m. position of protons:

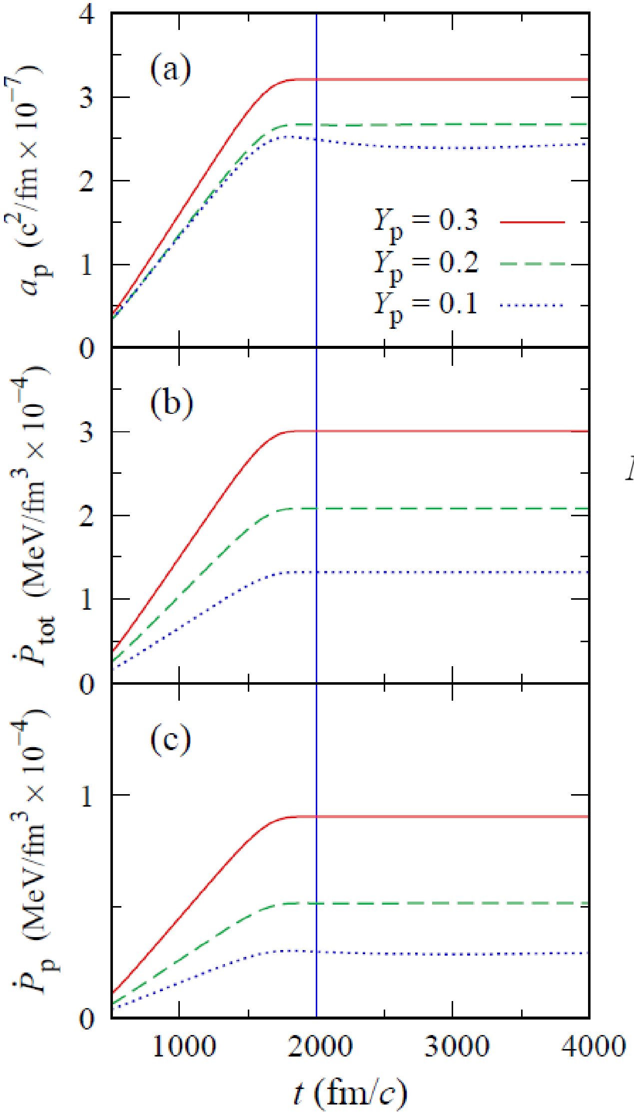
$$Z(t) = \frac{1}{a}\int_0^a z\, n_{\rm p}(z,t)\, dz$$

Momentum of nucleons:

$$P_q(t) = \hbar\int_0^a j_q(z,t)\, dz$$

Total momentum:

$$P_{\rm tot}(t) = P_{\rm n}(t) + P_{\rm p}(t)$$

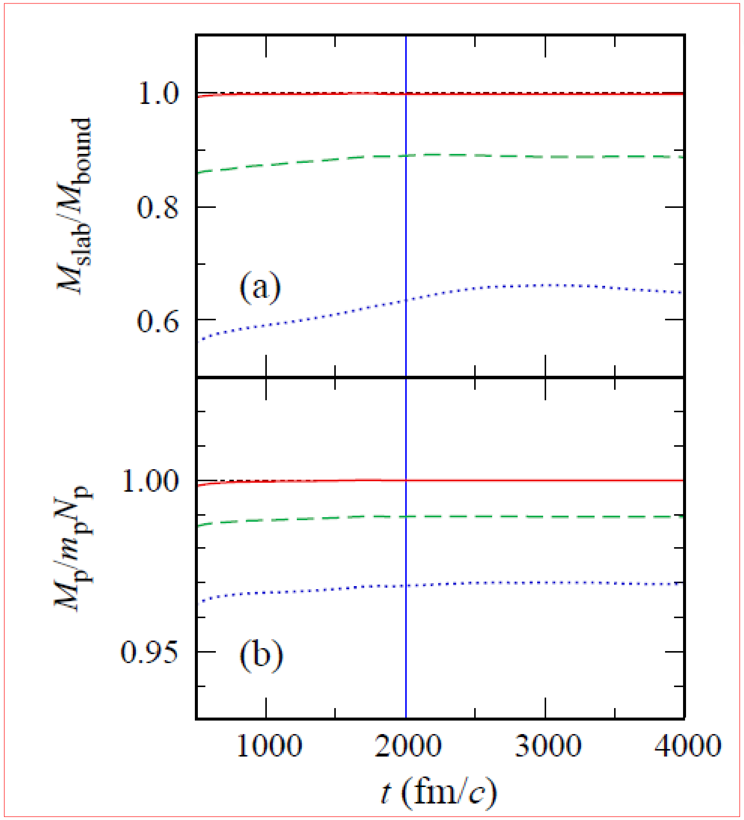


$$M_{\rm slab} = \dot{P}_{\rm tot}/a_{\rm p}$$

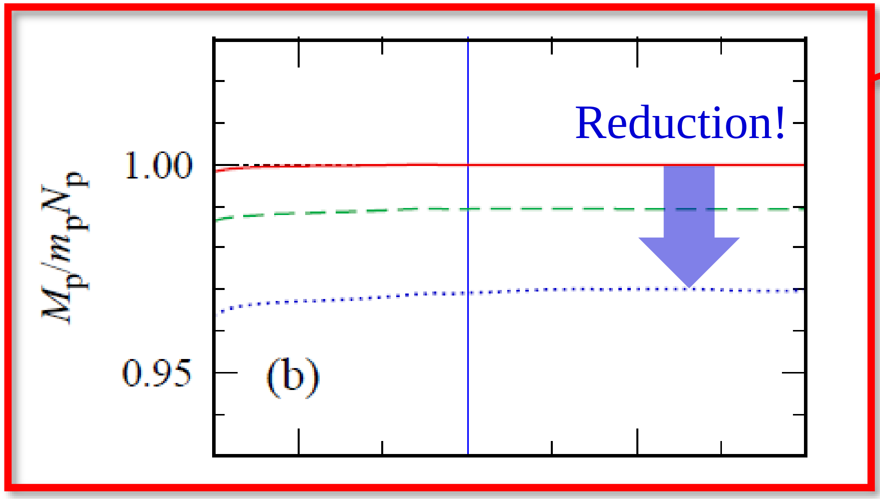
$$M_{\rm p} = \dot{P}_{\rm p}/a_{\rm p}$$

✓ For neutron-dripped slabs, we find significant ***reduction*** of the collective mass!

➢ **What is the origin of the reduction?**



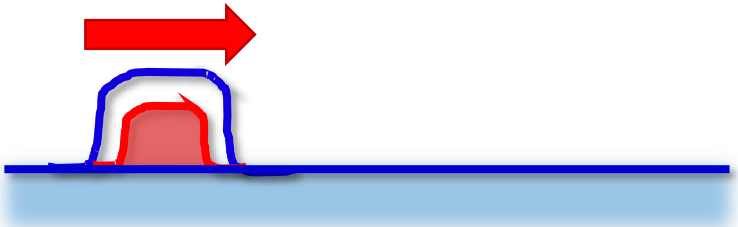
✓ Cause of the reduction of the collective mass of protons: **the density-dependent "microscopic" effective mass**



Collective mass of protons

$$M_\mathrm{p} \le m_\mathrm{p} N_\mathrm{p} \approx m_\mathrm{p}^{\oplus}[n_\mathrm{n}^\mathrm{b.g.}] N_\mathrm{p}$$

Protons and bound neutrons move together



There must be a velocity lag between protons and background neutrons!

The continuity equation within Skyrme TDDFT reads:

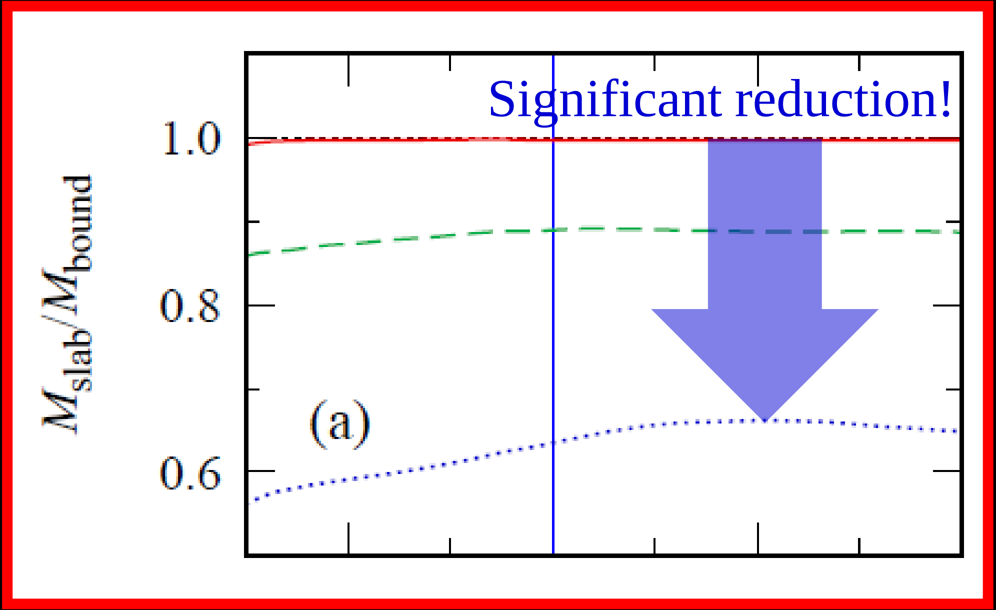
$$\frac{\partial \rho_q(\boldsymbol{r},t)}{\partial t} + \hbar\, \boldsymbol{\nabla} \cdot \boldsymbol{p}_q(\boldsymbol{r},t) = 0$$

where

$$\boldsymbol{p}_q(\boldsymbol{r},t) = \boldsymbol{j}_q(\boldsymbol{r},t) + \underset{\substack{+1 \text{ for protons} \\ -1 \text{ for neutrons}}}{q} \frac{2m_q}{\hbar^2}(C_0^\tau - C_1^\tau) n_\mathrm{n}(\boldsymbol{r},t) n_\mathrm{p}(\boldsymbol{r},t) \underbrace{\left(\frac{\boldsymbol{j}_\mathrm{p}(\boldsymbol{r},t)}{n_\mathrm{p}(\boldsymbol{r},t)} - \frac{\boldsymbol{j}_\mathrm{n}(\boldsymbol{r},t)}{n_\mathrm{n}(\boldsymbol{r},t)} \right)}_{\text{velocity difference}}$$

Then, what is the cause of the reduction of the collective mass of the slab?

→ an "anti-entrainment" effect!



It can **not** be explained solely by the microscopic effective mass.

"Anti-entrainment" effect

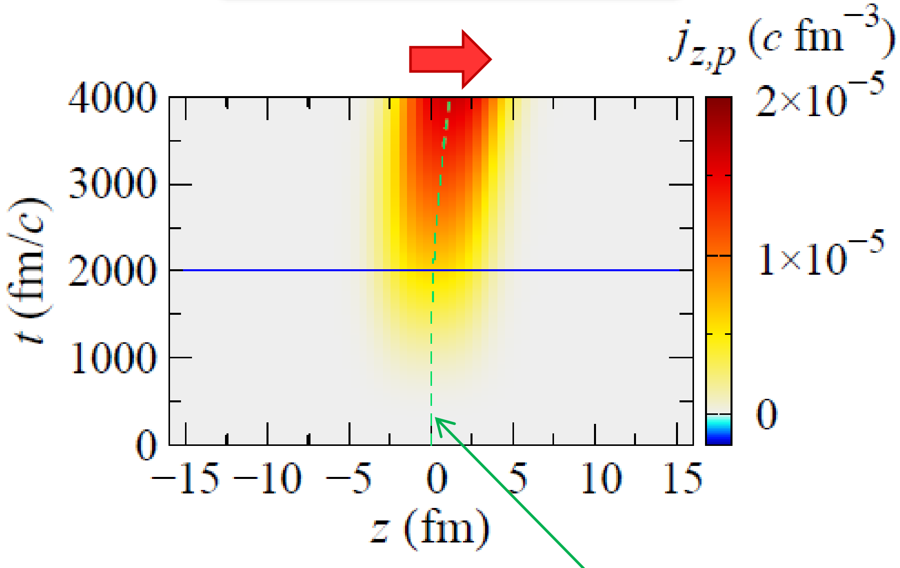
K.S., S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

Current density:

$$j_{z,q}(z,t) = \frac{\hbar}{m_q}\sum_{\alpha,\boldsymbol{k}}^{\text{occ.}} \mathrm{Im}\big[\psi^{(q)*}_{\alpha\boldsymbol{k}}(\boldsymbol{r},t)\nabla\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r},t)\big] = \frac{\hbar}{m_q}\frac{1}{aN_{k_z}}\sum_{\alpha,k_z}\int\frac{k_\parallel}{\pi}\,\mathrm{Im}\big[u^{(q)*}_{\alpha\boldsymbol{k}}(z,t)(\partial_z+ik_z)u^{(q)}_{\alpha\boldsymbol{k}}(z,t)\big]\,\theta(\mu_q-\varepsilon^{(q)}_{\alpha\boldsymbol{k}})\,dk_\parallel$$

- ✓ Protons inside the slab move toward the direction of the external force, as expected.

Proton current density



C.m. position of protons

“*Anti-entrainment*” *effect*

<u>K.S.</u>, S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

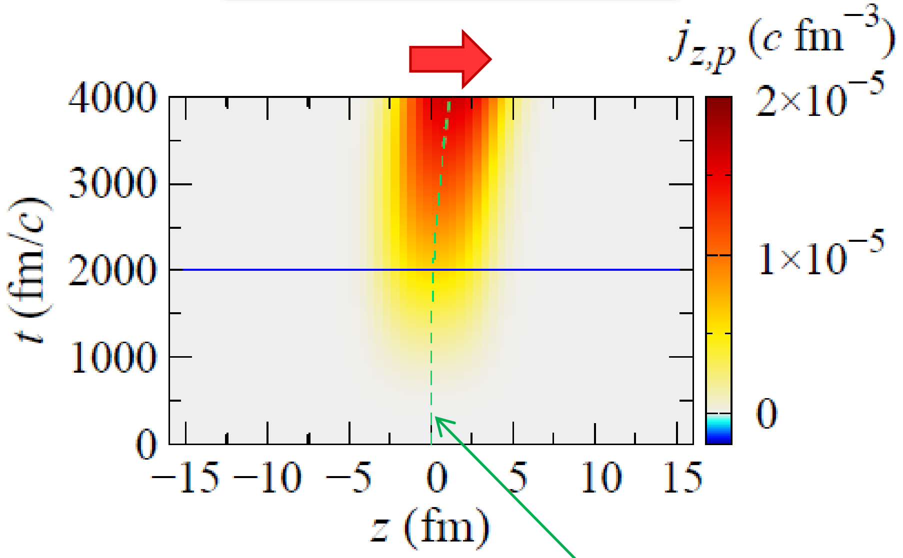
Current density:

$$j_{z,q}(z,t) = \frac{\hbar}{m_q}\sum_{\alpha,\boldsymbol{k}}^{\text{occ.}} \mathrm{Im}\left[\psi^{(q)*}_{\alpha\boldsymbol{k}}(\boldsymbol{r},t)\nabla\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r},t)\right] = \frac{\hbar}{m_q}\frac{1}{aN_{k_z}}\sum_{\alpha,k_z}\int \frac{k_\parallel}{\pi}\,\mathrm{Im}\left[u^{(q)*}_{\alpha\boldsymbol{k}}(z,t)(\partial_z+ik_z)u^{(q)}_{\alpha\boldsymbol{k}}(z,t)\right]\theta(\mu_q-\varepsilon^{(q)}_{\alpha\boldsymbol{k}})\,dk_\parallel$$

✓ Dripped neutrons outside the slab move toward the opposite direction!

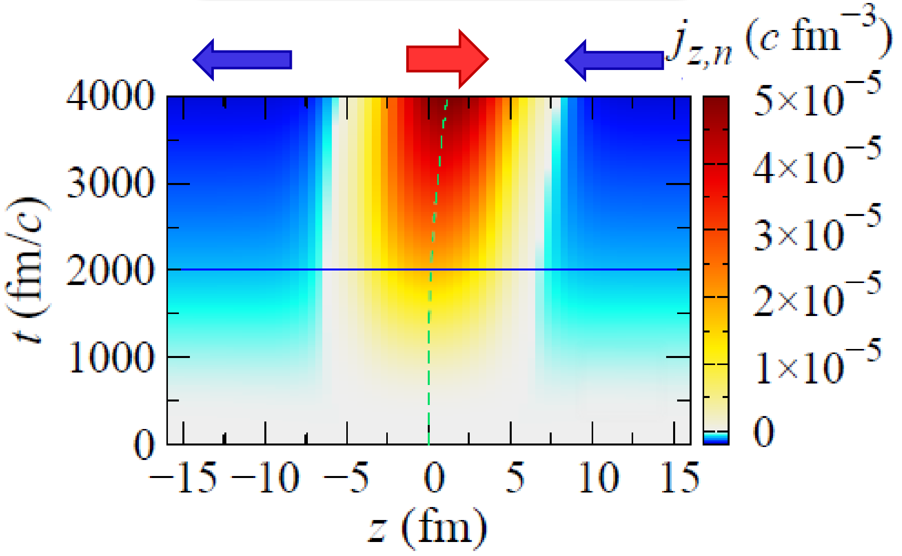
Since it reduces P_{tot} and $\dot{P}_{\mathrm{tot}}$, $M_{\mathrm{slab}} = \dot{P}_{\mathrm{tot}}/a_{\mathrm{p}}$ is reduced

Proton current density



C.m. position of protons

Neutron current density



$$\left(m^{\star\,-1}_{\mathrm{n},\alpha\boldsymbol{k}}\right)_{\mu\nu} = \frac{1}{\hbar^2}\frac{\partial^2\varepsilon^{(\mathrm{n})}_{\alpha\boldsymbol{k}}}{\partial k_\mu \partial k_\nu}$$

"Anti-entrainment" effect

<u>K.S.</u>, S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

Current density:

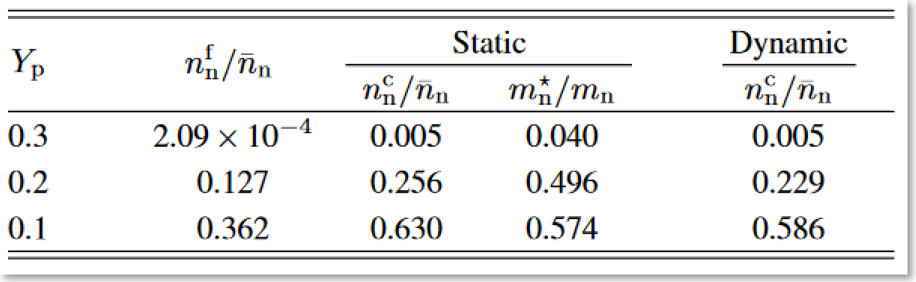
$$j_{z,q}(z,t) = \frac{\hbar}{m_q}\sum_{\alpha,\boldsymbol{k}}^{\text{occ.}} \mathrm{Im}\big[\psi_{\alpha\boldsymbol{k}}^{(q)*}(\boldsymbol{r},t)\nabla\psi_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r},t)\big] = \frac{\hbar}{m_q}\frac{1}{aN_{k_z}}\sum_{\alpha,k_z}\int\frac{k_\parallel}{\pi}\,\mathrm{Im}\big[u^{(q)*}_{\alpha\boldsymbol{k}}(z,t)(\partial_z+ik_z)u^{(q)}_{\alpha\boldsymbol{k}}(z,t)\big]\,\theta(\mu_q-\varepsilon^{(q)}_{\alpha\boldsymbol{k}})\,dk_\parallel$$

✓ Dripped neutrons outside the slab move toward the opposite direction!

Since it reduces P_{tot} and $\dot{P}_{\text{tot}}$, $M_{\text{slab}} = \dot{P}_{\text{tot}}/a_{\text{p}}$ is reduced

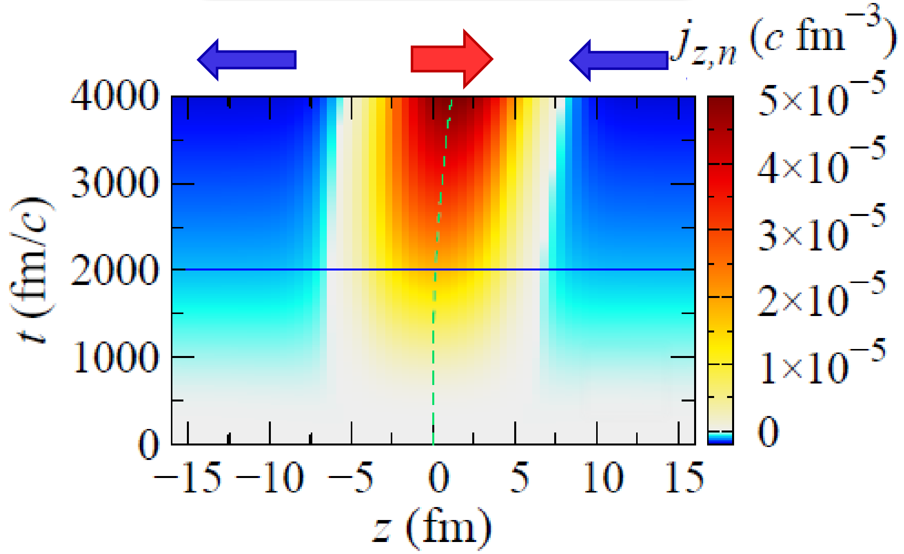
Reduction of M_{slab}
→ enhancement of n_{c}
→ reduction of m^*

We interpret it as an "anti-entrainment" effect



Y_{p}	$n^{\text{f}}_{\text{n}}/\bar{n}_{\text{n}}$	Static		Dynamic
		$n^{\text{c}}_{\text{n}}/\bar{n}_{\text{n}}$	$m^{\star}_{\text{n}}/m_{\text{n}}$	$n^{\text{c}}_{\text{n}}/\bar{n}_{\text{n}}$
0.3	2.09×10^{-4}	0.005	0.040	0.005
0.2	0.127	0.256	0.496	0.229
0.1	0.362	0.630	0.574	0.586

Neutron current density



$$\left(m^{\star\,-1}_{\text{n},\alpha\boldsymbol{k}}\right)_{\mu\nu} = \frac{1}{\hbar^2}\frac{\partial^2\varepsilon^{(\text{n})}_{\alpha\boldsymbol{k}}}{\partial k_\mu\partial k_\nu}$$

At the frontiers in nuclear physics III:

Neutron-star merger, gravitational wave, and nucleosynthesis

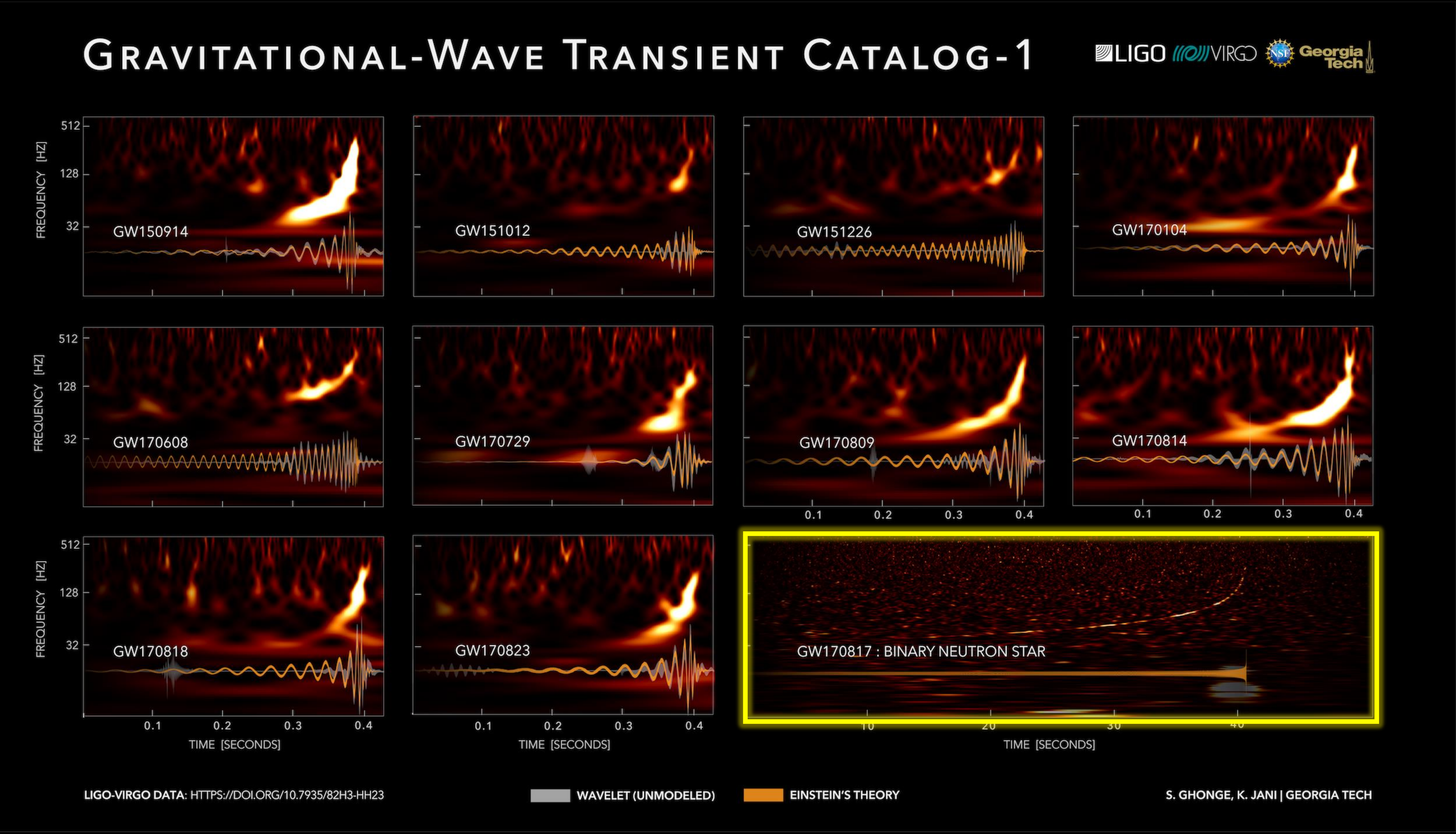
Fe is the most stable!

Well, then how were elements heavier than iron produced!?

One of the unsolved problems in Physics

but, we have learned a lot!

Dawn of a new era of the multi-messenger astronomy



GRAVITATIONAL-WAVE TRANSIENT CATALOG-1

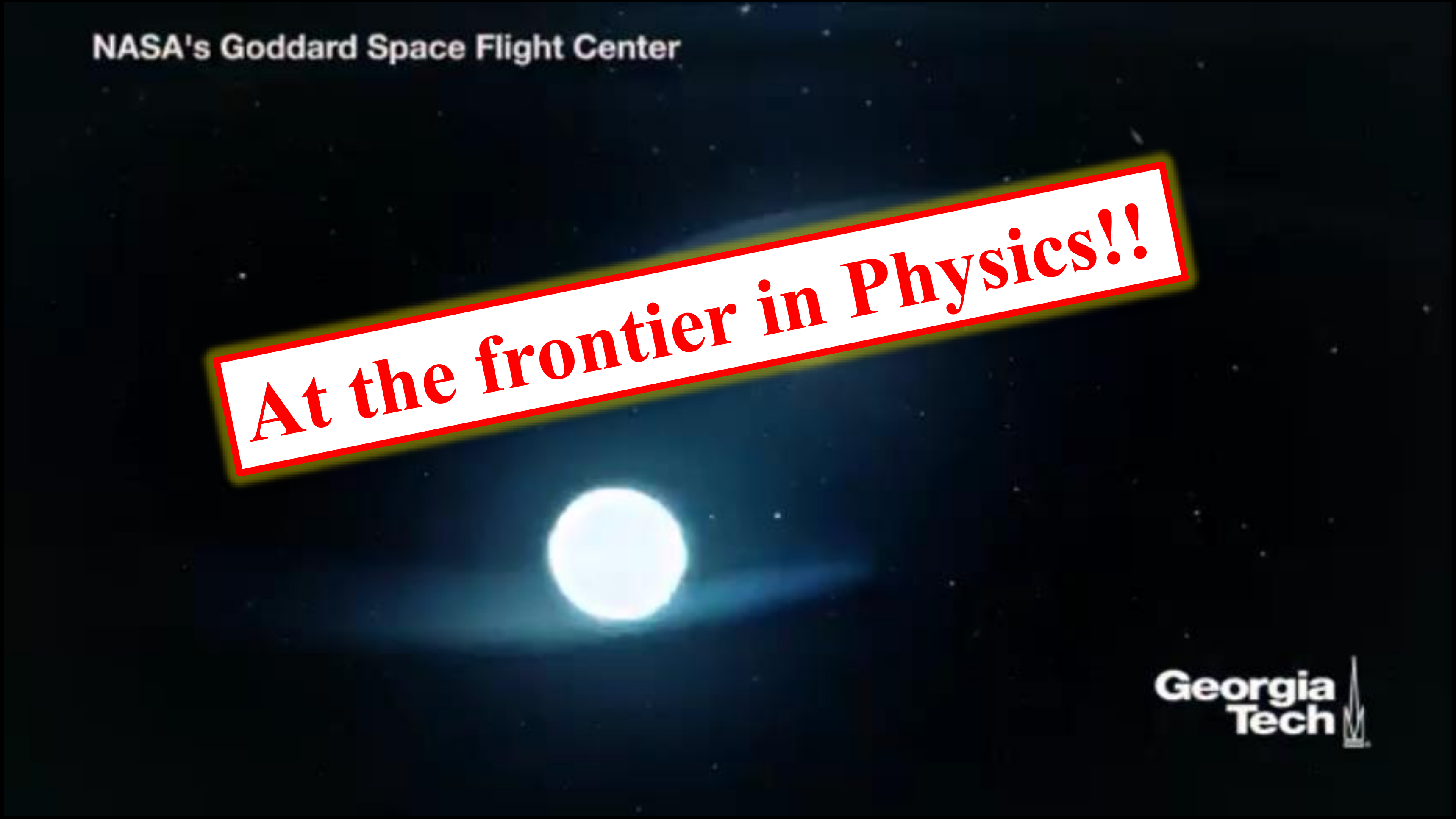
LIGO-VIRGO DATA: HTTPS://DOI.ORG/10.7935/82H3-HH23

S. GHONGE, K. JANI | GEORGIA TECH

Neutron Star Merger!!

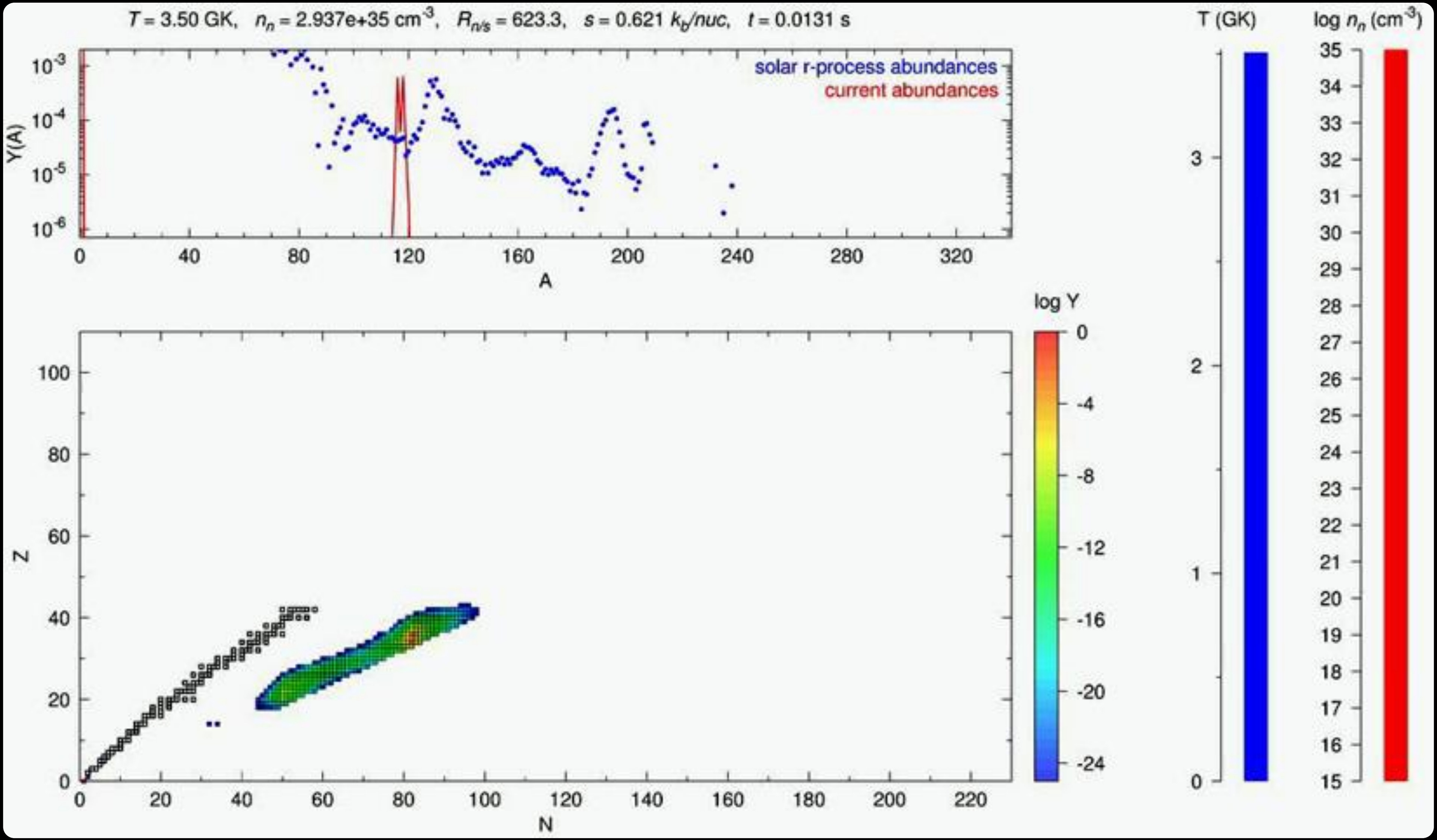
Relevant to gravitational waves, nucleosynthesis, as well as neutron stars

Gravitational waves and gamma rays were detected on Aug. 17, 2017



At the frontier in Physics!!

Simulation of r-process nucleosynthesis during a neutron-star merger



NUCLEAR MASSES AND THEIR IMPACT IN R-PROCESS NUCLEOSYNTHESIS: https://youtu.be/T44B9j3Vzxw

Summary

Nuclear physics - from fundamentals to applications to our life

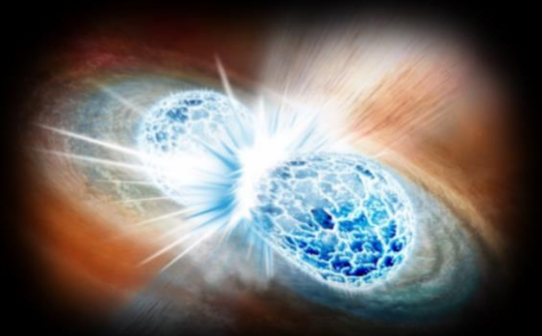
Strong + EM + Weak + Gravity

Neutron stars

n, p, e^-, μ^-
hyperons
meson condensates
quark matter

Accelerator science
Detector systems

Equation of state and structure
Superfluidity/superconductivity
Thermal evolution
Supernova explosions

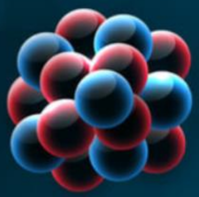


Neutron star merger
Gravitational waves
Origin of the elements

Nuclear security

Strong + EM + Weak

Structure
Reactions
Decays



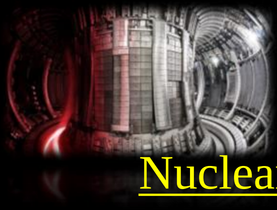
Atomic Nuclei

neutrons and protons

Nuclear force

Physics beyond the Standard Model
$0\nu\beta\beta$, CP violation (EDM, CN resonance),
X11, Axion, Dark matter, ...

JET, Oxfordshire, UK



A reactor in Switzerland

Nuclear power

CNAO, Pavia, Italy

Medical physics

MRI, Hadron therapy,
Nuclear medicine

Radiometric dating

e.g. ^{14}C ($\lesssim$60,000 year)

Strong force

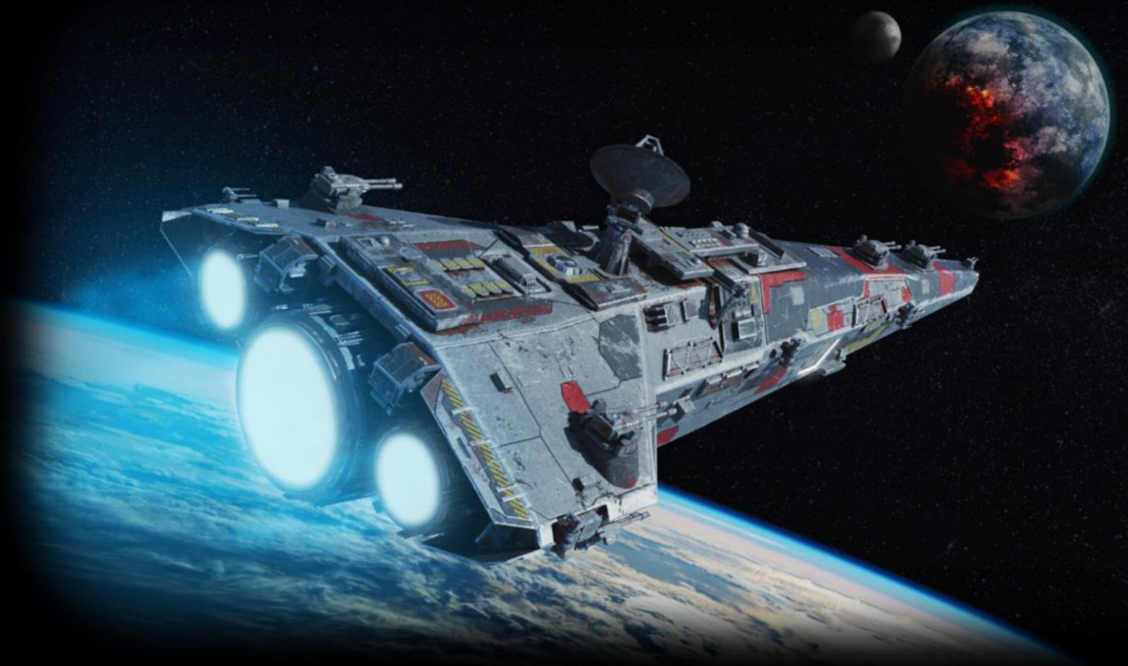
QCD

quarks and gluons

*Each of all pictures has been linked to its source URL.

I hope you enjoyed our exciting adventure!

with a magic of DFT and TDDFT..

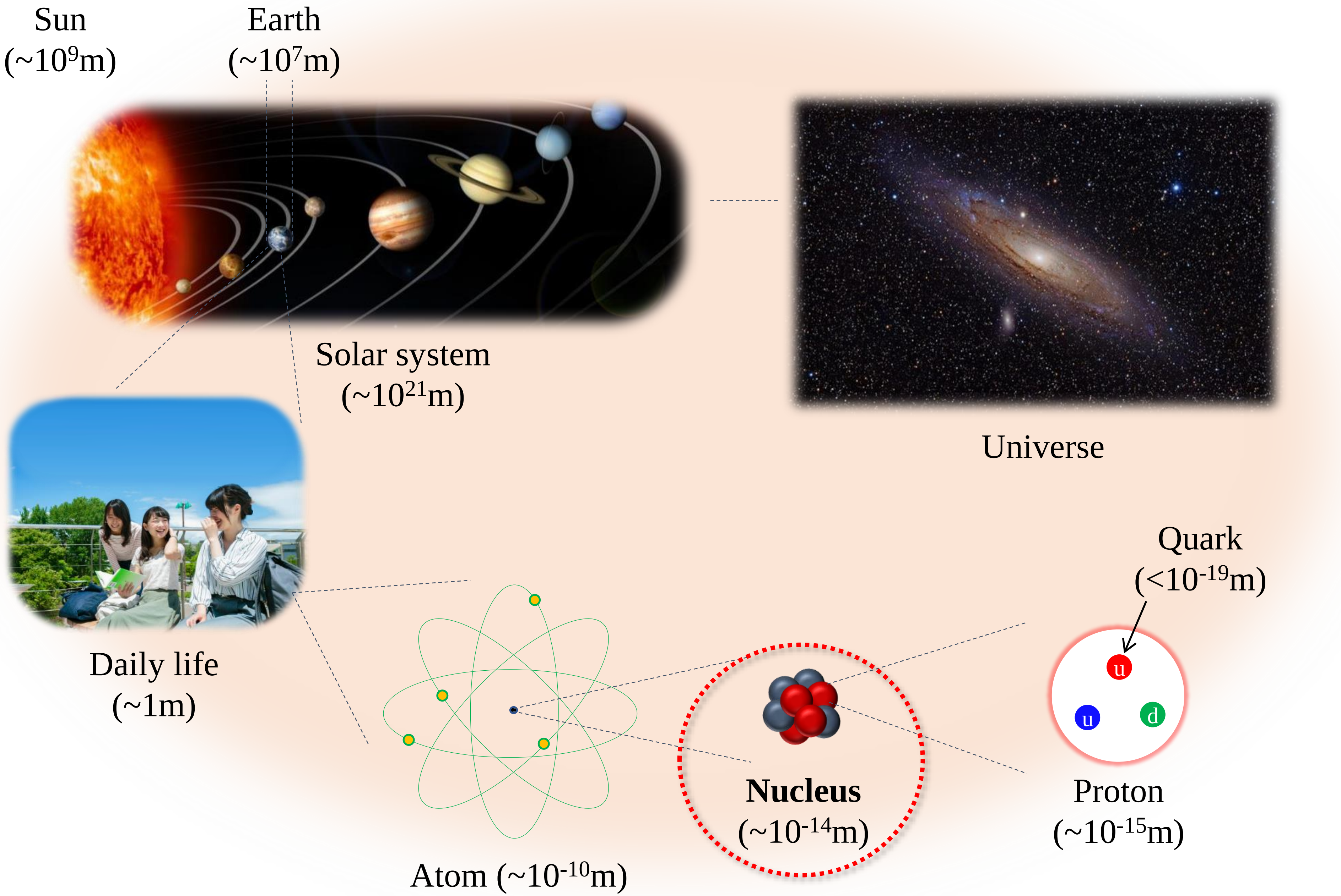


“Flat Earth” by iStock; “Fanciful view of ship sailing over edge of Earth” by Georgia Studies Images; “Star Wars Spaceship” by Valérian Pierret @Artstation

The transdisciplinary character is
one of the fascinating points of Nuclear Physics

What I showed today are only a tiny part of the huge field! :)

Takeaway message



Sun
(~10^{9}m)

Earth
(~10^{7}m)

Solar system
(~10^{21}m)

Universe

Daily life
(~1m)

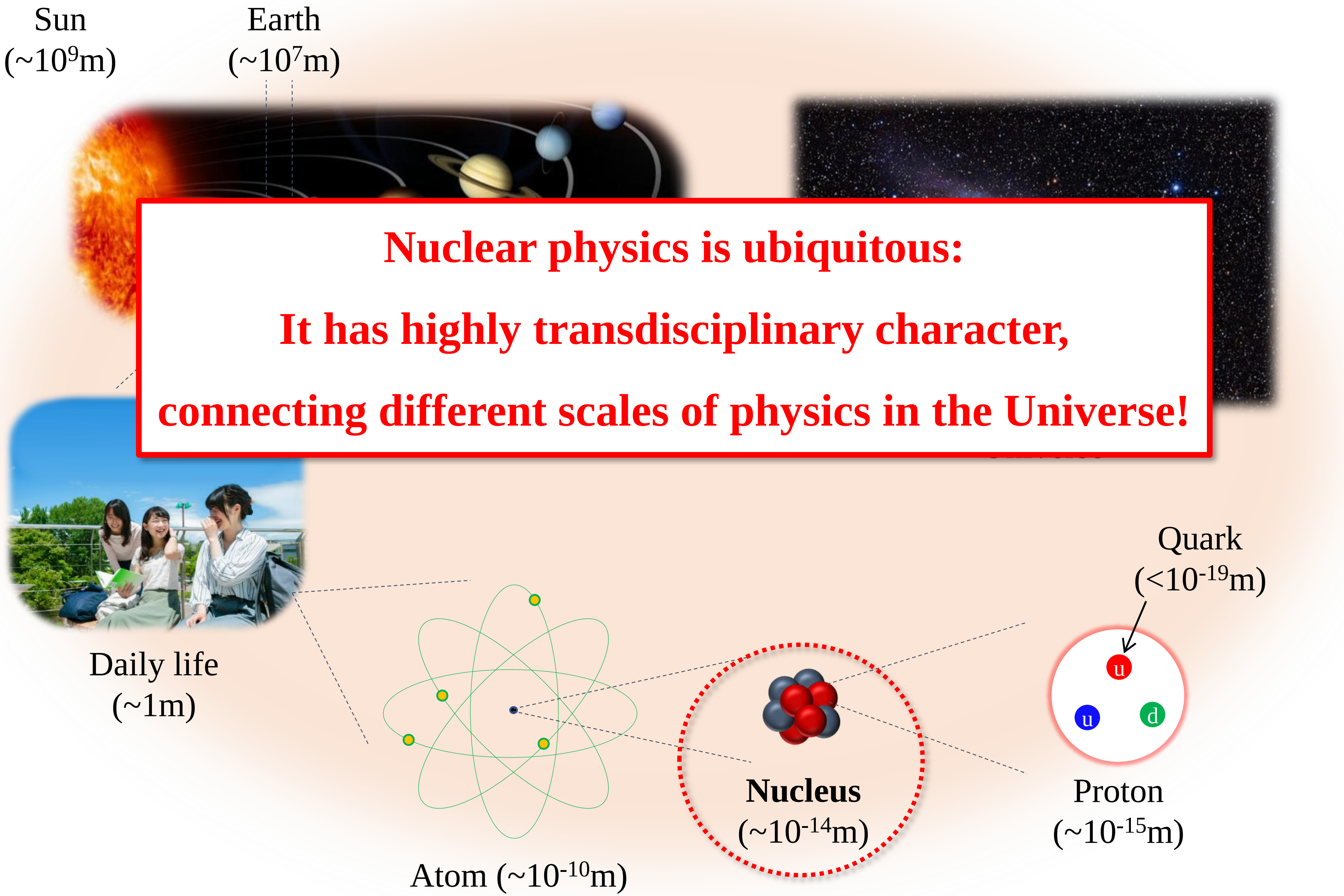
Quark
(<10^{-19}m)

Atom (~10^{-10}m)

Nucleus
(~10^{-14}m)

Proton
(~10^{-15}m)

Takeaway message



Nuclear physics is ubiquitous:

It has highly transdisciplinary character,

connecting different scales of physics in the Universe!

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About us: https://nuclphystitech.wordpress.com/

See also:

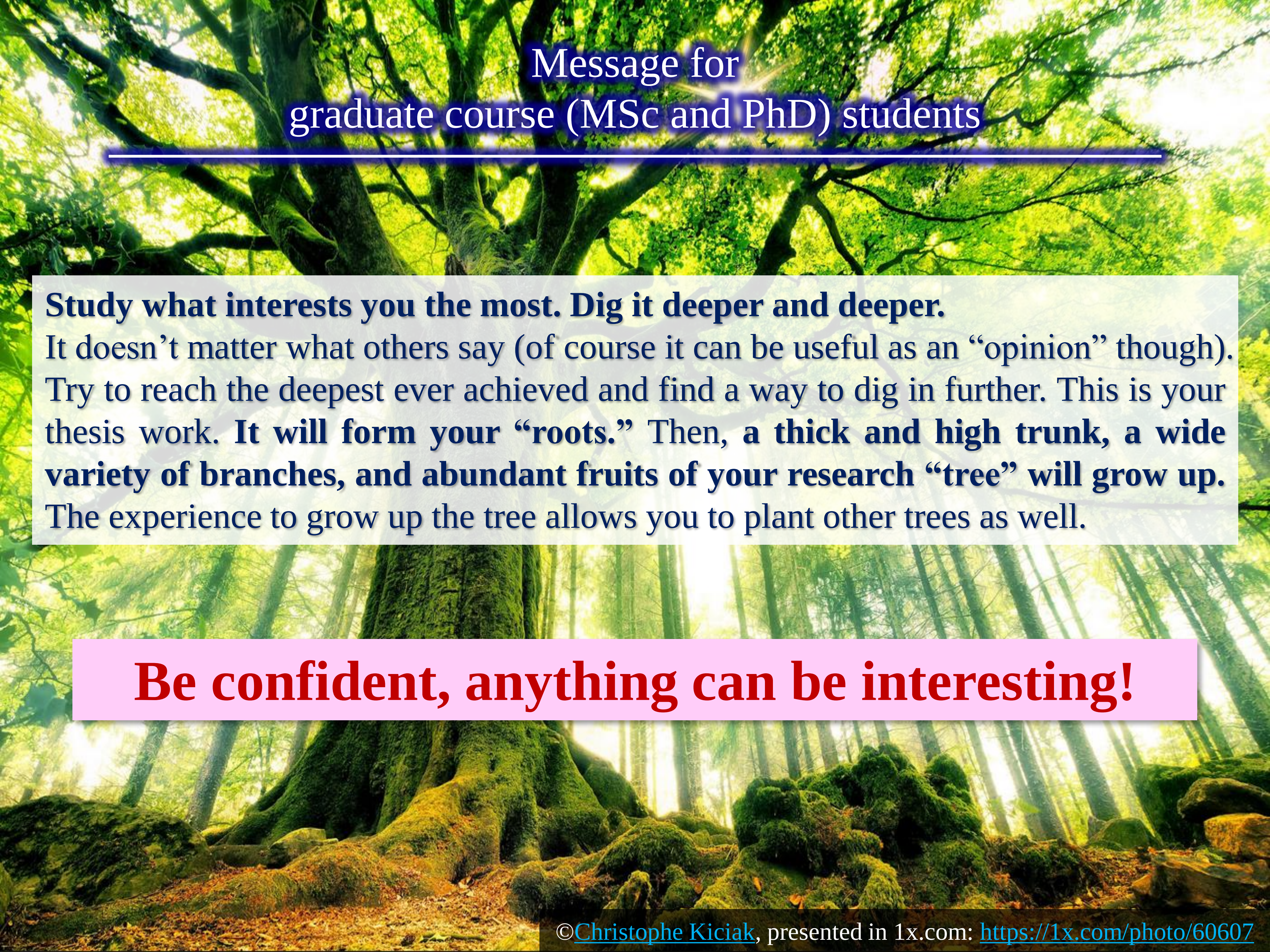
Message for undergraduate course (BSc) students

All the subjects you learn (e.g., classical mechanics, electromagnetism, analytical mechanics, thermodynamics, statistical mechanics, quantum mechanics, etc.) **are indispensable to explore the wonderful world of physics in the universe.** It’s like equipment for climbing. When completed, you’ll see the breathtaking beauty of the nature that may change the rest of your life! :)

Study hard, be ambitious, and have fun!

Photo by Patrick Hendry on Unsplash

Message for
graduate course (MSc and PhD) students



Study what interests you the most. Dig it deeper and deeper.
It doesn’t matter what others say (of course it can be useful as an “opinion” though). Try to reach the deepest ever achieved and find a way to dig in further. This is your thesis work. **It will form your “roots.”** Then, **a thick and high trunk, a wide variety of branches, and abundant fruits of your research “tree” will grow up.** The experience to grow up the tree allows you to plant other trees as well.

Be confident, anything can be interesting!

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