RCNP研究会「低エネルギー核物理と高エネルギー天文学で読み解く中性子星」
2022年8月4日(木) 14:00-14:30 (25+5min), 量子シミュレーション #2

Time-Dependent Density Functional Theory for the Inner Crust of Neutron Stars

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Hierarchy of Scales in the Universe

Cosmic Eye - Universe Size Comparison: https://youtu.be/8Are9dDbW24

Hierarchy of Scales in the Universe

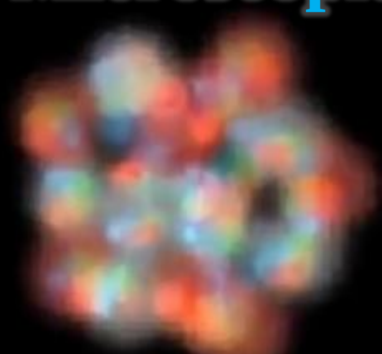
Neutron stars, NS merger, nucleosynthesis, GW, ...

Macroscopic

(Nuclear)Astrophysics

Nuclear structure, Equation of State (EoS) Superfluidity & Superconductivity Reaction rates, Fission fragments, ...	Neutron-star structure, Star quakes, GW Pulsar glitches, Cooling Stellar evolution, Nucleosynthesis, ...

Microscopic



Nuclear Many-Body Problem

↔ 10 fm

Cosmic Eye - Universe Size Comparison: https://youtu.be/8Are9dDbW24

Hierarchy of Scales in the Universe

Neutron stars, NS merger, nucleosynthesis, GW, ...

Macroscopic

(Nuclear)Astrophysics

Nuclear structure,
Superfluid
Reaction ra

Our Mission:

To establish a concrete microscopic foundation of macroscopic models

ture, Star quakes, GW
Cooling
osynthesis, ...

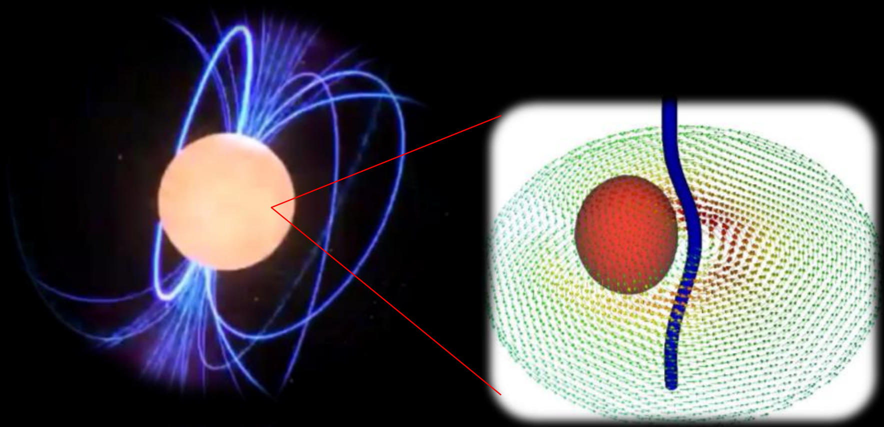
Nuclear Many-Body Problem

10 fm

Cosmic Eye - Universe Size Comparison: https://youtu.be/8Are9dDbW24

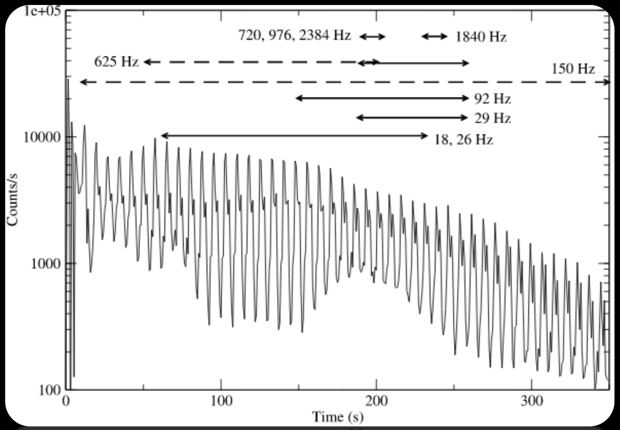
Today, I will talk about:

Dynamics of <u>quantum vortices</u> of superfluid neutrons



⟺ Neutron-star glitches

<u>Time-dependent band theory</u> for the inner crust of neutron stars



⟺ Quasi-periodic oscillations

(TD)DFT in a tiny nutshell

What is DFT?

A theory which gives us access to the *exact* solution

Equivalent!
(for a special EDF)

$$\hat{H}\Psi(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_N) = E\Psi(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_N)$$

Kohn-Sham equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + v_{\mathrm{KS}}[\rho(\boldsymbol{r})]\right]\phi_i(\boldsymbol{r}) = \varepsilon_i\,\phi_i(\boldsymbol{r})$$

EDF — This is the key!

$$v_{\mathrm{KS}}[\rho(\boldsymbol{r})] = \frac{\delta\mathcal{E}[\rho]}{\delta\rho} \qquad \rho(\boldsymbol{r}) = \sum_{i=1}^{N}|\phi_i(\boldsymbol{r})|^2$$

Energy can also be written as a functional of density

$$E[\rho] = \langle\Psi[\rho]|\hat{H}|\Psi[\rho]\rangle$$

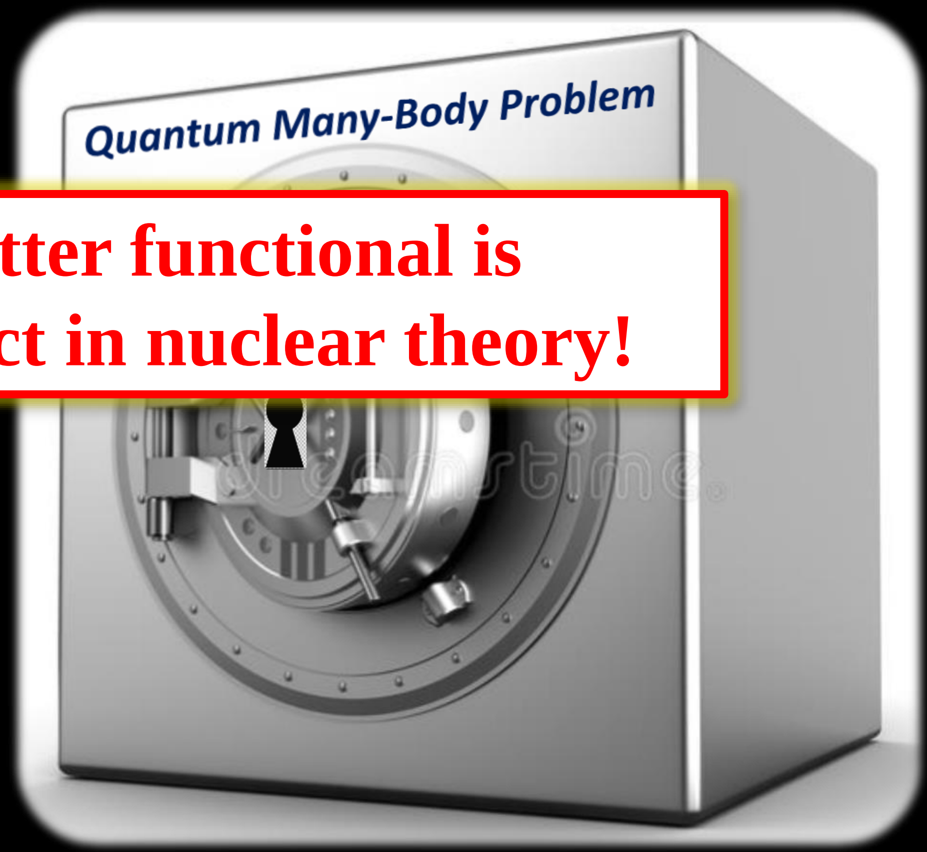
w.f. is a functional of density

P. Hohenberg and W. Kohn, Phys. Rev. B **136**, 864 (1964)

CAUTION!

The existence was proven, but its shape is unknown..

“Inverse Kohn-Sham”

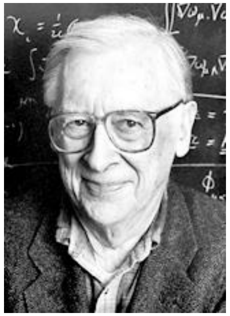


Developing a better functional is an important subject in nuclear theory!

Great Success of the Density Functional Theory

The Nobel Prize in Chemistry 1998

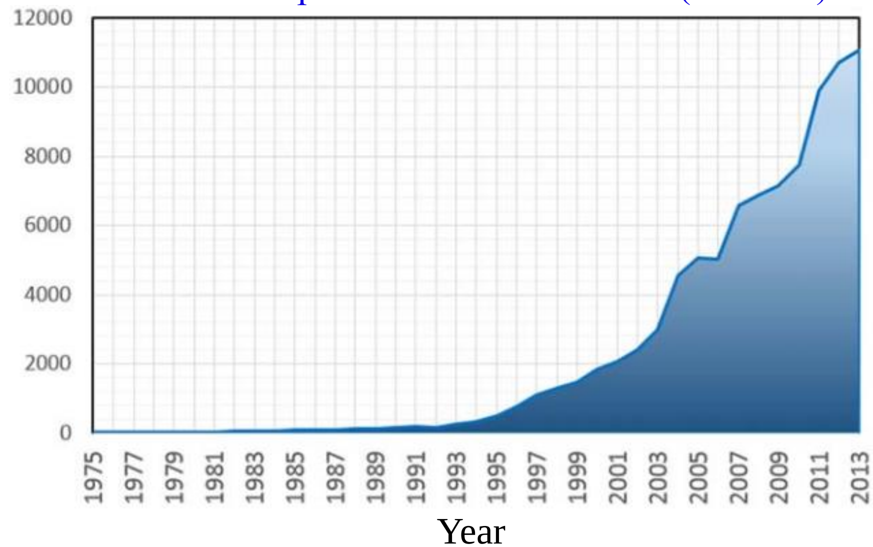
Walter Kohn



John Pople

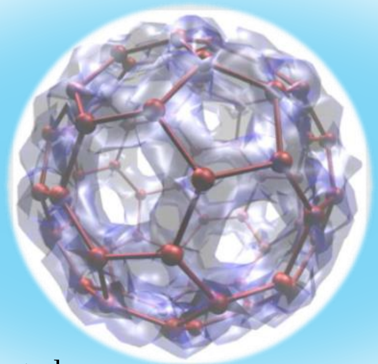
©https://www.nobelprize.org

Number of publications with “DFT” (till 2013)



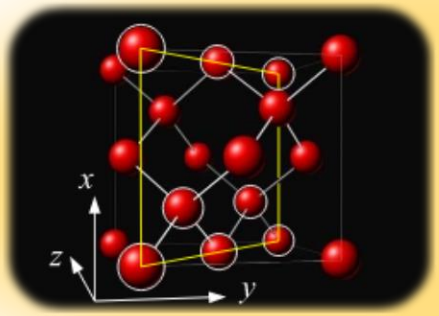
A. Galano and J.R. Alvarez-Idadoy, J. Compt. Chem. **35**, 2019 (2014)

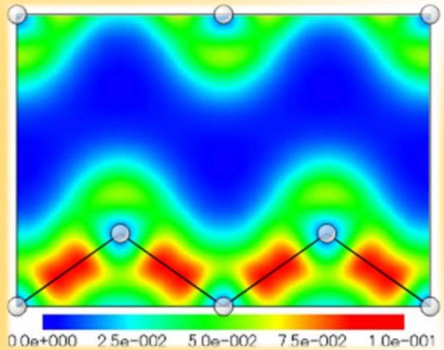
Fullerene: C_{60}



C-Z. Gao et al.,
J. Phys. B: At. Mol. Opt. Phys. **48**, 105102 (2015)

Si crystal

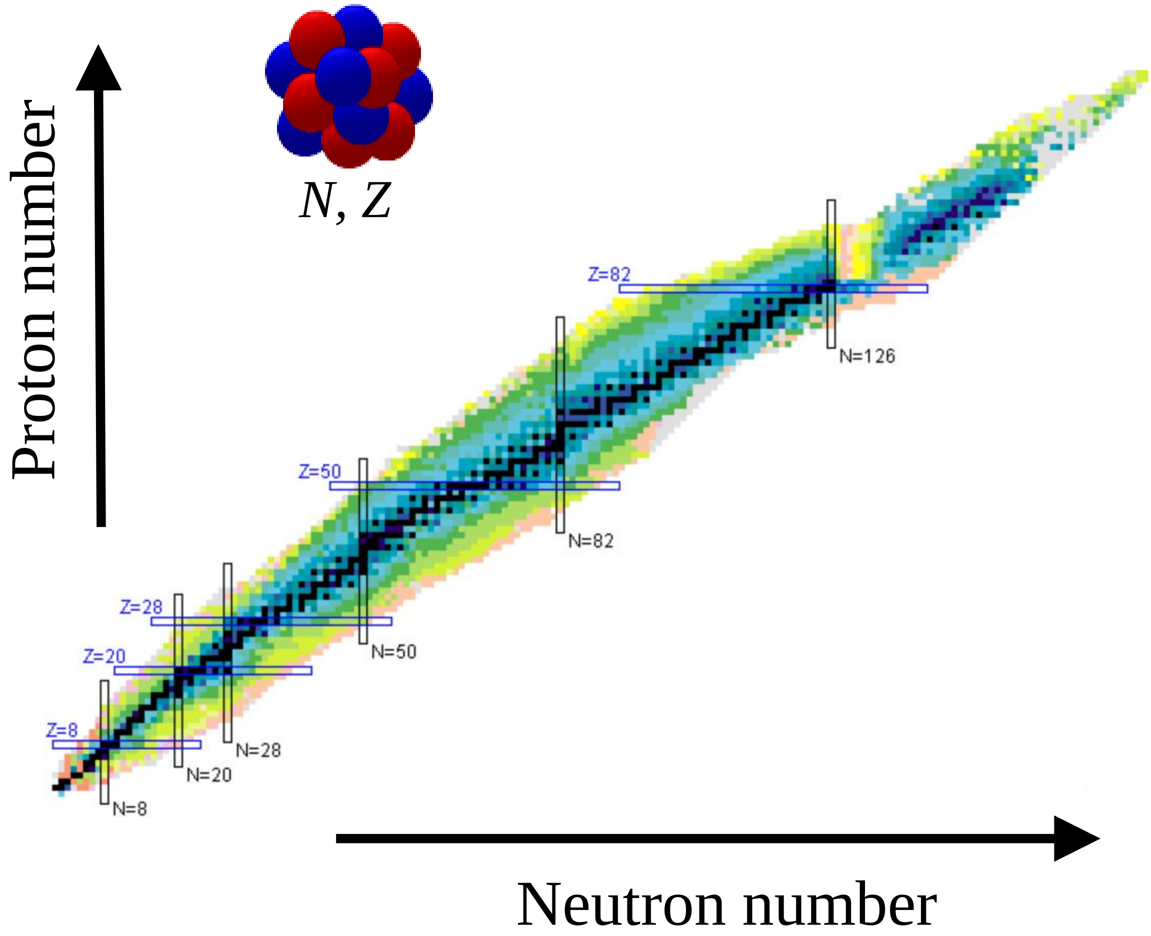




Y. Shinohara, K. Yabana, Y. Kawashita, J.-I. Iwata, T. Otobe, and G. F. Bertsch, Phys. Rev. B **82**, 155110 (2010)

The seminal papers on DFT
- P. Hohenberg and W. Kohn, Phys. Rev. **136**, B864 (1964) ➧ **19,015 citations!**
- W. Kohn and L.J. Sham, Phys. Rev. **140**, A1133 (1965) ➧ **24,384 citations!**

All nuclei can be described with a *single* EDF



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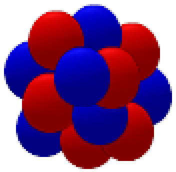
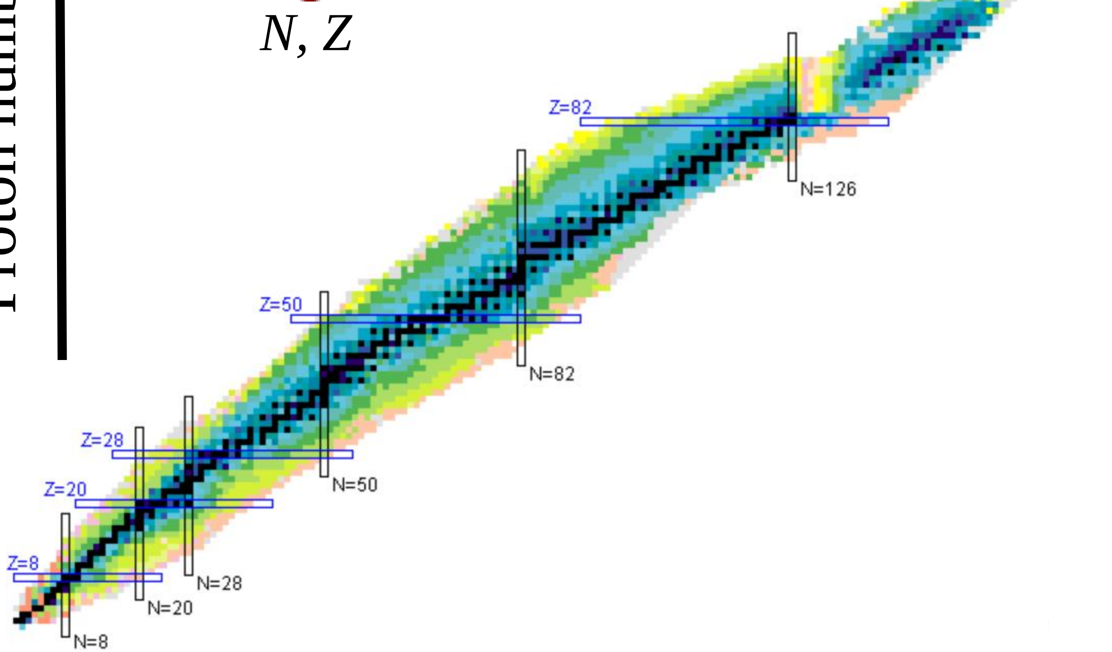
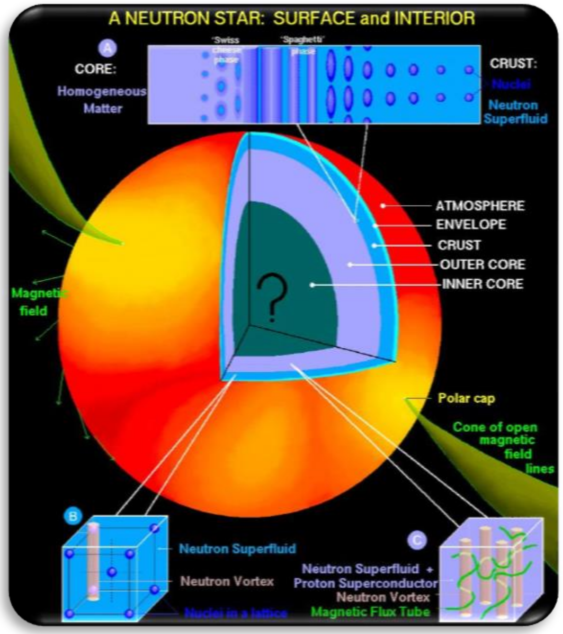


N, Z

Proton number



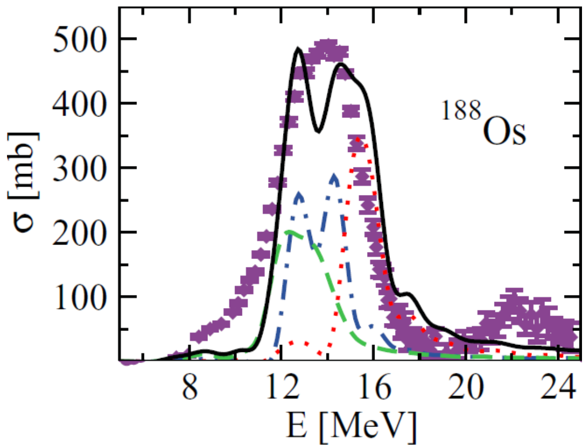
Neutron star



Neutron number

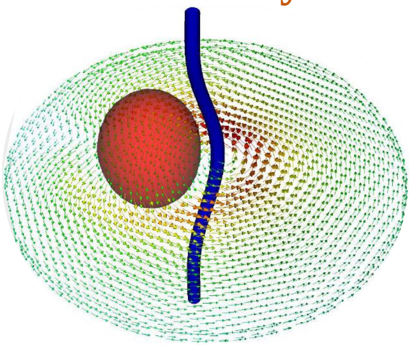
TDDFT is a versatile tool!!

IVGDR



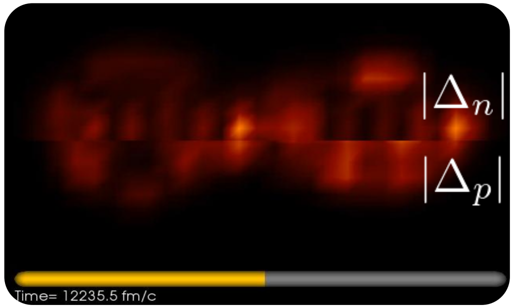
Phys. Rev. C **84**, 051309(R) (2011)
I. Stetcu, A. Bulgac, P. Magierski, and K.J. Roche

Vortex-nucleus dynamics



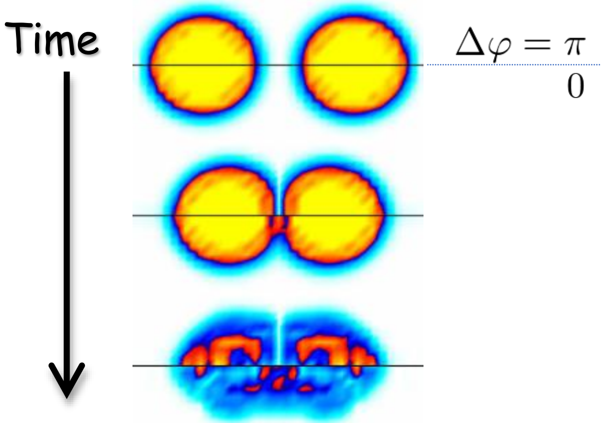
Phys. Rev. Lett. **117**, 232701 (2016)
G. Wlazłowski, K.S., P. Magierski, A. Bulgac, and M.M. Forbes

Induced fission of ^{240}Pu



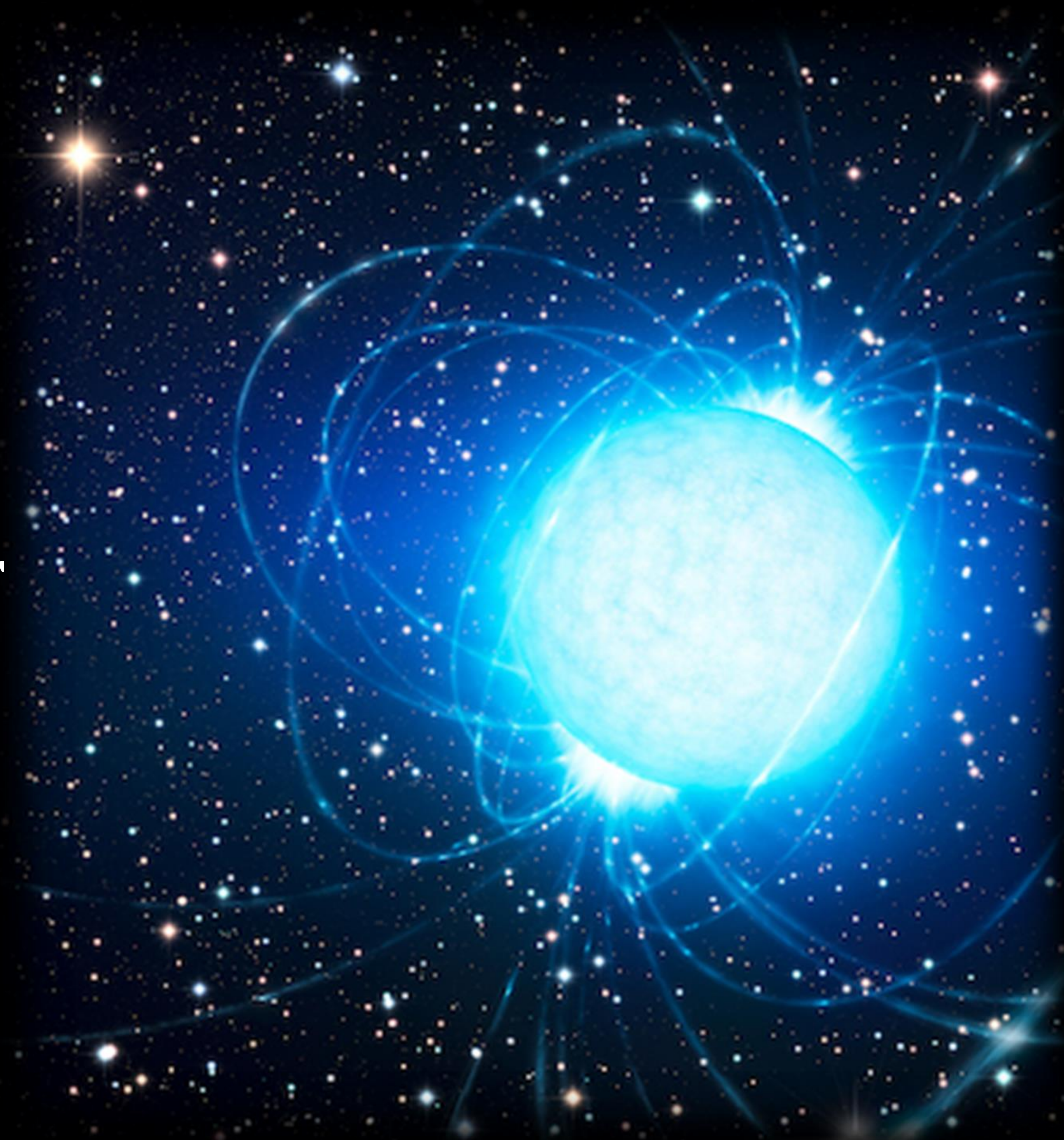
Phys. Rev. Lett. **116**, 122504 (2016)
A. Bulgac, P. Magierski, K.J. Roche, and I. Stetcu

Low-energy heavy-ion reactions



Phys. Rev. Lett. **119**, 042501 (2017)
P. Magierski, K.S., and G. Wlazłowski

Neutron-star “glitch”

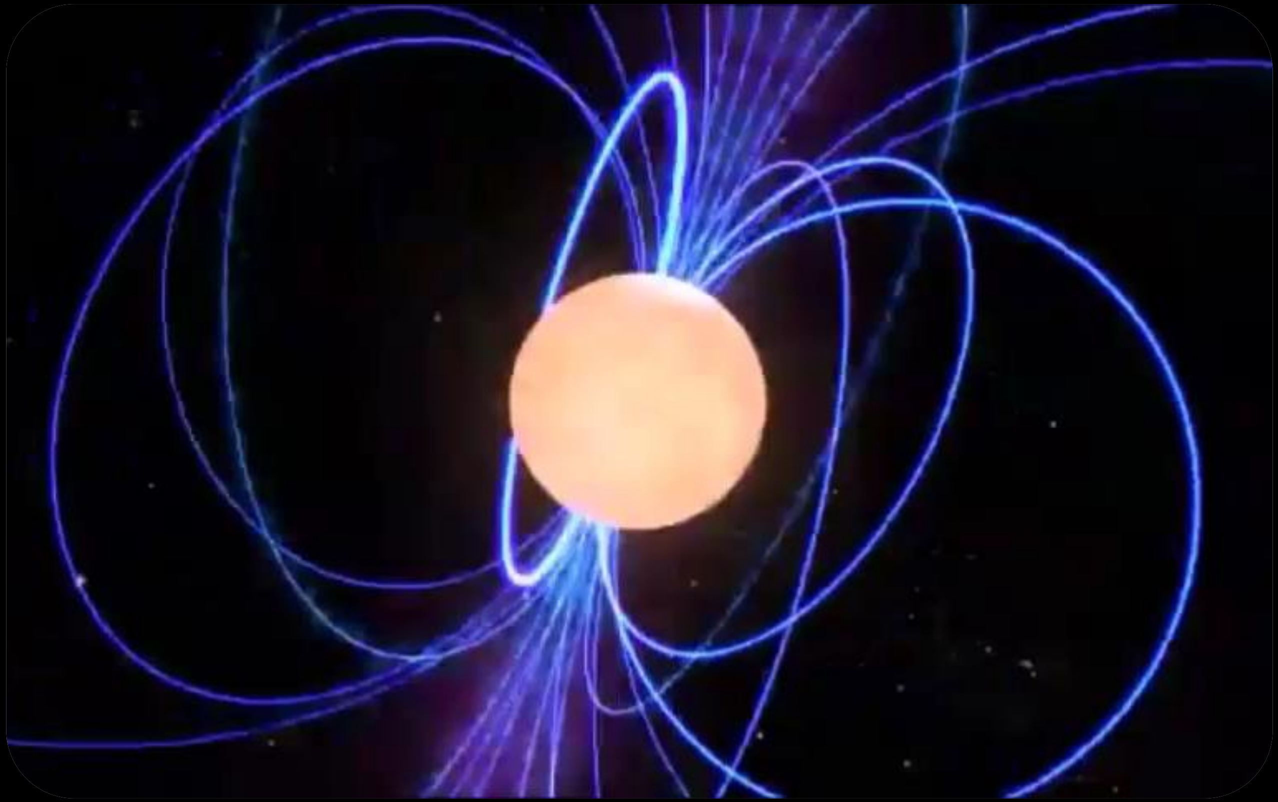


Picture: https://astronomy.com/magazine/ask-astro/2017/12/stellar-magnets

What is the glitch?

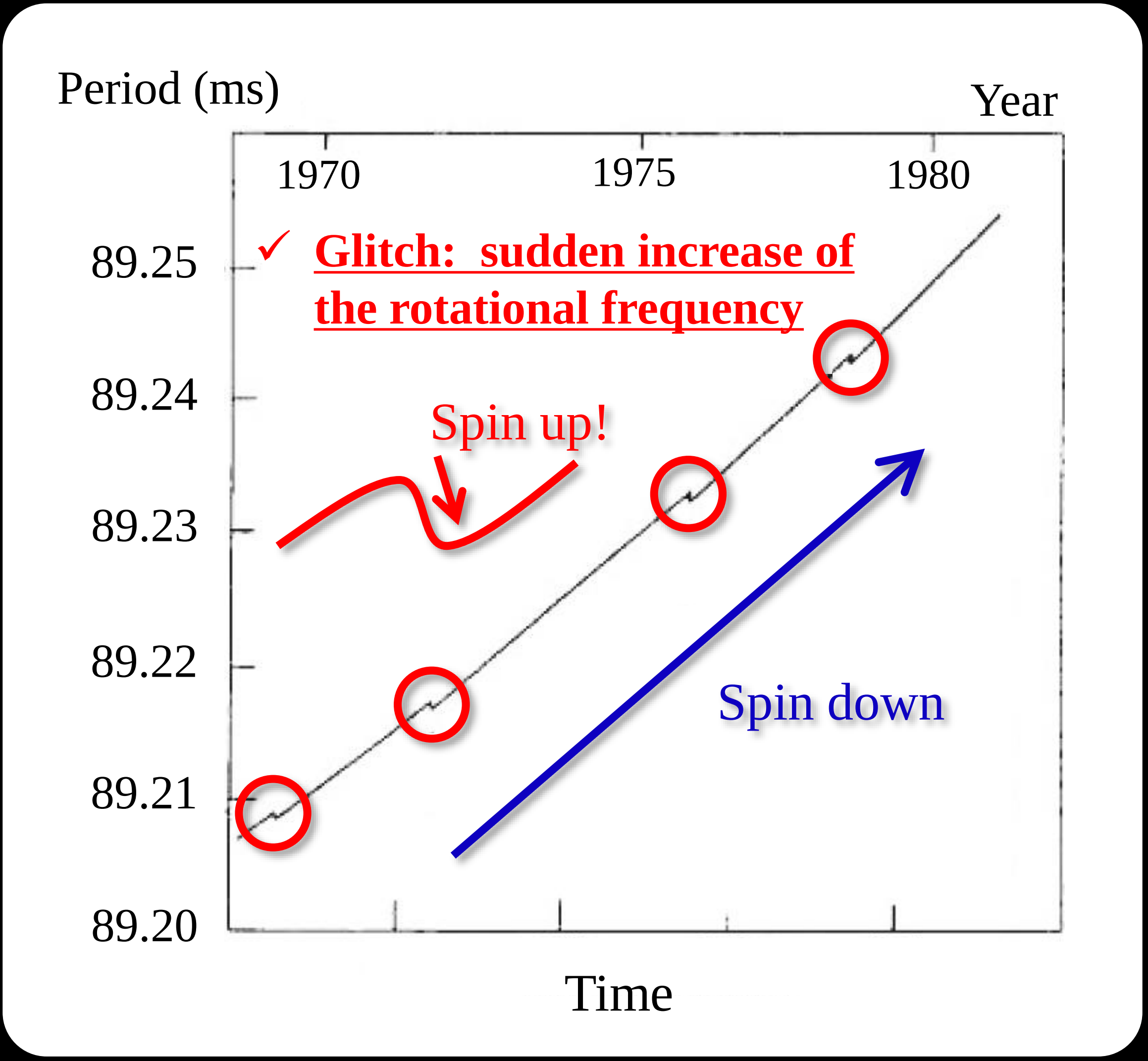
Pulsar - a rotating neutron star

- ✓ First discovery in August 1967 → "Little Green Man" LGM-1 → PSR B1919+21
- ✓ Since then, more than 2650 pulsars have been observed
- ✓ It gradually spins down due to the EM radiation



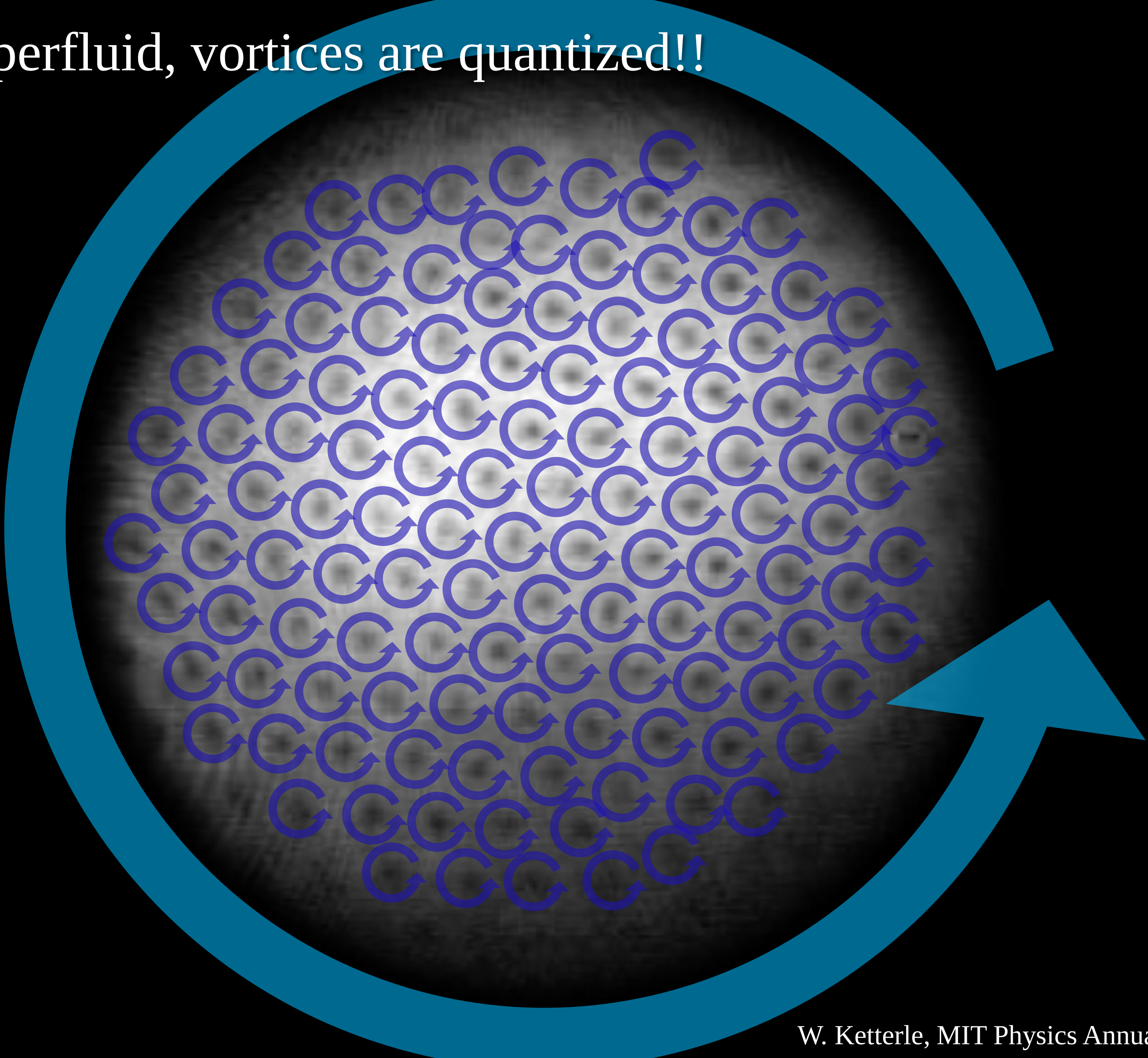
Typical example: the Vela pulsar

- *Irregularity* has been observed from continuous monitoring of the pulsation period



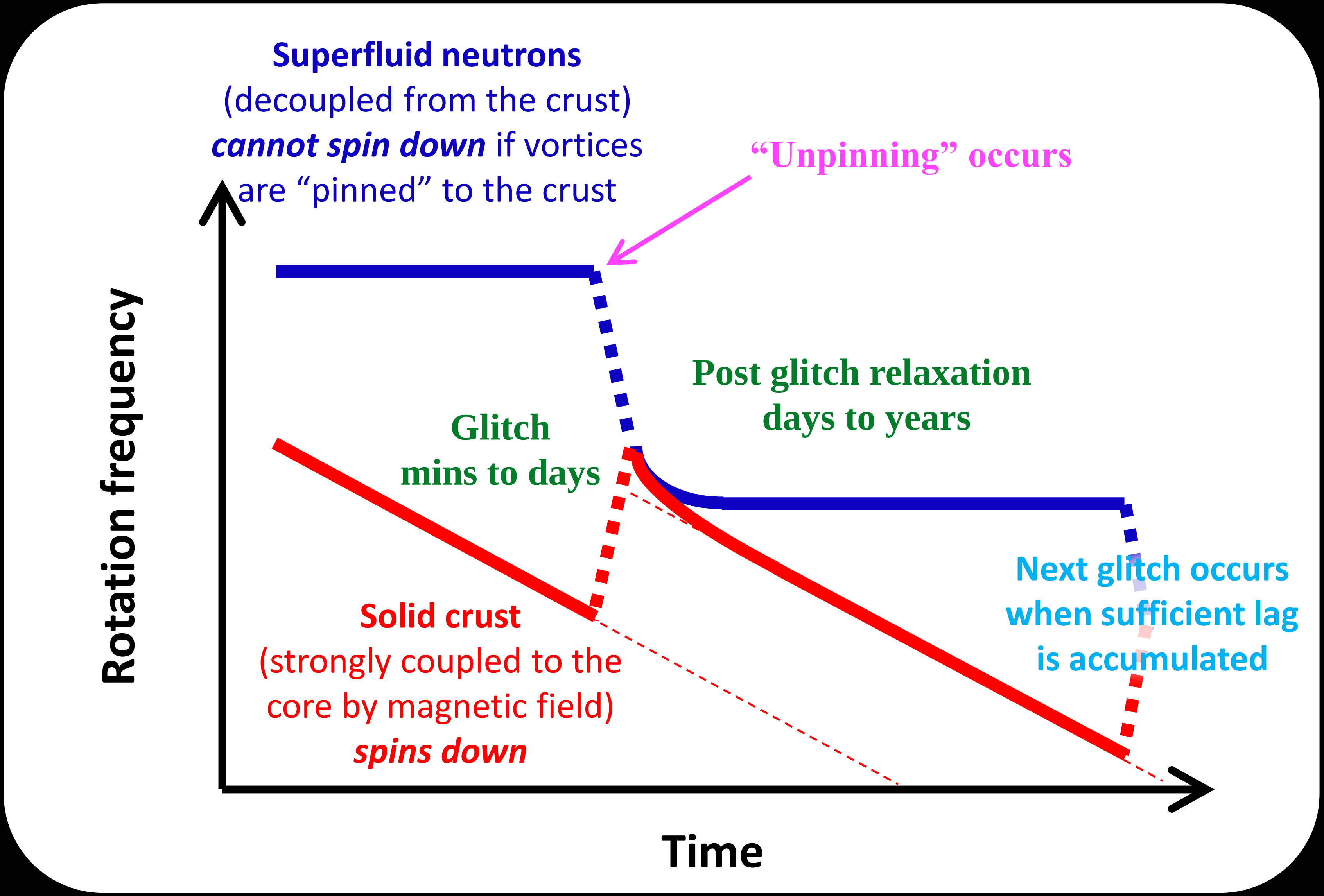
In daily life, a vortex is continuous..

In superfluid, vortices are quantized!!



W. Ketterle, MIT Physics Annual. 2001

The vortex mediated glitch: Naive picture



To fully understand the glitches, we need to clarify:

Glitch dynamics

How do vortices move?

and, of course,
details of NS matter..

Pinning mechanism

How are vortices pinned?

Trigger mechanism

How are vortices unpinned?

We attacked this problem using
the state-of-the-art microscopic nuclear theory

We attack this problem with HPC on GPU supercomputers with TDDFT for superfluid systems, TDSLDA!

TDSLDA: TDDFT with local treatment of pairing

Kohn-Sham scheme is extended for non-interacting quasiparticles

- TDSLDA equations (formally equivalent to TDHFB or TD-BdG equations)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow\uparrow}(\boldsymbol{r},t) & h_{\uparrow\downarrow}(\boldsymbol{r},t) & 0 & \Delta(\boldsymbol{r},t) \\ h_{\downarrow\uparrow}(\boldsymbol{r},t) & h_{\downarrow\downarrow}(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & -h^*_{\uparrow\uparrow}(\boldsymbol{r},t) & -h^*_{\uparrow\downarrow}(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & 0 & -h^*_{\downarrow\uparrow}(\boldsymbol{r},t) & -h^*_{\downarrow\downarrow}(\boldsymbol{r},t) \end{pmatrix} \begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix}$$

$$h_\sigma = \frac{\delta E}{\delta n_\sigma}$$: s.p. Hamiltonian

$$\Delta = -\frac{\delta E}{\delta \nu^*}$$: pairing field

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_k<E_c} |v_{k,\sigma}(\boldsymbol{r},t)|^2$$: number density

$$\nu(\boldsymbol{r},t) = \sum_{E_k<E_c} u_{k,\uparrow}(\boldsymbol{r},t)v^*_{k,\downarrow}(\boldsymbol{r},t)$$: anomalous density

$$\boldsymbol{j}_\sigma(\boldsymbol{r},t) = \hbar\sum_{E_k<E_c} \mathrm{Im}[v^*_{k,\sigma}(\boldsymbol{r},t)\boldsymbol{\nabla}v_{k,\sigma}(\boldsymbol{r},t)]$$: current

A large number (10^4-10^6) of 3D coupled non-linear PDEs have to be solved!!

# of qp orbitals ~ # of grid points

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Supercomputing!!

$$h_\sigma = \frac{\delta E}{\delta n_\sigma}$$: s.p. Hamiltonian

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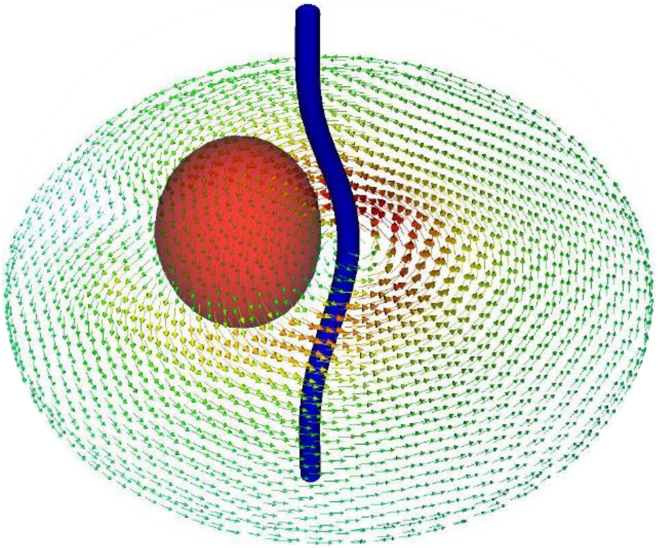
Vortex-nucleus dynamics within TDSLDA



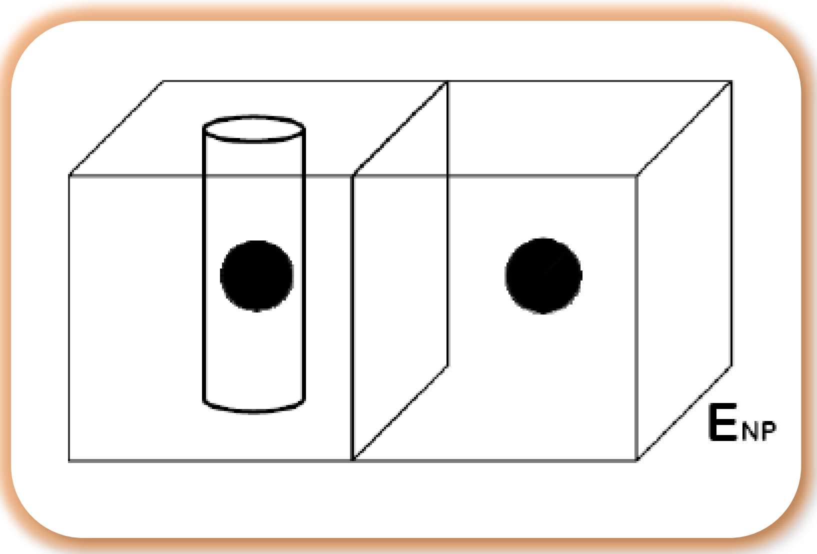
G. Wlazłowski, K. Sekizawa, P. Magierski, A. Bulgac, and M.M. Forbes,
Phys. Rev. Lett. **117**, 232701 (2016)

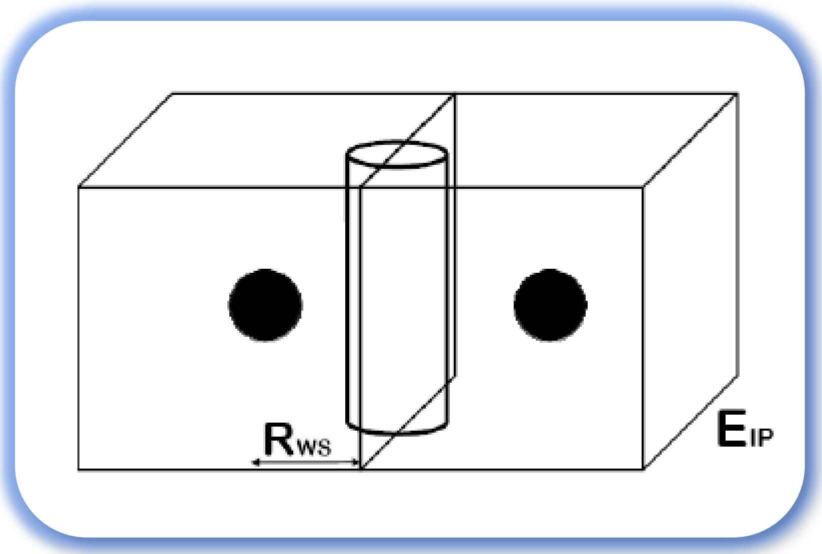
A key to understand the glitches is:

Vortex pinning mechanism in the inner crust of neutron stars

Q. Is the vortex-nucleus interaction

Attractive? or Repulsive?

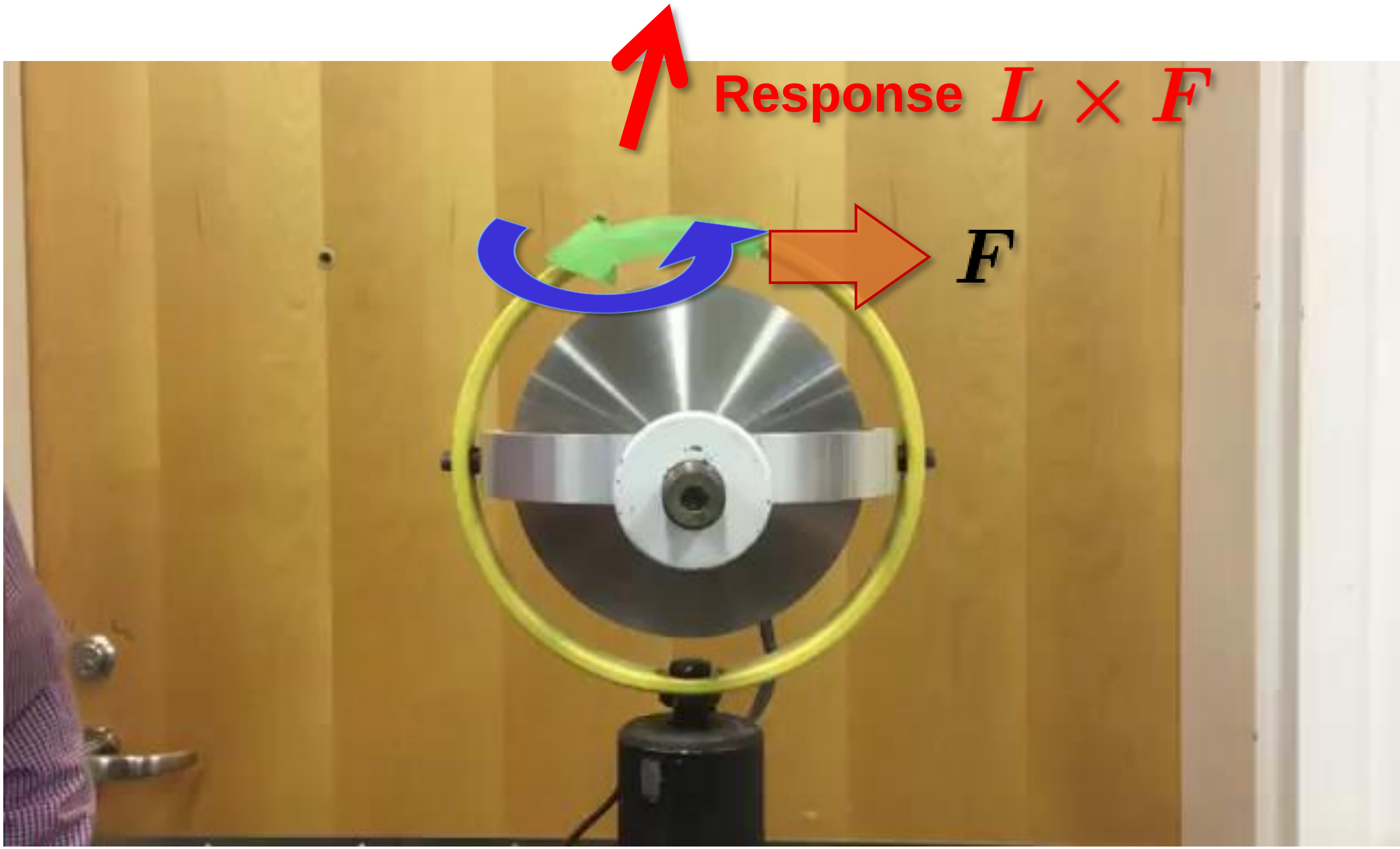




"Nuclear pinning"

"Interstitial pinning"

Response of a spinning gyroscope when pushed



We performed 3D, dynamical simulations by TDDFT with superfluidity

TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}u_i(\mathbf{r})\\ v_i(\mathbf{r})\end{pmatrix} = \begin{pmatrix}h(\mathbf{r}) & \Delta(\mathbf{r})\\ \Delta^*(\mathbf{r}) & -h(\mathbf{r})\end{pmatrix}\begin{pmatrix}u_i(\mathbf{r})\\ v_i(\mathbf{r})\end{pmatrix}$$

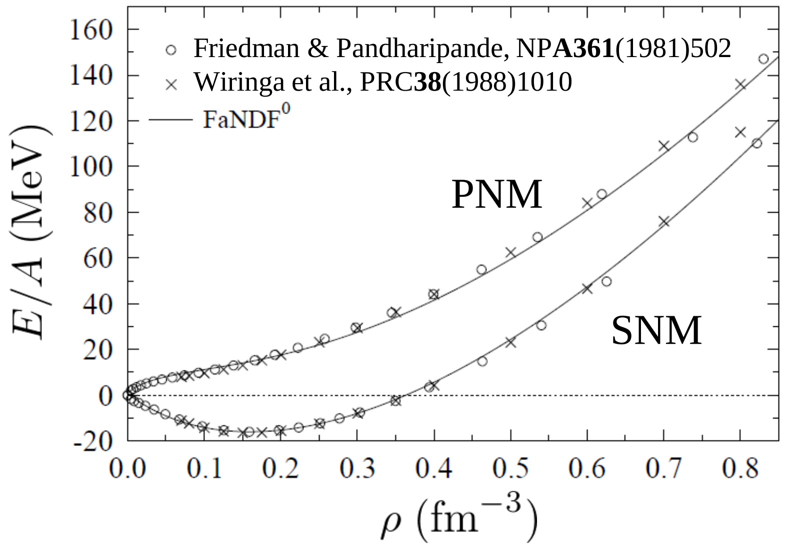
Energy density functional (EDF)

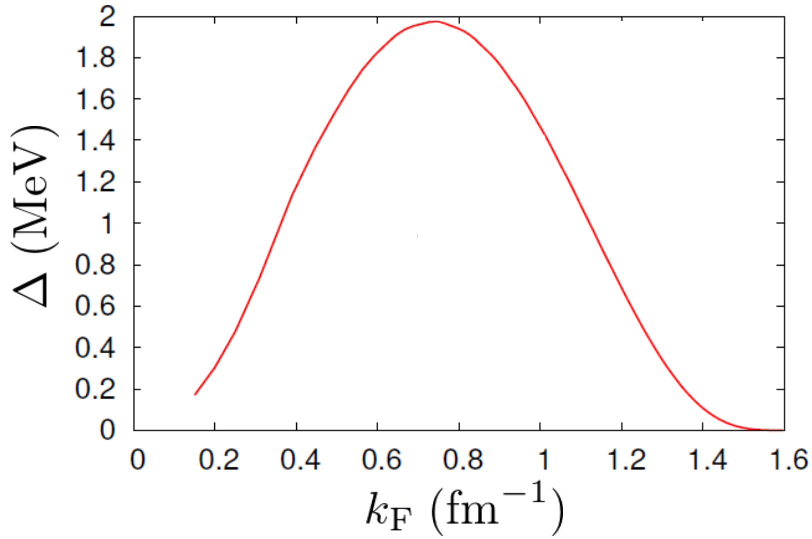
$$\mathcal{E}(\mathbf{r}) = \mathcal{E}_0(\mathbf{r}) + \mathcal{E}_{\rm pair}(\mathbf{r})$$

$\mathcal{E}_0(\mathbf{r})$: Fayans EDF (FaNDF^0) w/o LS

$$\mathcal{E}(\boldsymbol{r}) = \sum_{q=n,p} g[\rho_q(\boldsymbol{r})]|\nu_q(\boldsymbol{r})|^2$$

S.A. Fayans, JETP Lett. **68**, 169 (1998)





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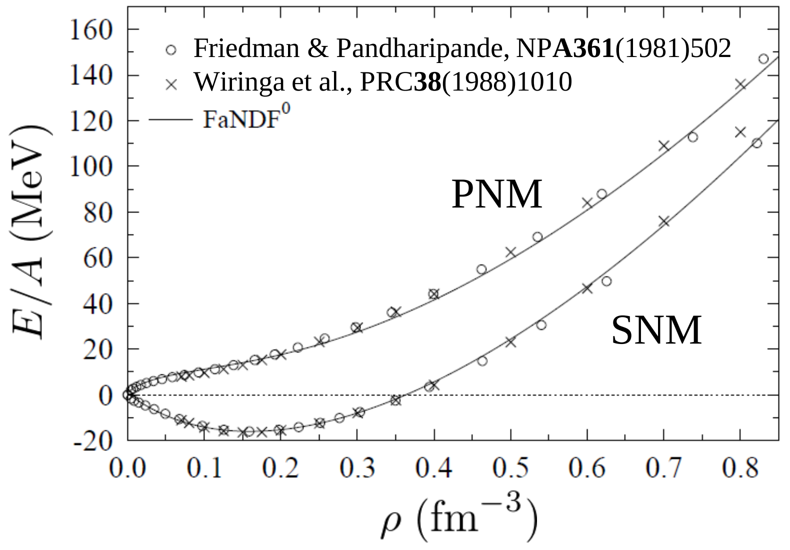

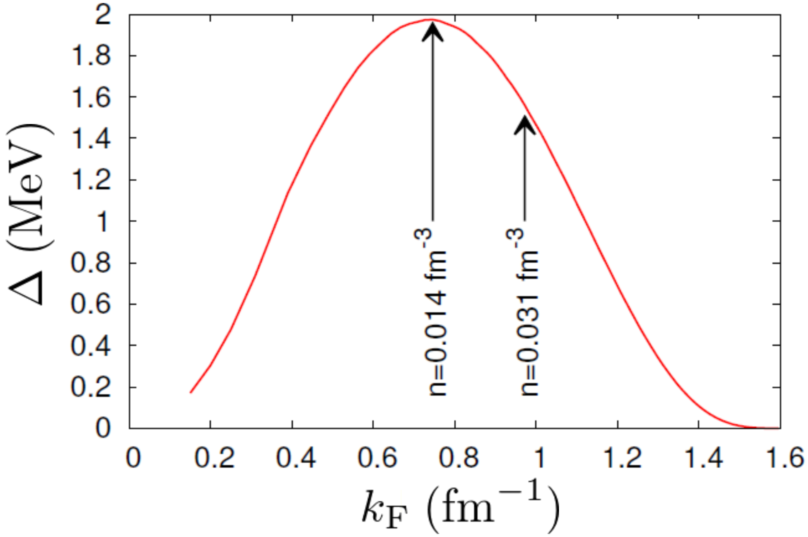

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Computational details

$75\text{ fm} \times 75\text{ fm} \times 60\text{ fm}$

$(50 \times 50 \times 40,\ \Delta x = 1.5\text{ fm})$

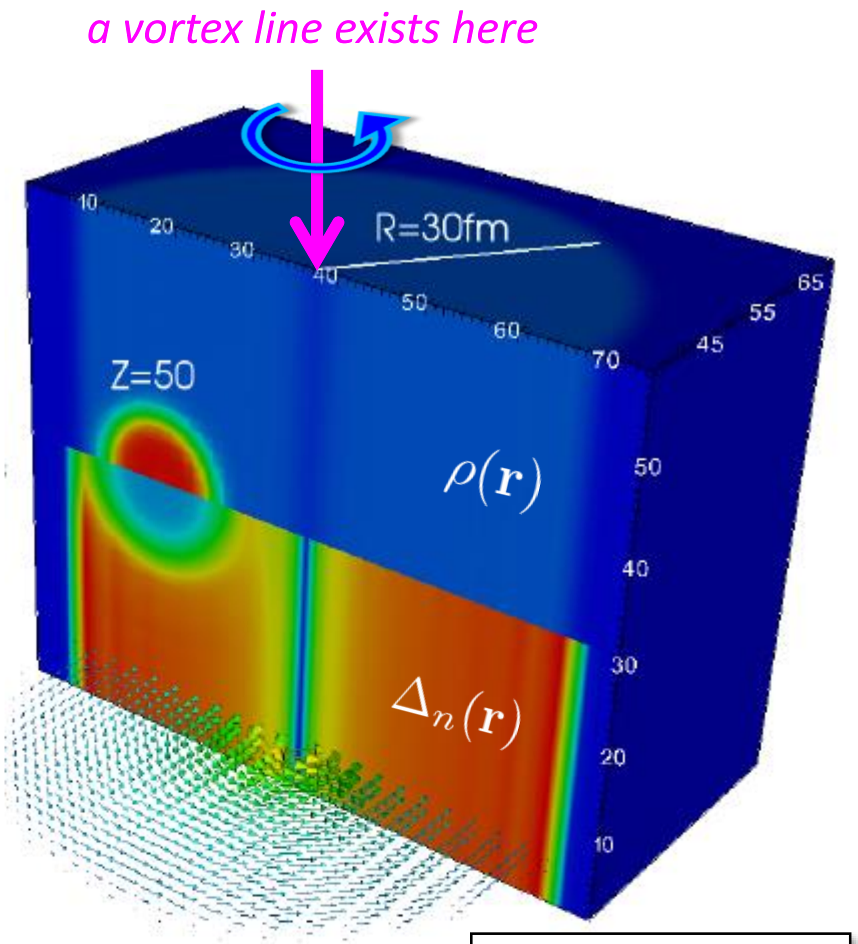
$k_c = \pi/\Delta x > k_F \qquad k_F = (3\pi^2\rho_n)^{1/3}$

Nuclear impurity: $Z = 50$

$\rho_n \simeq 0.014\text{ fm}^{-3}$ $(N \simeq 2{,}530)$

$\rho_n \simeq 0.031\text{ fm}^{-3}$ $(N \simeq 5{,}714)$

# of quasi-particle w.f. $\approx 100{,}000$



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Method - TDSLDA

We performed 3D, dynamical simulations by TDDFT with superfluidity

☐ TDSLDA equations (or TDHFB, TD-BdG)

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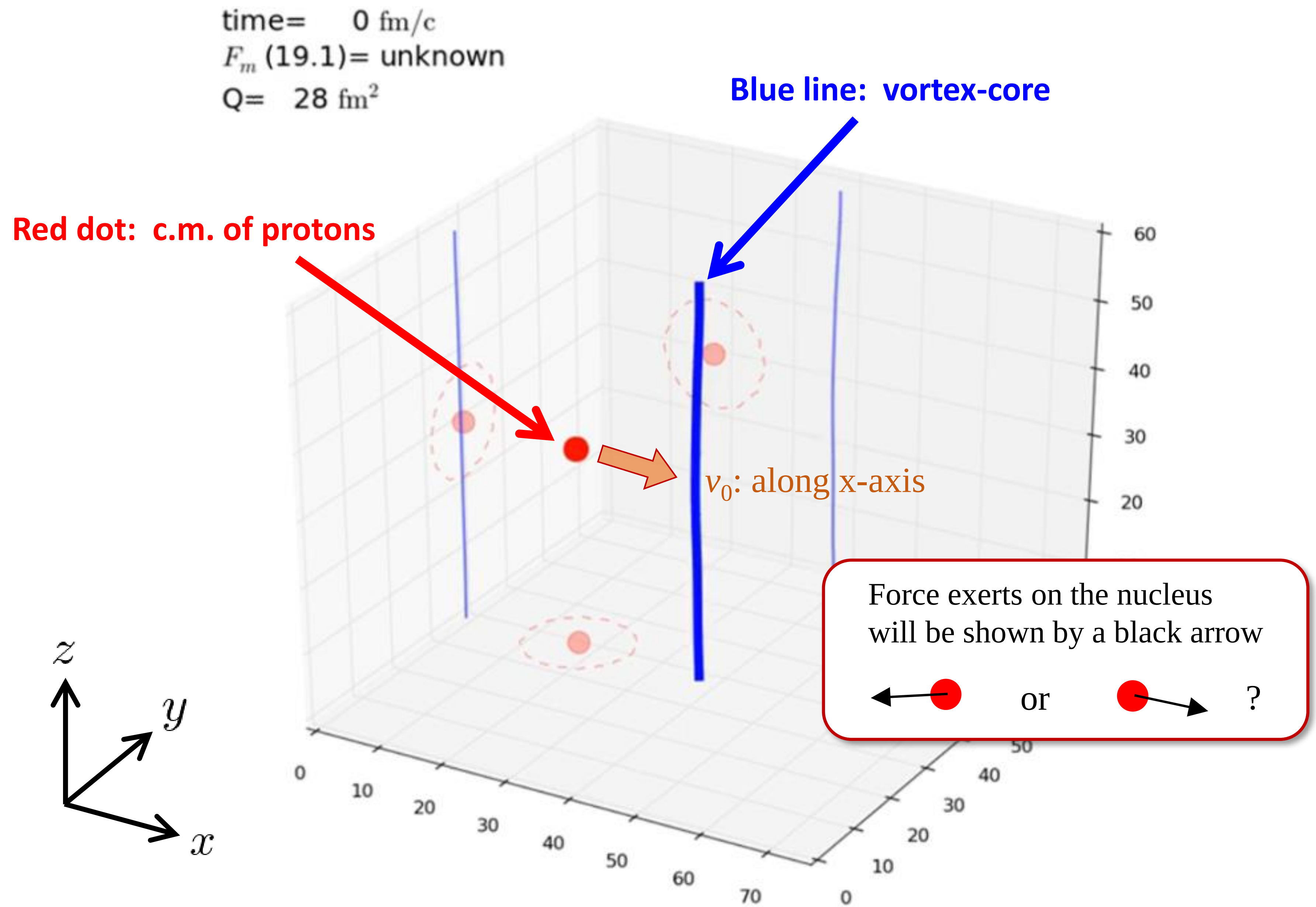
MPI+GPU
→ 48h w/ 200GPUs
for 10,000 fm/c

TITAN, Oak Ridge

NERSC Edison, Berkeley

HA-PACS, Tsukuba

Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$



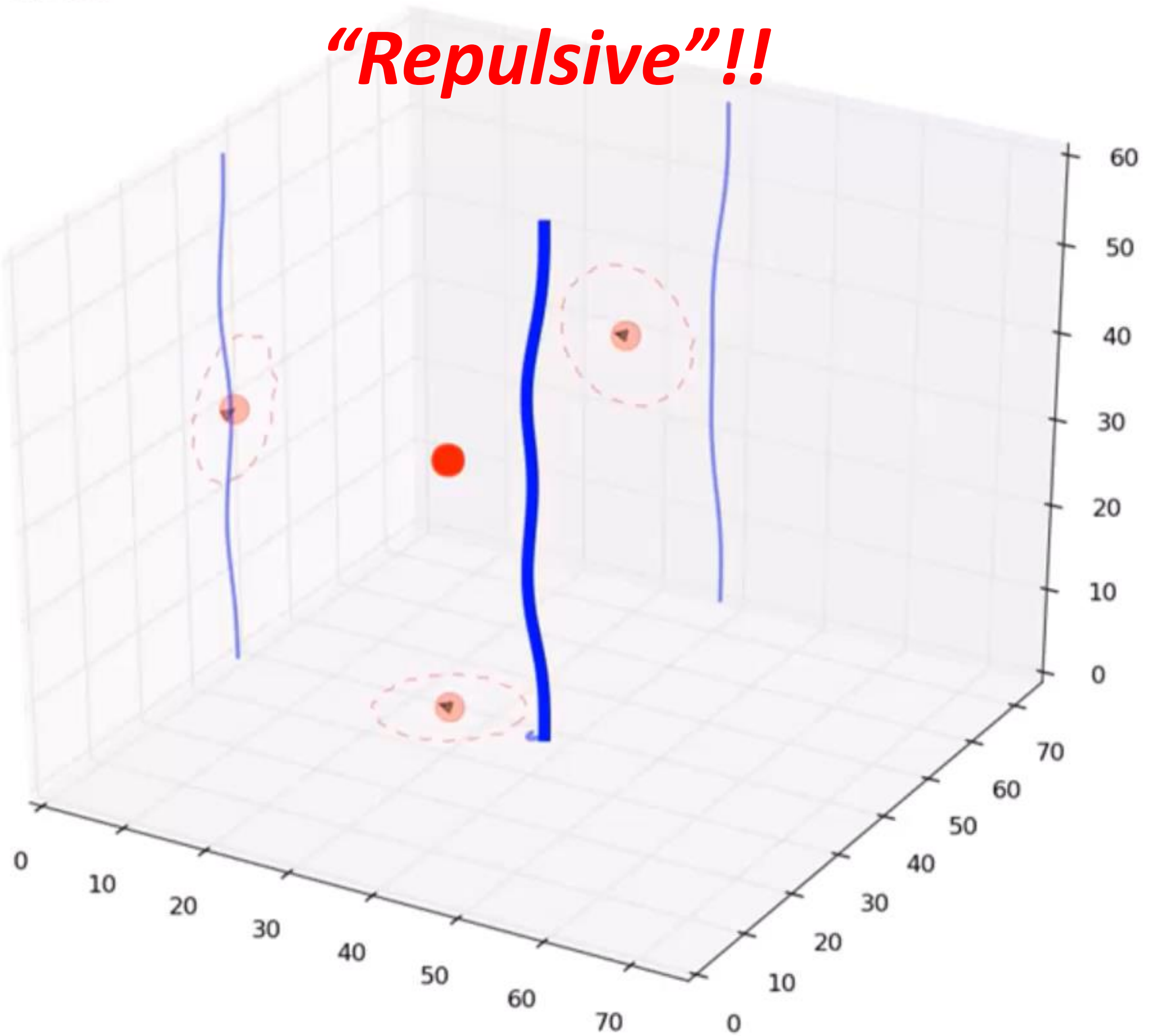
Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$

time= 8032 fm/c

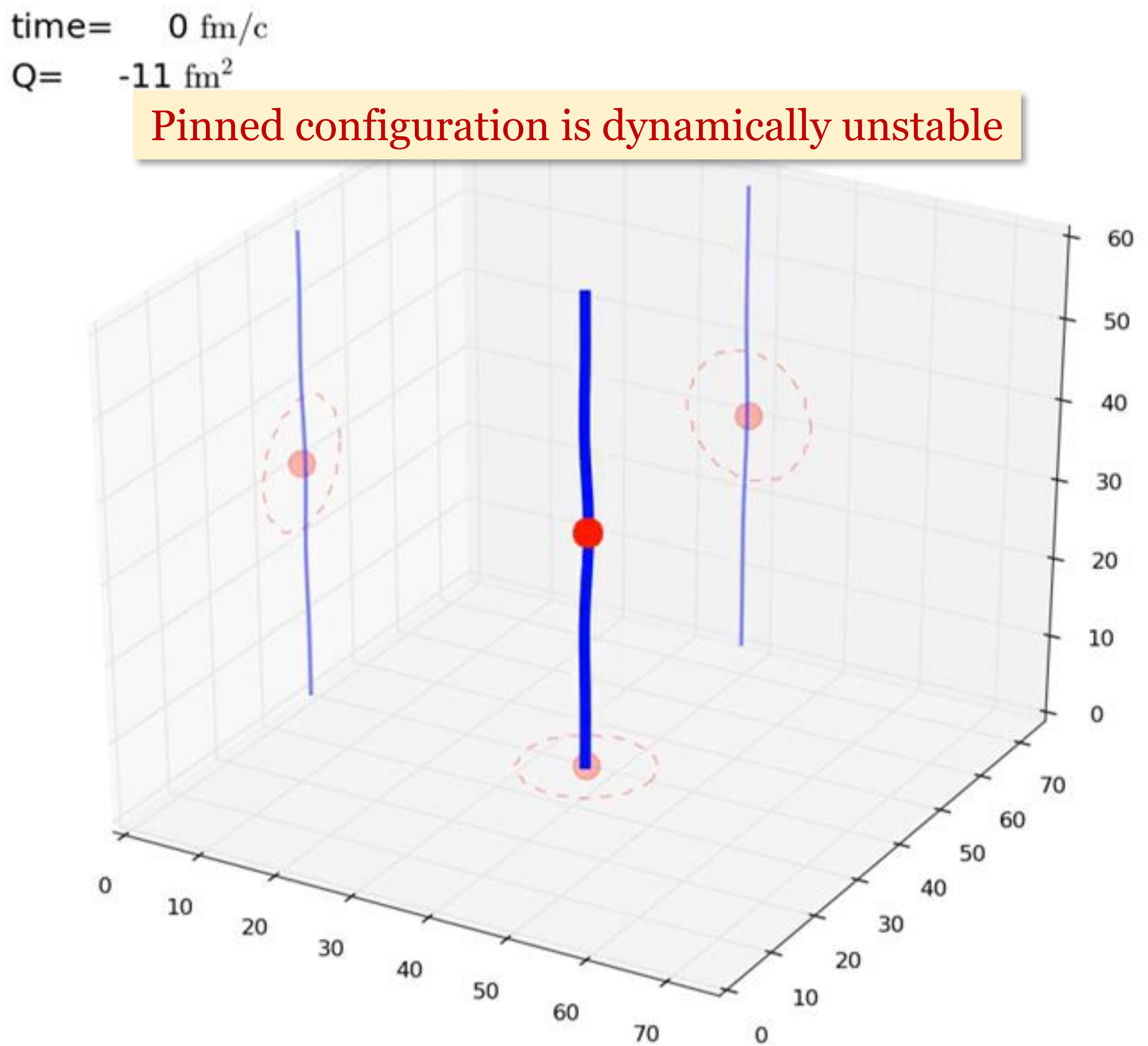
F_m (10.6)= 0.17 MeV/fm

Q= 13 fm^2

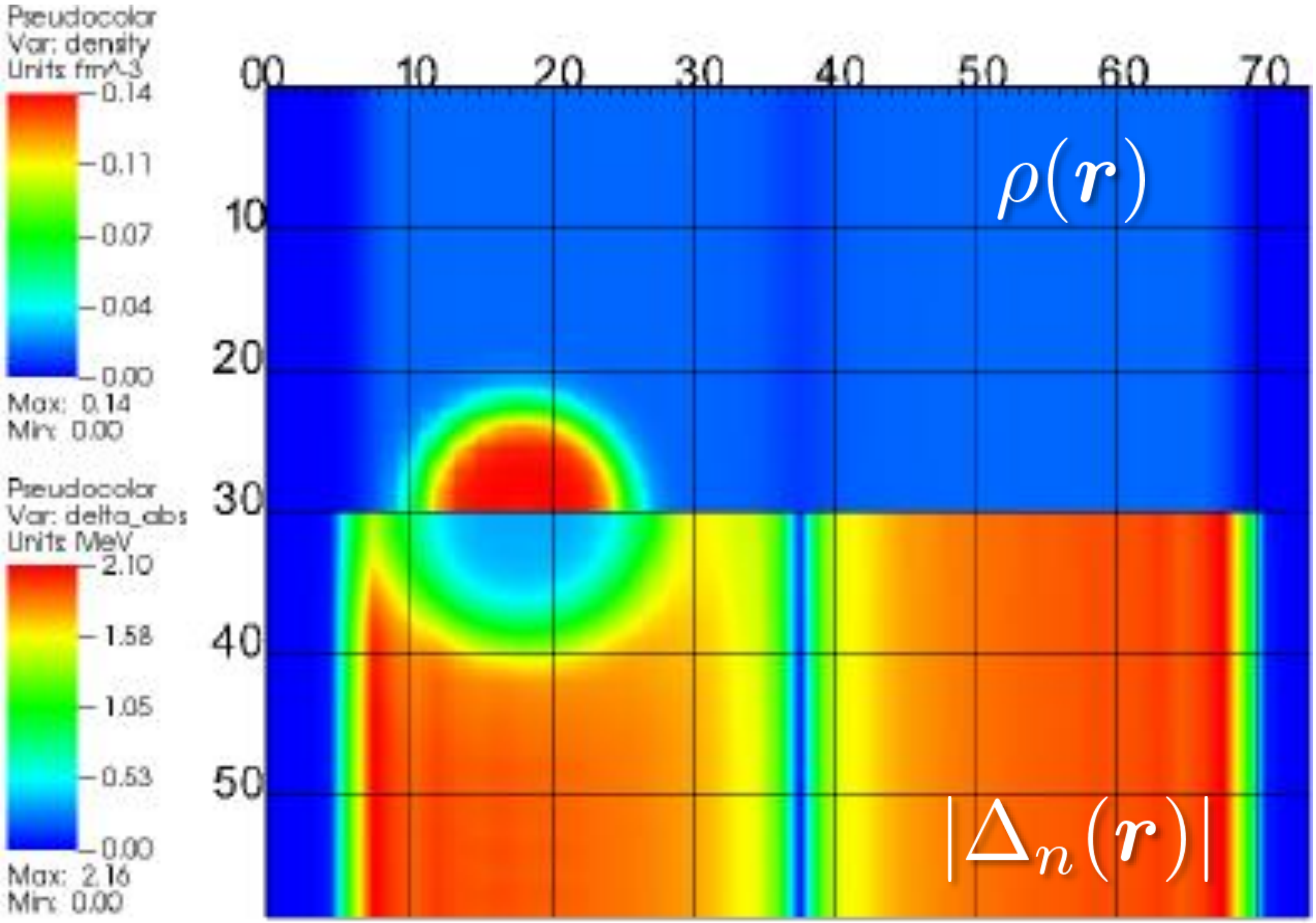
"Repulsive"!!



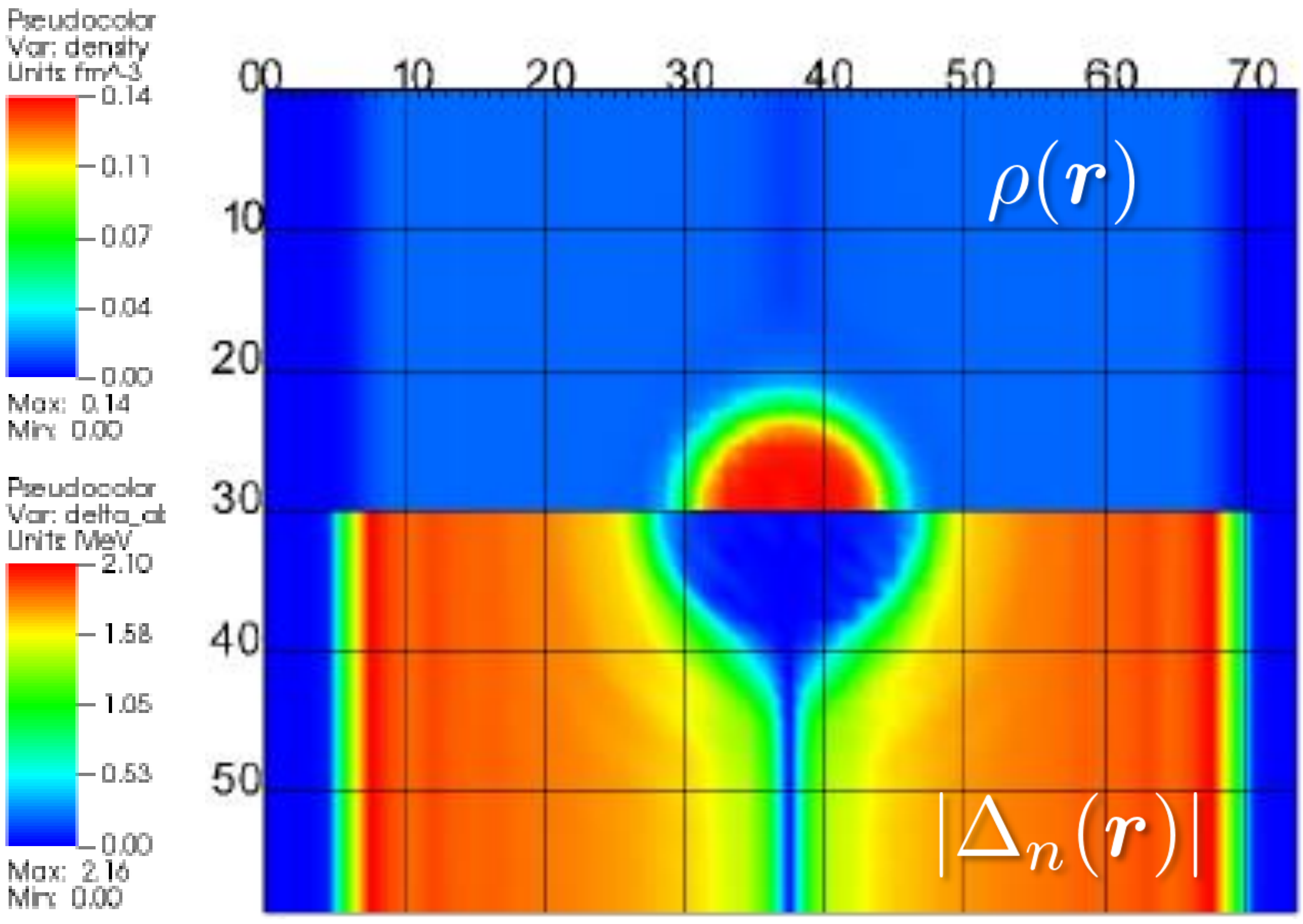
Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$



"Unpinned configuration"

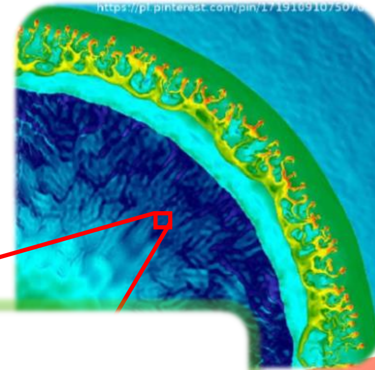


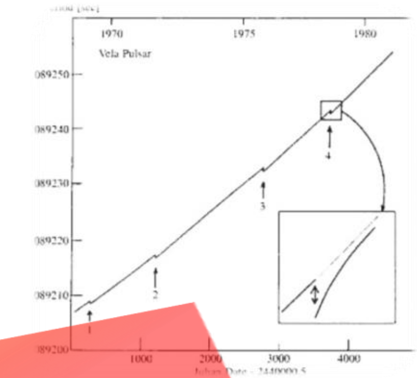
"Pinned configuration"



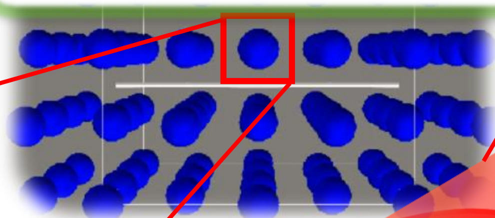
Our goal and strategy

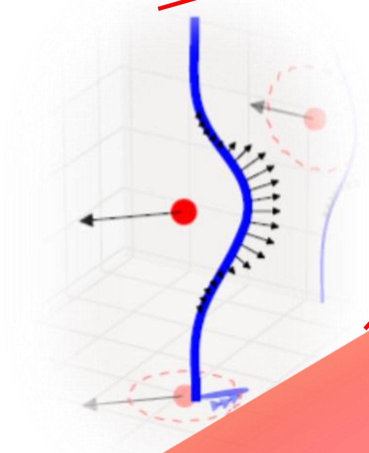
Goal: Unveil the mechanism of glitches





New collaboration started:
Nicolaus Copernicus Astronomical Centre
B. Haskell et al.





10^{4}m

Macroscopic

- observations
- hydrodynamics

~10^{-10}m

Mesoscopic

- dynamics of *vortices* in a lattice of *nuclei* (e.g. filament model)

Provide model ingredients

10^{-15}-10^{-13}m

Microscopic

Nuclear Physics!!

- vortex-nucleus dynamics from *neutrons and protons*

Time-Dependent Band Theory for the Inner Crust of Neutron Stars

https://www.newsweek.com/nuclear-pasta-neutron-star-strongest-material-universe-1127491

The rest of the talk is based on one of my most recent publications:

PHYSICAL REVIEW C **105**, 045807 (2022)

Time-dependent extension of the self-consistent band theory for neutron star matter: Anti-entrainment effects in the slab phase

Kazuyuki Sekizawa,[1,2,*] Sorataka Kobayashi,[3] and Masayuki Matsuo[4,†]

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[2]*Nuclear Physics Division, Center for Computational Sciences, University of Tsukuba, Ibaraki 305-8577, Japan*
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(Received 28 December 2021; accepted 4 April 2022; published 25 April 2022)

in collaboration with

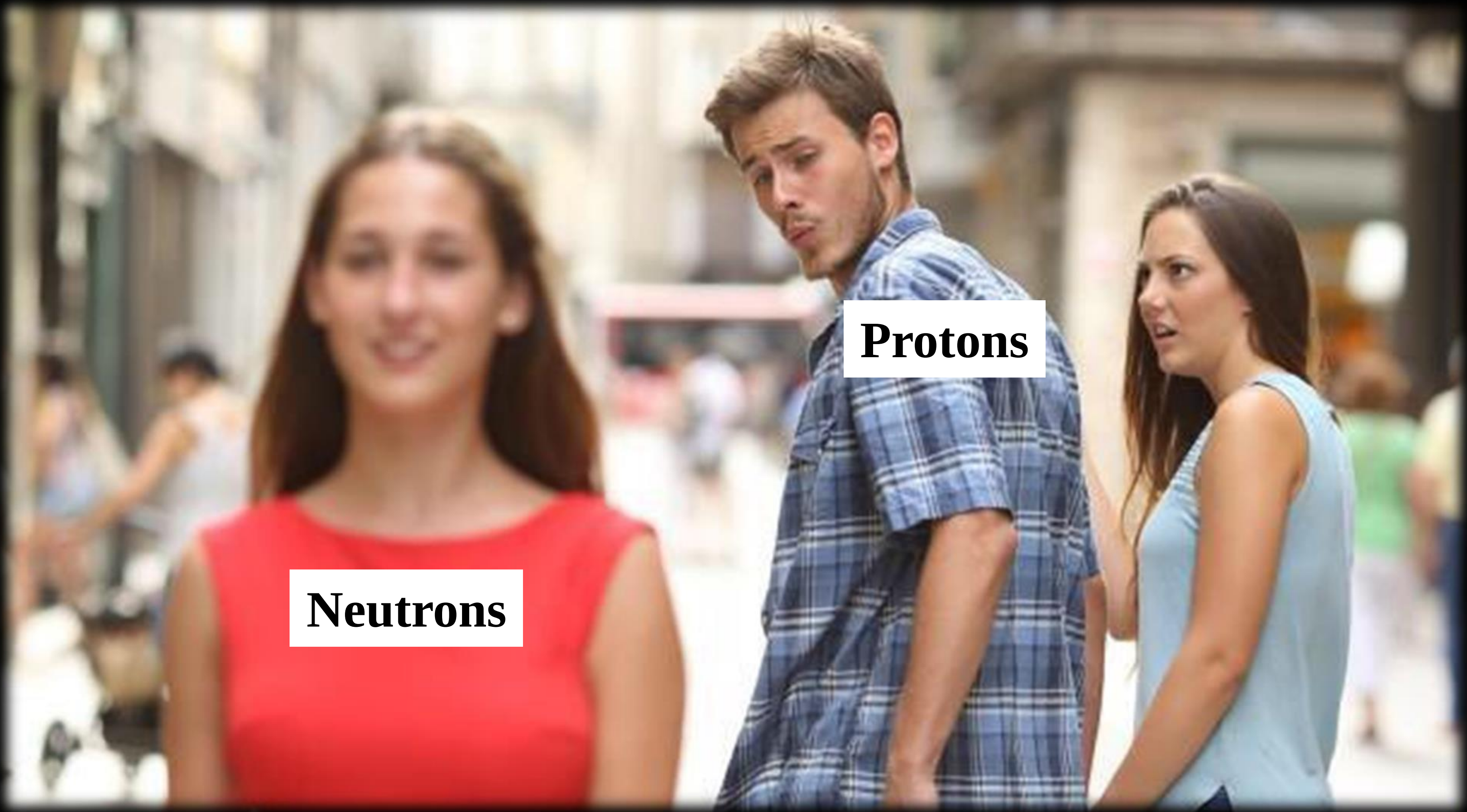
Sorataka Kobayashi
(Finished MSc in Mar. 2019)

Masayuki Matsuo

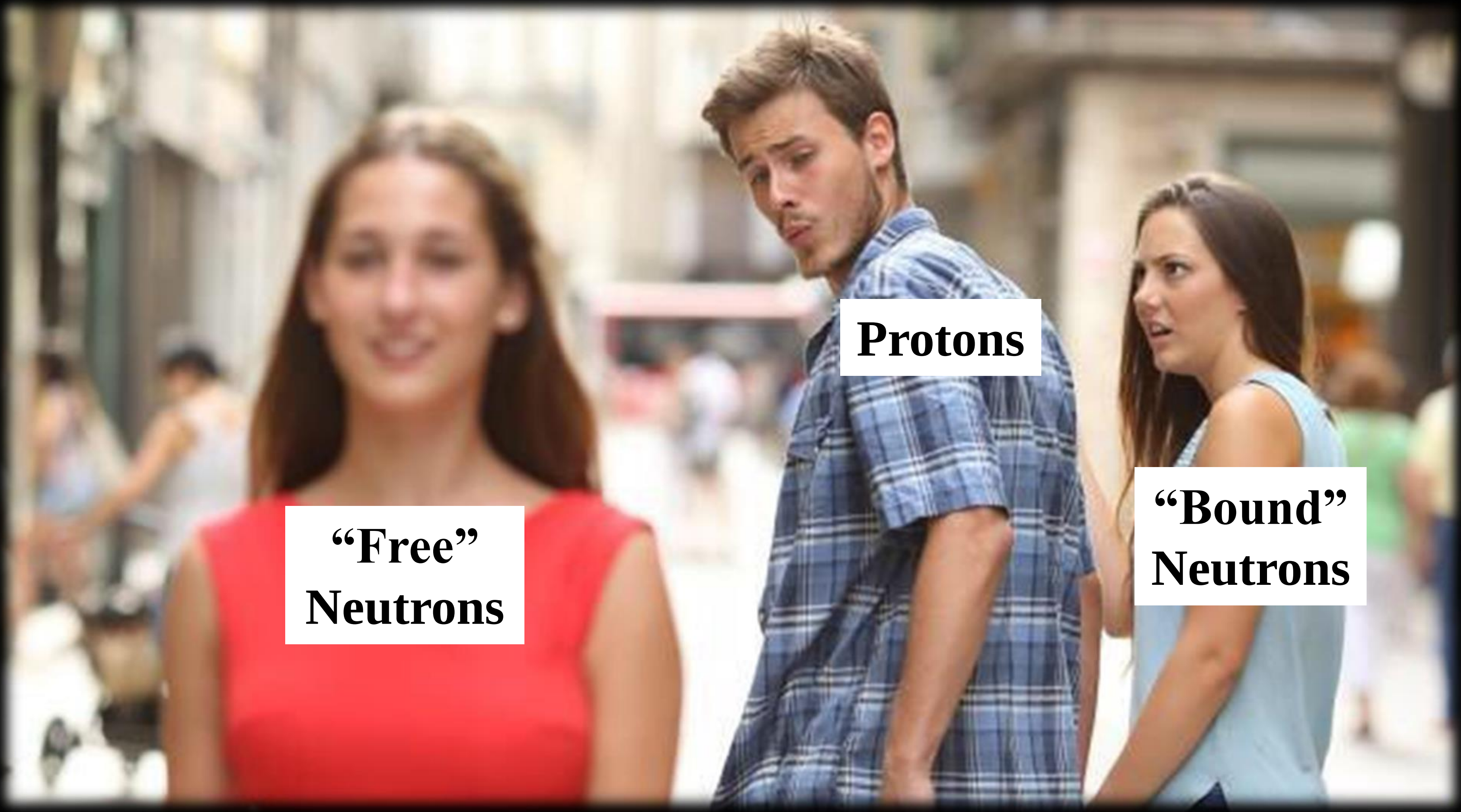
Kenta Yoshimura (M1)

What is the “entrainment” effect?

“Entrainment” is a phenomenon between two species (particles, gases, fluids, etc.), where a motion of one component attracts the other.



“Entrainment” is a phenomenon between two species (particles, gases, fluids, etc.), where a motion of one component attracts the other.



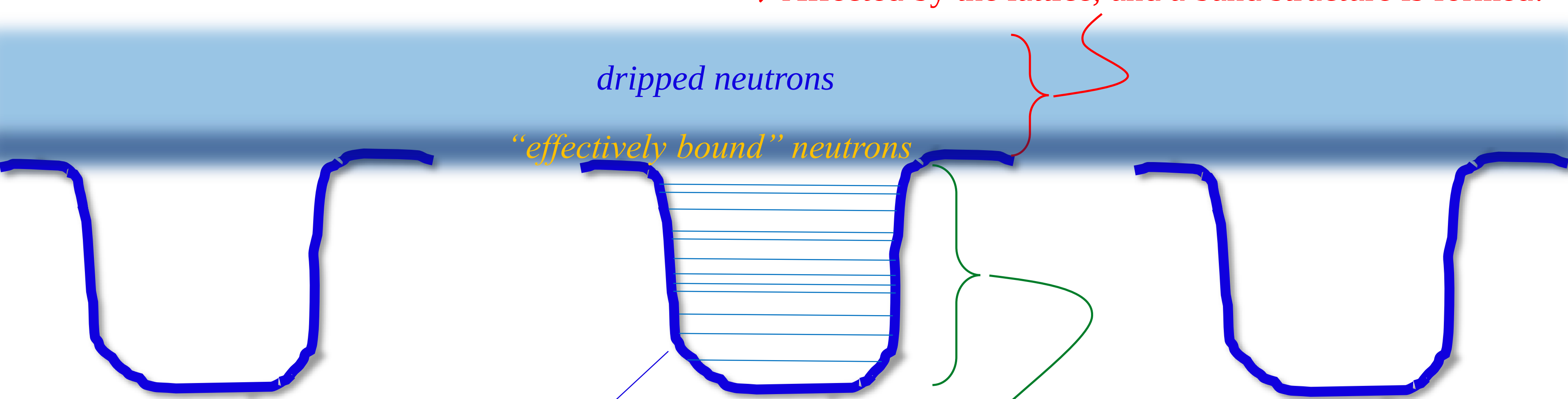
- **Part of dripped neutrons are "effectively bound" (immobilized) by the periodic structure (due to Bragg scatterings), resulting in a larger effective mass**

$$m_{\rm n} n_{\rm n}^{\rm f} = m_{\rm n}^{\star} n_{\rm n}^{\rm c}$$

$n_{\rm n}^{\rm c}$: Conduction neutron number density (neutrons that can actually flow)

$m_{\rm n}^{\star}$: (Macroscopic) Effective mass

Dripped neutrons extend spatially
→ Affected by the lattice, and a band structure is formed!



Entrainment leads:
→ reduction of n_c
→ enhancement of m^*

Potential for neutrons

Bound orbitals are well **localized**
→ Not affected by the lattice

The "entrainment effect" is still a debatable problem

- The first consideration for 1D, square-well potential — K. Oyamatsu and Y. Yamada, NPA**578**(1994)184

- Band calculations for slab (1D) and rod (2D) phases — B. Carter, N. Chamel, and P. Haensel, NPA**748**(2005)675
  - ➡ Entrainment effects are **weak** for the slab & rod phases:

$$\frac{m^\star}{m} \sim \begin{cases} 1.02-1.03 & \text{for the slab phase} \\ 1.11-1.40 & \text{for the rod phase} \end{cases}$$

- Band calculations for cubic-lattice (3D) phases

  N. Chamel, NPA**747**(2005)109 (2005); NPA**773**(2006)263; PRC**85**(2012)035801; J. Low Temp. Phys. **189**, 328 (2017)
  - ➡ **Significant** entrainment effects were found in a low-density region:

$$\frac{m^\star}{m} \gtrsim 10 \text{ or more!} \quad \text{for the cubic lattice}$$

- The first *self-consistent* band calculation for the slab (1D) phase (based on DFT with a BCPM EDF)
  - ➡ "***Reduction***" of the effective mass was observed for the slab phase:

$$\frac{m^\star}{m} \sim 0.65-0.75 \quad \text{for the slab phase}$$

Yu Kashiwaba and T. Nakatsukasa, PRC**100**(2019)035804

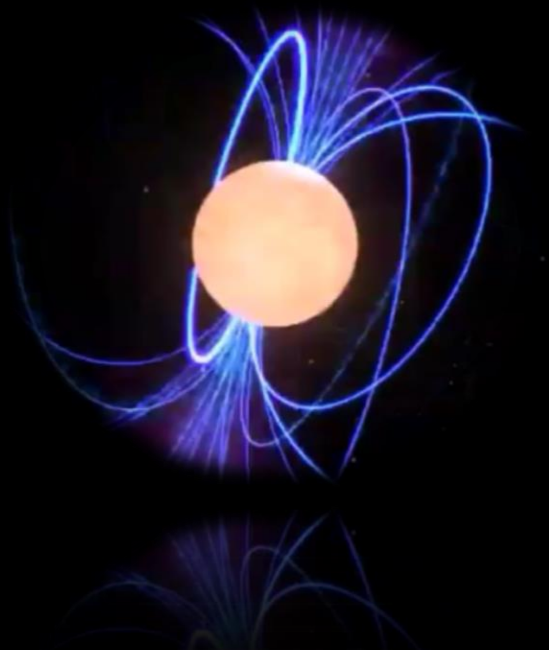
New!

- **Time-dependent extension of the self-consistent band theory (based on TDDFT with a Skyrme EDF)**
  - ➡ "***Reduction***" was observed, consistent with the Tsukuba group.

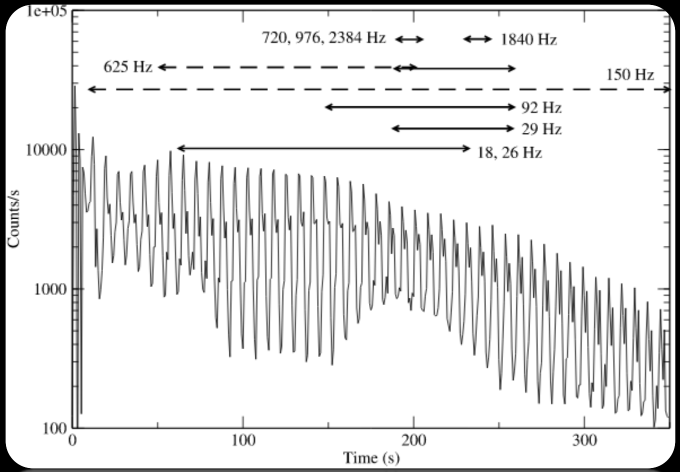
K. Sekizawa, S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

It may affect interpretation of various phenomena, e.g.:

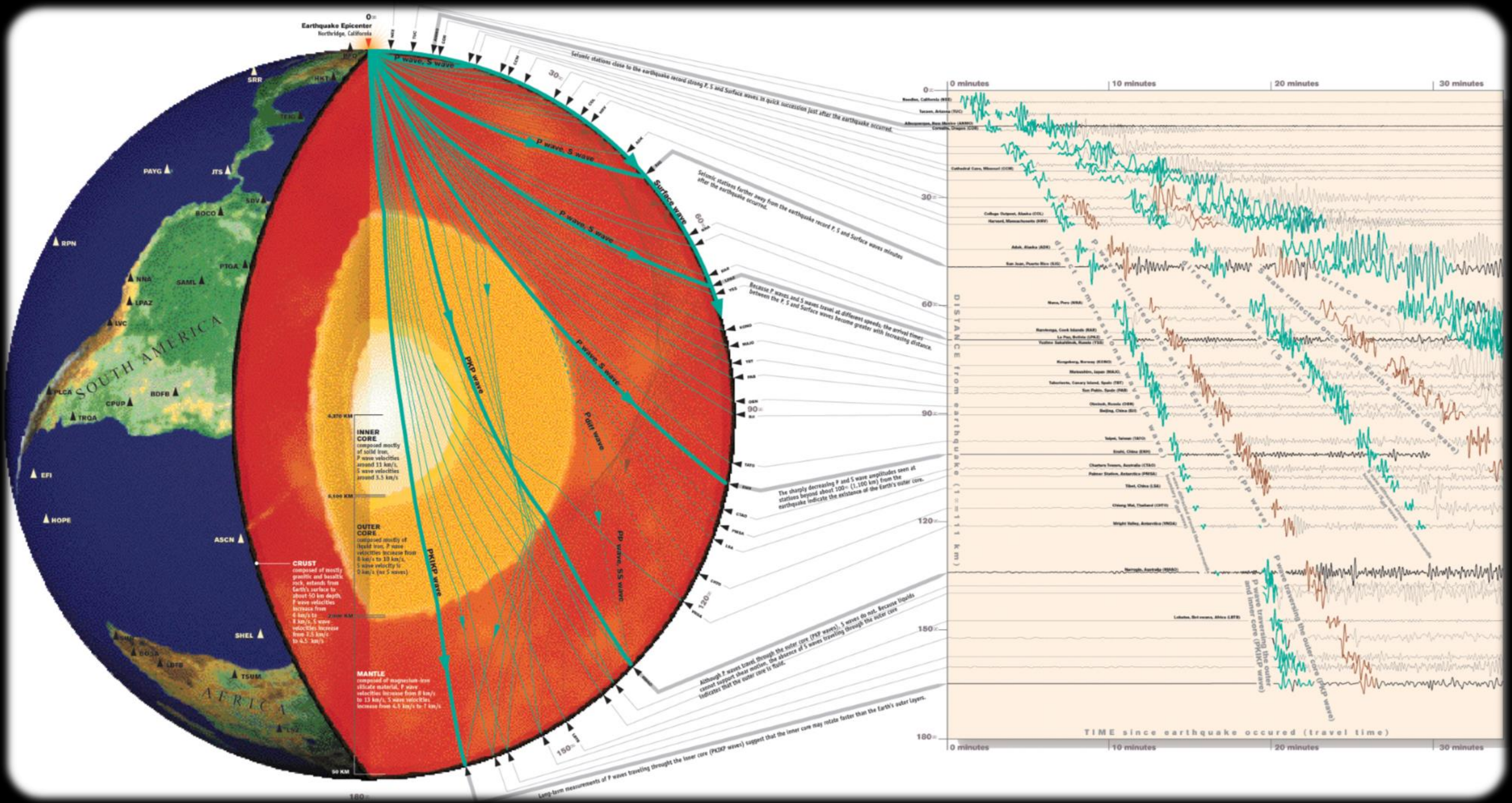
Neutron-star glitch



Quasi-periodic oscillation



Seismology (地震学): Studying inside of the Earth from earthquakes and their propagation



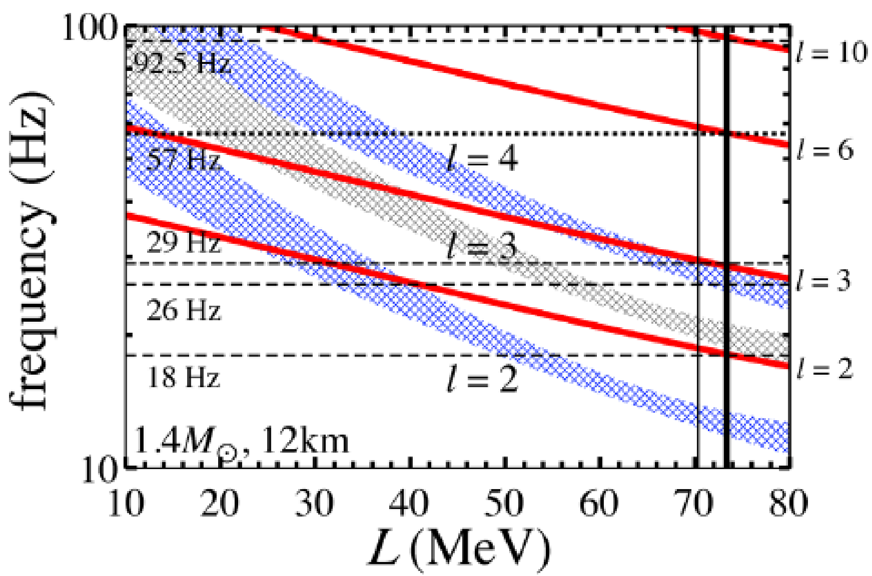
Picture taken from AusPass by Australian National University

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS **489**, 3022–3030 (2019)
Advance Access publication 2019 August 29
doi:10.1093/mnras/stz2385

Astrophysical implications of double-layer torsional oscillations in a neutron star crust as a lasagna sandwich

Hajime Sotani,[1★] Kei Iida[2] and Kazuhiro Oyamatsu[3]
[1] *Division of Science, National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan*
[2] *Department of Mathematics and Physics, Kochi University, 2-5-1 Akebono-cho, Kochi 780-8520, Japan*
[3] *Department of Human Informatics, Aichi Shukutoku University, 2-9 Katahira, Nagakute, Aichi 480-1197, Japan*



- Many (~30) observed QPO frequencies, and prediction by a Bayesian analysis, have been nicely explained by torsional oscillations of tube–bubble or sphere–cylinder layer

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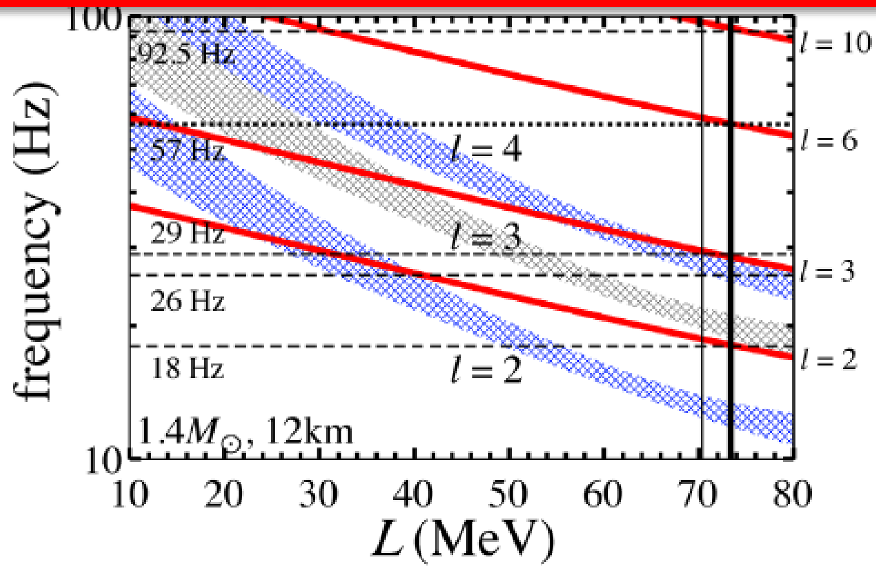
The interpretation could be affected by the entrainment effects!



- Many (~30) observed QPO frequencies, and prediction by a Bayesian analysis, have been nicely explained by torsional oscillations of tube–bubble or sphere–cylinder layer

Recently we have developed:

Time-Dependent Band Theory based on TDDFT

Structure and dynamics of infinite neutron-star matter can be described microscopically taking full account of periodicity of crystalline structure
(*i.e.* band structure effects)

As the first step, it has been applied to the Slab phase!

We employ the Skyrme-Kohn-Sham DFT with the Bloch boundary condition

✓ The Bloch boundary condition for single-particle orbitals

$$\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) = \frac{1}{\sqrt{V}} u^{(q)}_{\alpha\boldsymbol{k}}(z) e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \qquad u^{(q)}_{\alpha\boldsymbol{k}}(z+na) = u^{(q)}_{\alpha\boldsymbol{k}}(z)$$

Periodicity of the slabs

α: Band index $\boldsymbol{k}$: Bloch wave vector q: Isospin (n or p) a: Period of the slabs

✓ Skyrme EDF

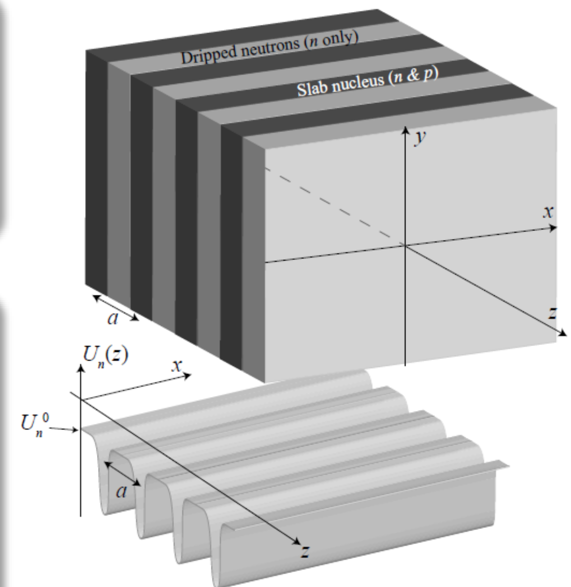
$$\frac{E}{A} = \frac{1}{N_{\rm b}} \int_0^a \left(\frac{\hbar^2}{2m}\tau(z) + \sum_{t=0,1} \left[C^\rho_t[n] n_t^2(z) + C^{\Delta\rho}_t n_t(z) \partial_z^2 n_t(z) + C^\tau_t \left(n_t(z)\tau_t(z) - \boldsymbol{j}_t^2(z) \right) \right] + \mathcal{E}^{(p)}_{\rm Coul}(z) \right) dz$$

Number density: $n_q(z) = 2 \sum_{\alpha,\boldsymbol{k}}^{\rm occ.} \left|\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r})\right|^2$

Kinetic density: $\tau_q(z) = 2 \sum_{\alpha,\boldsymbol{k}}^{\rm occ.} \left|\nabla\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r})\right|^2$

Current (momentum) density: $\boldsymbol{j}_q(z) = 2 \sum_{\alpha,\boldsymbol{k}}^{\rm occ.} {\rm Im}\left[\psi^{(q)*}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) \nabla \psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r})\right]$

*Uniform background electrons are assumed for the charge neutrality condition: $n_e = \bar{n}_p$



Picture from PRC**100**(2019)035804

✓ Skyrme-Kohn-Sham equations

Note: While we deal with 3D slabs, the equations to be solved are 1D!

$$\hat{h}^{(q)}(z)\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) = \varepsilon^{(q)}_{\alpha\boldsymbol{k}}\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) \quad \Rightarrow \quad \left(\hat{h}^{(q)}(z) + \hat{h}^{(q)}_{\boldsymbol{k}}(z)\right) u^{(q)}_{\alpha\boldsymbol{k}}(z) = \varepsilon^{(q)}_{\alpha\boldsymbol{k}} u^{(q)}_{\alpha\boldsymbol{k}}(z)$$

Ordinary single-particle Hamiltonian:

$$\hat{h}^{(q)}(z) = -\nabla\cdot\frac{\hbar^2}{2m^\oplus_q(z)}\nabla + U^{(q)}(z) + \frac{1}{2i}\left[\nabla\cdot\boldsymbol{I}^{(q)}(z) + \boldsymbol{I}^{(q)}(z)\cdot\nabla\right]$$

Additional (k-dependent) term:

$$\hat{h}^{(q)}_{\boldsymbol{k}}(z) = \frac{\hbar^2\boldsymbol{k}^2}{2m^\oplus_q(z)} + \hbar\boldsymbol{k}\cdot\hat{\boldsymbol{v}}^{(q)}(z)$$

Velocity operator:

$$\hat{\boldsymbol{v}}^{(q)}(z) \equiv \frac{1}{i\hbar}\left[\boldsymbol{r}, \hat{h}^{(q)}(z)\right]$$

Proton fraction:

$$Y_\mathrm{p} = \frac{\bar{n}_\mathrm{p}}{\bar{n}_\mathrm{n} + \bar{n}_\mathrm{p}}$$

Average nucleon density:

$$\bar{n}_q = \frac{1}{a}\int_0^a n_q(z)dz$$

Single-particle energy:

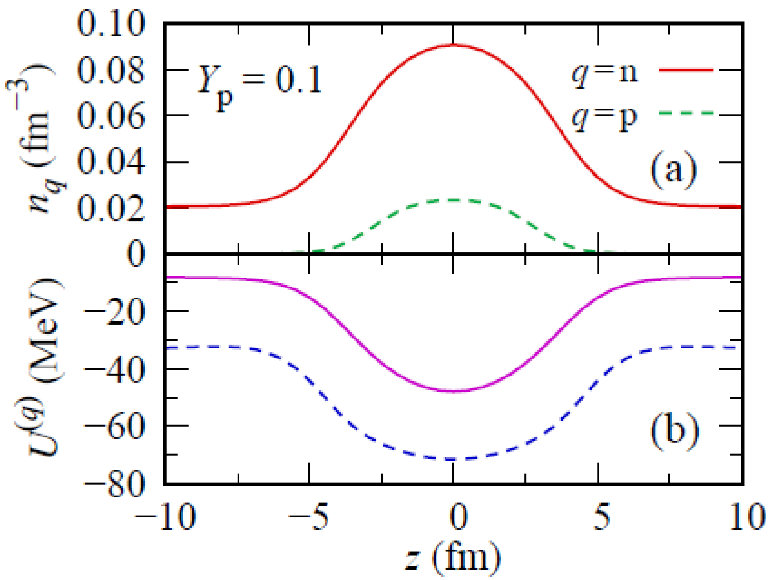
$$\varepsilon^{(q)}_{\alpha\boldsymbol{k}} = e^{(q)}_{\alpha\boldsymbol{k}} + \varepsilon^{(q)}_{\mathrm{kin}\text{-}xy,\alpha\boldsymbol{k}} \approx \frac{\hbar^2 k_\parallel^2}{2m} \qquad k_\parallel = \sqrt{k_x^2 + k_y^2}$$

z-component

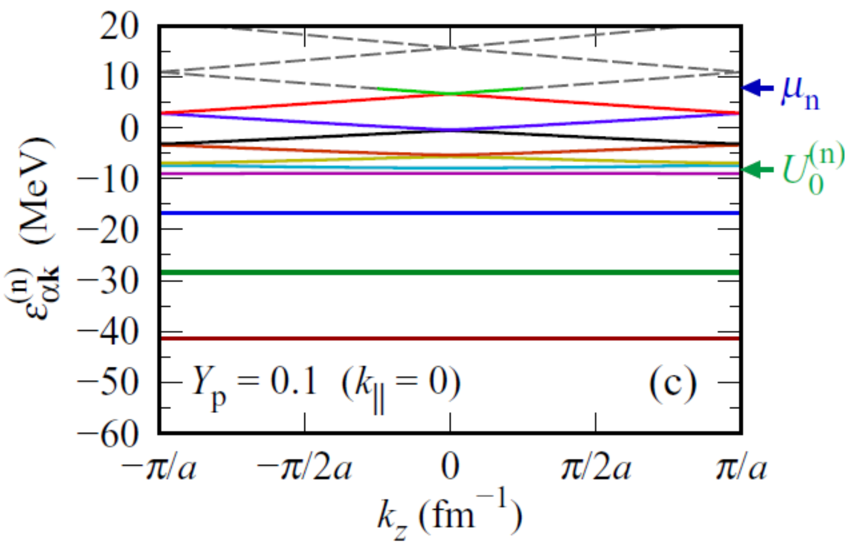
✓ Dripped neutrons show band structure (k_z dependence)

Y_p = 0.1, n_B = 0.4 fm^{-3}: Neutron-dripped slab

Density and potential

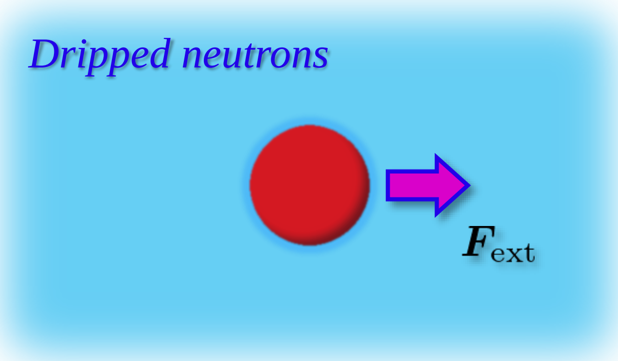


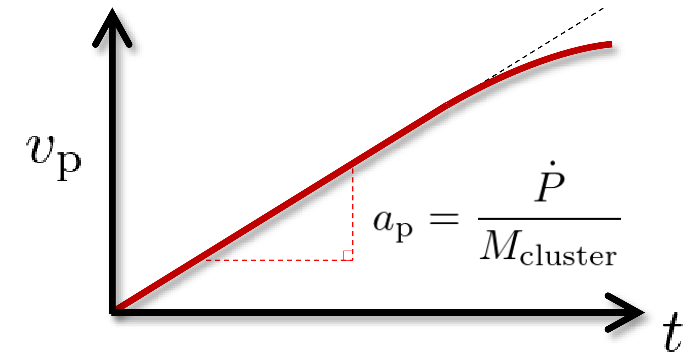
Neutron single-particle energies



✓ The collective mass is extracted from **acceleration motion under constant force**

The real-time method: Idea





How to introduce spatially-uniform electric field

✓ TDKS equation in a "velocity gauge"

Spatially-uniform Vector potential

$$i\hbar \frac{\partial \tilde{u}^{(q)}_{\alpha\boldsymbol{k}}(z,t)}{\partial t} = \Big(\hat{h}^{(q)}(z,t) + \hat{h}^{(q)}_{\boldsymbol{k}(t)}(z,t)\Big)\tilde{u}^{(q)}_{\alpha\boldsymbol{k}}(z,t) \qquad \boldsymbol{k}(t) = \boldsymbol{k} + \frac{e}{\hbar c}A_z(t)\hat{\boldsymbol{e}}_z$$

Gauge transformation for the Bloch orbitals:

$$\tilde{u}^{(q)}_{\alpha\boldsymbol{k}}(z,t) = \exp\Big[-\frac{ie}{\hbar c}A_z(t)z\Big]u^{(q)}_{\alpha\boldsymbol{k}}(z,t)$$

Electric field:

$$E_z(t) = -\frac{1}{c}\frac{dA_z}{dt}$$

k-dependent term:

$$\hat{h}^{(q)}_{\boldsymbol{k}}(z) = \frac{\hbar^2\boldsymbol{k}^2}{2m^{\oplus}_q(z)} + \hbar\boldsymbol{k}\cdot\hat{\boldsymbol{v}}^{(q)}(z)$$

Velocity operator:

$$\hat{\boldsymbol{v}}^{(q)}(z) \equiv \frac{1}{i\hbar}\big[\boldsymbol{r}, \hat{h}^{(q)}(z)\big]$$

cf. K. Yabana and G.F. Bertsch, Phys. Rev. B **54**, 4484 (1996); G.F. Bertch *et al.*, Phys. Rev. B **62**, 7998 (2000)

Results: The collective mass

K. Sekizawa, S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

Acceleration:

$$a_p = \frac{d^2 Z}{dt^2}$$

C.m. position of protons:

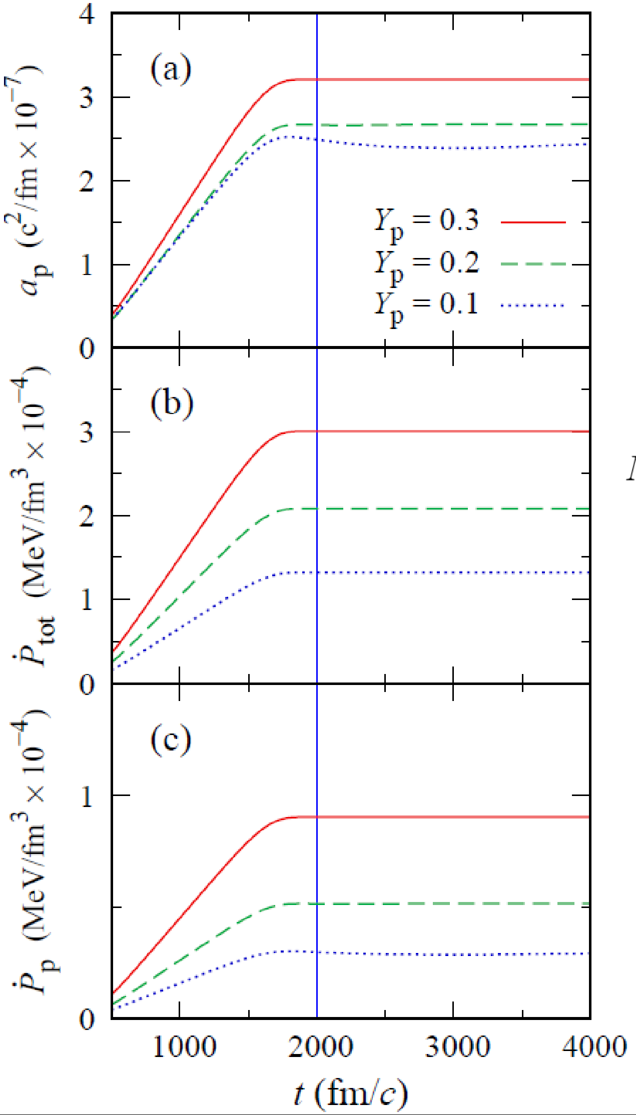
$$Z(t) = \frac{1}{a}\int_0^a z\, n_p(z,t)\, dz$$

Momentum of nucleons:

$$P_q(t) = \hbar \int_0^a j_q(z,t)\, dz$$

Total momentum:

$$P_{tot}(t) = P_n(t) + P_p(t)$$

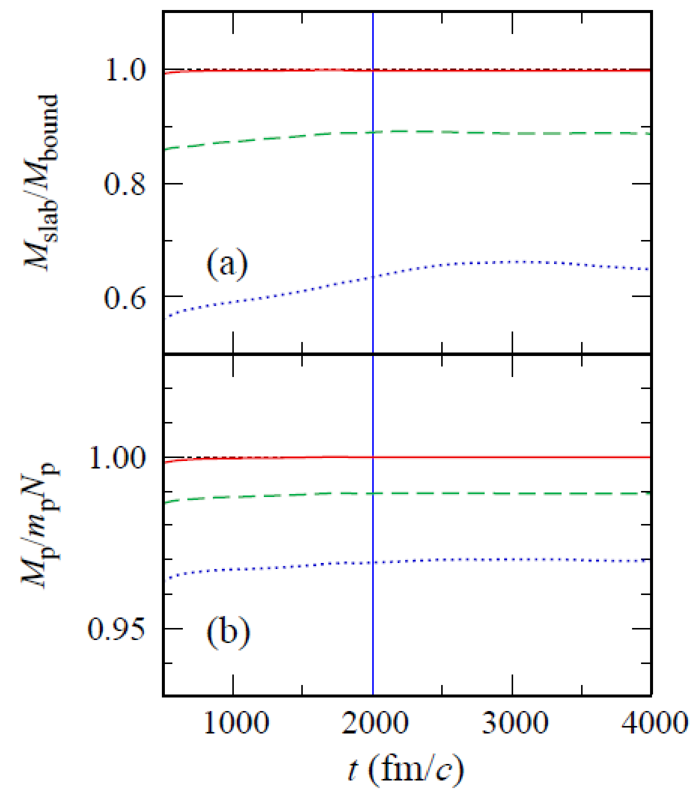


$$M_{slab} = \dot{P}_{tot}/a_p$$

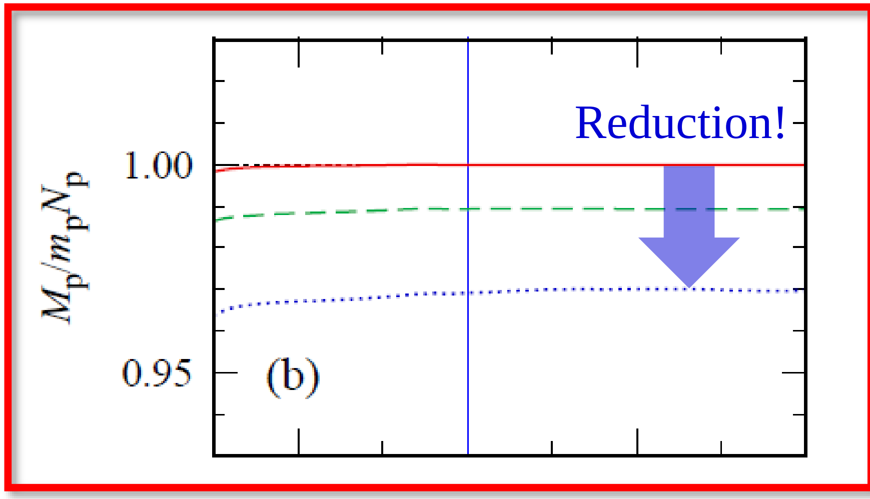
$$M_p = \dot{P}_p/a_p$$

✓ For neutron-dripped slabs, we find significant ***reduction*** of the collective mass!

➢ **What is the origin of the reduction?**



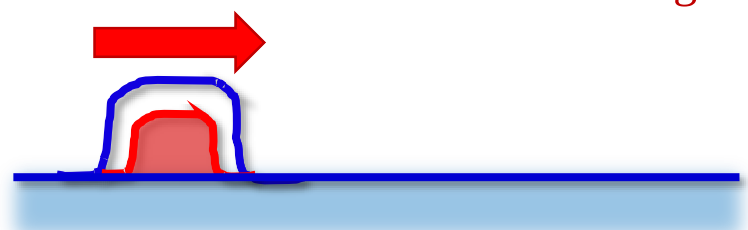
✓ Cause of the reduction of the collective mass of protons:
the density-dependent "microscopic" effective mass



Collective mass of protons

$$M_{\rm p} \leq m_{\rm p} N_{\rm p} \approx m_{\rm p}^{\oplus}[n_{\rm n}^{\rm b.g.}] N_{\rm p}$$

Protons and bound neutrons move together



There must be a velocity lag between protons and background neutrons!

The continuity equation within Skyrme TDDFT reads:

$$\frac{\partial \rho_q(\boldsymbol{r},t)}{\partial t} + \hbar\, \boldsymbol{\nabla} \cdot \boldsymbol{p}_q(\boldsymbol{r},t) = 0$$

where

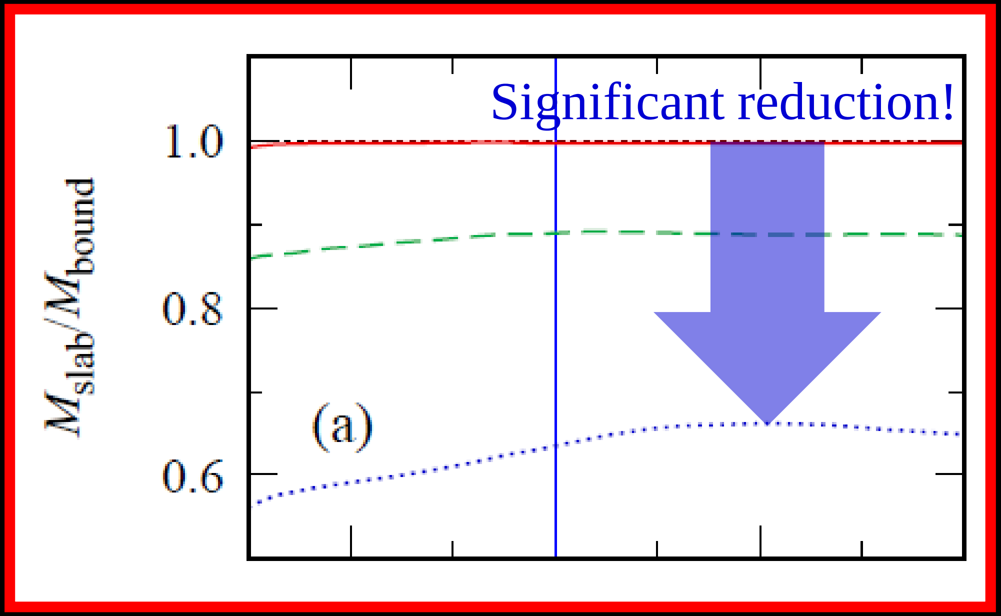
$$\boldsymbol{p}_q(\boldsymbol{r},t) = \boldsymbol{j}_q(\boldsymbol{r},t) + q\frac{2m_q}{\hbar^2}(C_0^\tau - C_1^\tau) n_{\rm n}(\boldsymbol{r},t) n_{\rm p}(\boldsymbol{r},t)\left(\frac{\boldsymbol{j}_{\rm p}(\boldsymbol{r},t)}{n_{\rm p}(\boldsymbol{r},t)} - \frac{\boldsymbol{j}_{\rm n}(\boldsymbol{r},t)}{n_{\rm n}(\boldsymbol{r},t)}\right)$$

+1 for protons
-1 for neutrons

velocity difference

Then, what is the cause of the reduction of the collective mass of the slab?

→ an "anti-entrainment" effect!



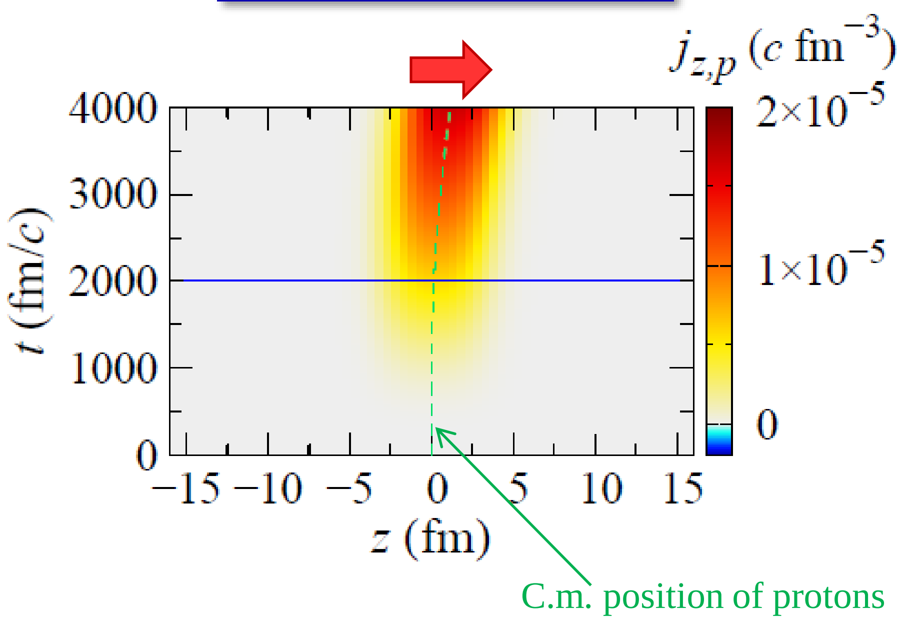
It can **not** be explained solely by the microscopic effective mass.

Current density:

$$j_{z,q}(z,t) = \frac{\hbar}{m_q}\sum_{\alpha,\boldsymbol{k}}^{\text{occ.}} \text{Im}\left[\psi_{\alpha\boldsymbol{k}}^{(q)*}(\boldsymbol{r},t)\nabla\psi_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r},t)\right] = \frac{\hbar}{m_q}\frac{1}{aN_{k_z}}\sum_{\alpha,k_z}\int\frac{k_\parallel}{\pi}\text{Im}\left[u_{\alpha\boldsymbol{k}}^{(q)*}(z,t)(\partial_z + ik_z)u_{\alpha\boldsymbol{k}}^{(q)}(z,t)\right]\theta(\mu_q - \varepsilon_{\alpha\boldsymbol{k}}^{(q)})\,dk_\parallel$$

✓ Protons inside the slab move toward the direction of the external force, as expected.

Proton current density



"Anti-entrainment" effect

K. Sekizawa, S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

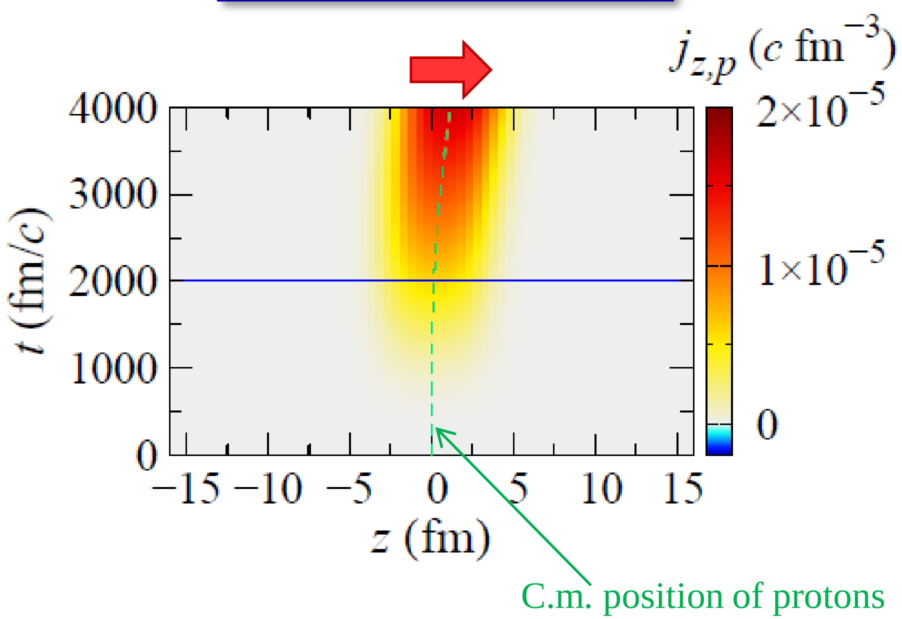
Current density:

$$j_{z,q}(z,t) = \frac{\hbar}{m_q}\sum_{\alpha,\boldsymbol{k}}^{\text{occ.}} \mathrm{Im}\big[\psi_{\alpha\boldsymbol{k}}^{(q)*}(\boldsymbol{r},t)\nabla\psi_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r},t)\big] = \frac{\hbar}{m_q}\frac{1}{aN_{k_z}}\sum_{\alpha,k_z}\int\frac{k_\parallel}{\pi}\,\mathrm{Im}\big[u_{\alpha\boldsymbol{k}}^{(q)*}(z,t)(\partial_z+ik_z)u_{\alpha\boldsymbol{k}}^{(q)}(z,t)\big]\,\theta(\mu_q-\varepsilon_{\alpha\boldsymbol{k}}^{(q)})\,dk_\parallel$$

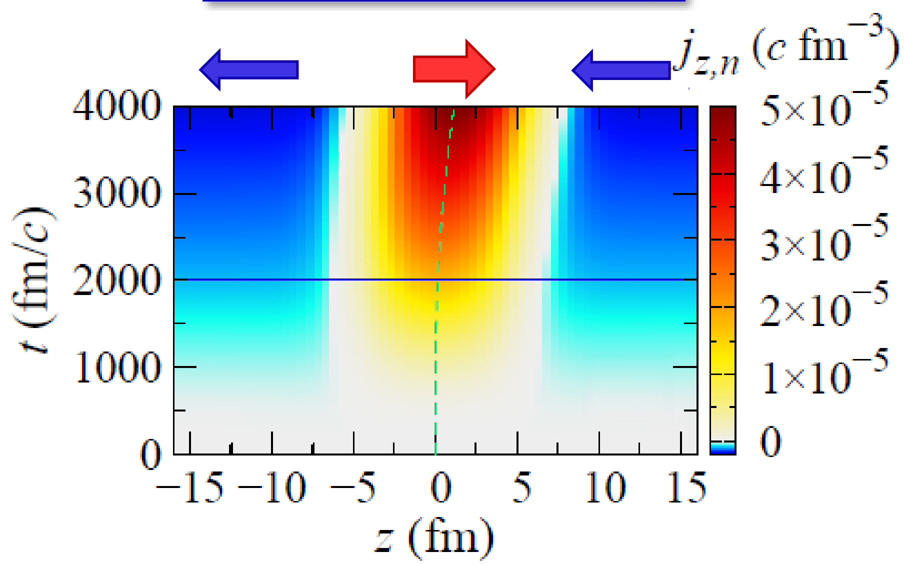
✓ Dripped neutrons outside the slab move toward the opposite direction!

Since it reduces $P_{\rm tot}$ and $\dot{P}_{\rm tot}$, $M_{\rm slab} = \dot{P}_{\rm tot}/a_{\rm p}$ is reduced

Proton current density



Neutron current density



$$\left(m^{\star\,-1}_{\mathrm{n},\alpha\boldsymbol{k}}\right)_{\mu\nu} = \frac{1}{\hbar^2}\frac{\partial^2\varepsilon_{\alpha\boldsymbol{k}}^{(\mathrm{n})}}{\partial k_\mu \partial k_\nu}$$

K. Sekizawa, S. Kobayashi, and M. Matsuo, PRC**105**(2022)045807

Current density:

$$j_{z,q}(z,t) = \frac{\hbar}{m_q} \sum_{\alpha,\boldsymbol{k}}^{\text{occ.}} \text{Im}\big[\psi^{(q)*}_{\alpha\boldsymbol{k}}(\boldsymbol{r},t)\nabla\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r},t)\big] = \frac{\hbar}{m_q}\frac{1}{aN_{k_z}}\sum_{\alpha,k_z}\int\frac{k_\parallel}{\pi}\,\text{Im}\big[u^{(q)*}_{\alpha\boldsymbol{k}}(z,t)(\partial_z + ik_z)u^{(q)}_{\alpha\boldsymbol{k}}(z,t)\big]\,\theta(\mu_q - \varepsilon^{(q)}_{\alpha\boldsymbol{k}})\,dk_\parallel$$

✓ Dripped neutrons outside the slab move toward the opposite direction!

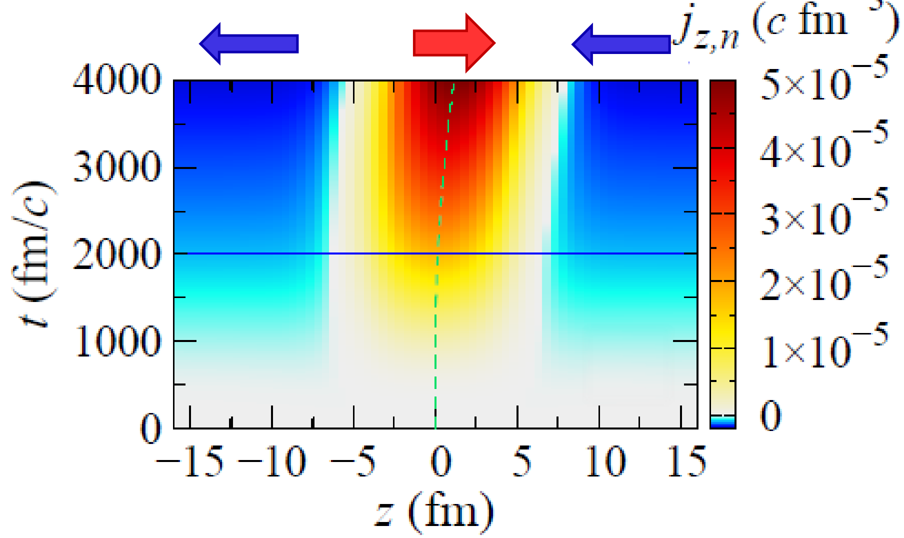
Since it reduces P_{tot} and $\dot{P}_{\text{tot}}$, $M_{\text{slab}} = \dot{P}_{\text{tot}}/a_{\text{p}}$ is reduced

Reduction of M_{slab}
→ enhancement of n_{c}
→ reduction of m^*

We interpret it as an "anti-entrainment" effect

Y_{p}	$n^{\text{f}}_{\text{n}}/\bar{n}_{\text{n}}$	Static		Dynamic
		$n^{\text{c}}_{\text{n}}/\bar{n}_{\text{n}}$	$m^{\star}_{\text{n}}/m_{\text{n}}$	$n^{\text{c}}_{\text{n}}/\bar{n}_{\text{n}}$
0.3	2.09×10^{-4}	0.005	0.040	0.005
0.2	0.127	0.256	0.496	0.229
0.1	0.362	0.630	0.574	0.586

Neutron current density



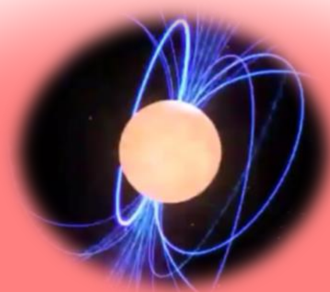
$$(m^{\star\,-1}_{\text{n},\alpha\boldsymbol{k}})_{\mu\nu} = \frac{1}{\hbar^2}\frac{\partial^2\varepsilon^{(\text{n})}_{\alpha\boldsymbol{k}}}{\partial k_\mu \partial k_\nu}$$

Summary

➢ **Definitely, all are rooted with the wonder of nuclear physics which is basically a quantum many-body problem! ;)**

Takeaway message

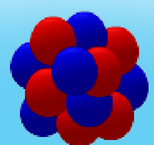
Physics of neutron stars



- Mass, radius, deformability, … (⇔ EoS, GW, …)
- Torsional oscillations (⇔ QPO, Pasta, GW, …)
- Superfluidity, cooling, glitches, …

Macroscopic, astro-

Physics of finite nuclei



Microscopic, finite-

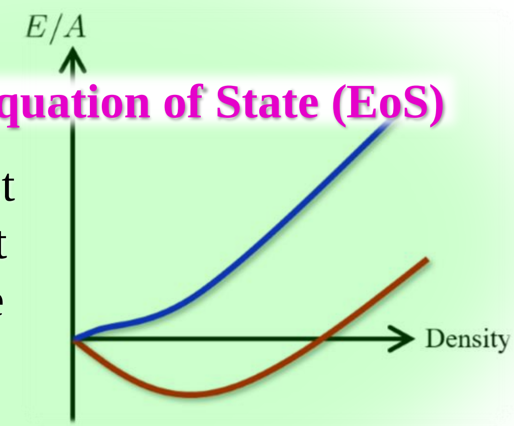
- Quantum many-body problem
- Nuclear force (⇔ EoS, Structure & Reactions)
- Mass/binding energy (⇔ Crust compositions)
- Excitation properties (⇔ GMR, GDR, EoS, …)
- Nuclear reactions (⇔ Stellar evolution, SNe, …)

Nuclear matter properties

Micro-to-macro, infinite-

- Outer crust
- Inner crust
- Outer core
- Inner core

Equation of State (EoS)



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