Webinar at Department of Physics, Ankara University
December 23rd, 2021

Exploring Quantum Many-Body Dynamics in Nuclear Systems: From Nuclei to Neutron Stars

Kazuyuki Sekizawa

Department of Physics, School of Science
Tokyo Institute of Technology

東京工業大学
Tokyo Institute of Technology

Brief personal history

1988: Born in Tokyo, grew up in Satte City in Saitama

2003-2006: Inagakuen Public High School

2006-2010: Tokyo University of Science (BSc)

2010-2015: University of Tsukuba (MSc-PhD)

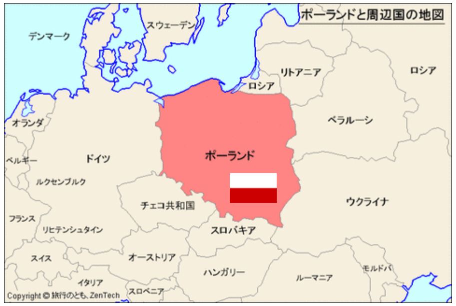
Apr. 2015-Aug. 2017: Warsaw University of Technology, Poland (Postdoc)

Sep. 2017-Dec. 2017: University of Washington, USA (Postdoc)

Jan. 2018-Mar. 2021: Niigata University (Assistant Professor, tenure track)

Apr. 2021-Present: Tokyo Institute of Technology (Associate Professor)

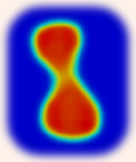
Main research field: Nuclear Theory



My brief research history

2012 MSc-PhD 2015 2018 2021 **Year**

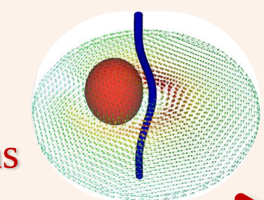
Low-energy heavy-ion reactions (MNT & QF) within TDDFT

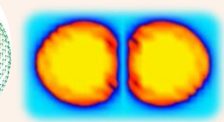


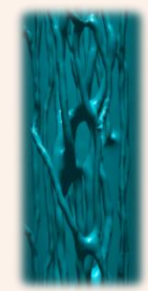
Applications & extensions of TDDFT: TDRPA, SMF, TDDFT+Langevin, ...

Developing a **time-dependent band theory** and its application to **neutron star** crusts

Superfluid dynamics in nuclear systems:
- quantum vortices in **neutron stars**
- solitonic excitations in nuclear collisions







Superfluid dynamics in spin-polarized **UFG**

Acronyms
MNT: Multi-nucleon transfer
QF: Quasi-fission
TDRPA: Time-Dependent Random Phase Approximation
SMF: Stochastic Mean-Field
UFG: Unitary Fermi Gas

From **quarks** to atomic nuclei

Standard model of the elementary particles

- ✓ Elementary particles: fundamental particles **without structure**
- ✓ Four forces: **strong, weak, electromagnetic**, and gravitational forces
- ✓ Particle physics explores an **ultimate theory** of the Universe

Higgs boson

Fermions

		1st gen.	2nd gen.	3rd gen.	Charge
Quarks		up u	charm c	top t	$+\frac{2}{3}$
		down d	strange s	bottom b	$-\frac{1}{3}$
Leptons		electron e	muon μ	tau τ	-1
		electron neutrino ν_e	muon neutrino ν_μ	tau neutrino ν_τ	0

Gauge bosons

Strong force

Gluons

EM force

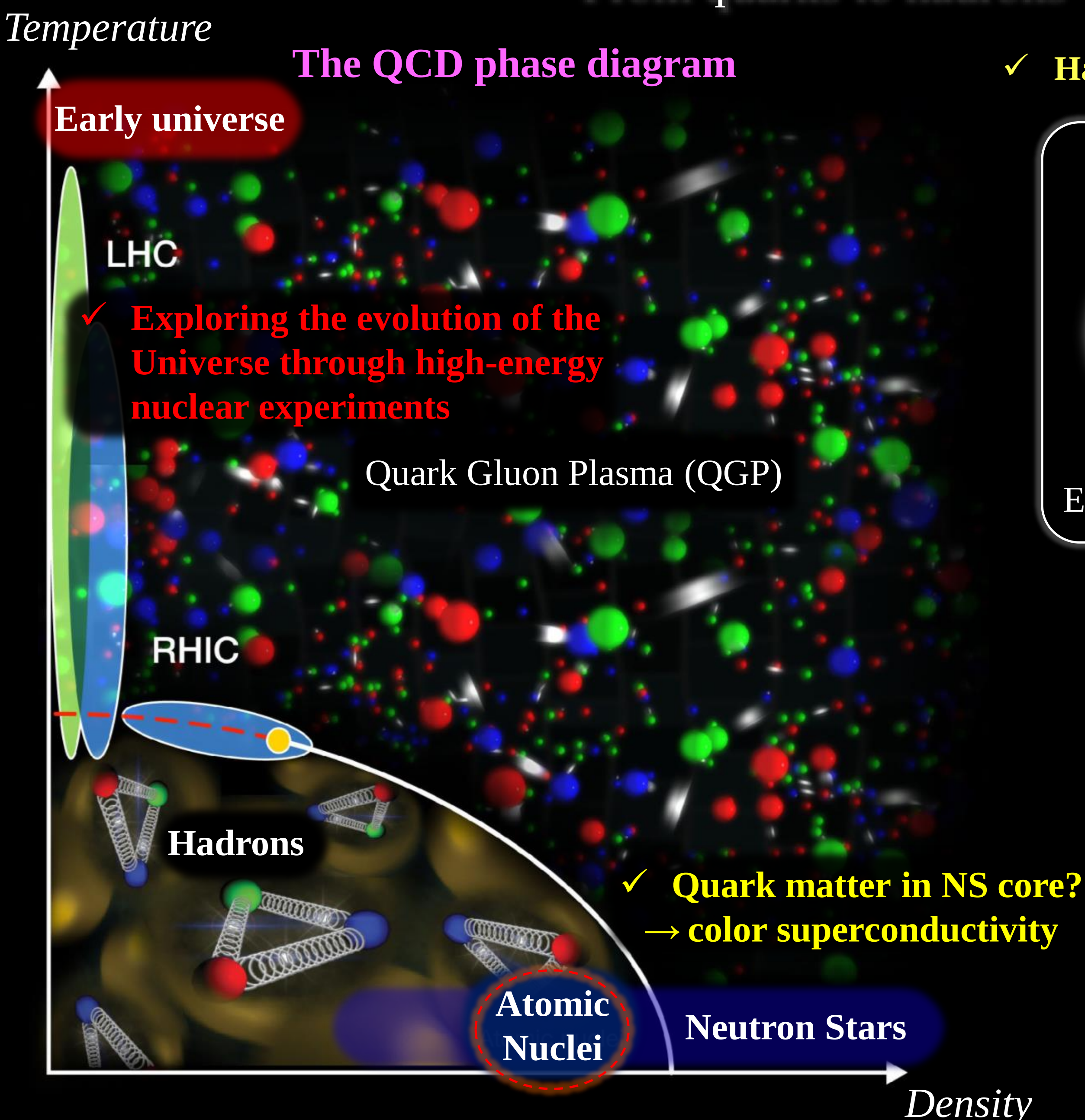
Photons

Weak force

$W^{\pm}$, Z^0 bosons

From quarks to hadrons

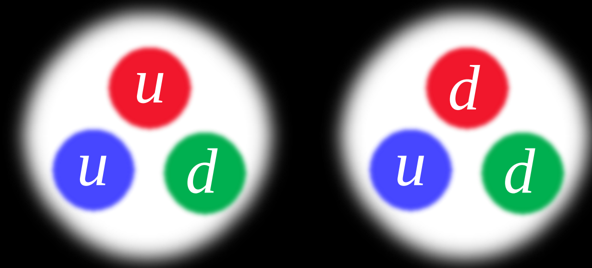
The QCD phase diagram



Left figure: https://www.bnl.gov/newsroom/news.php?a=24281

✓ **Hadrons:** composite particles of quarks

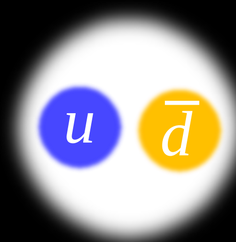
Baryons



E.g. Proton Neutron

➢ A nucleon (proton or neutron) is a baryon.

Mesons



E.g. Pion (π^+)

Energy scales and degrees of freedom

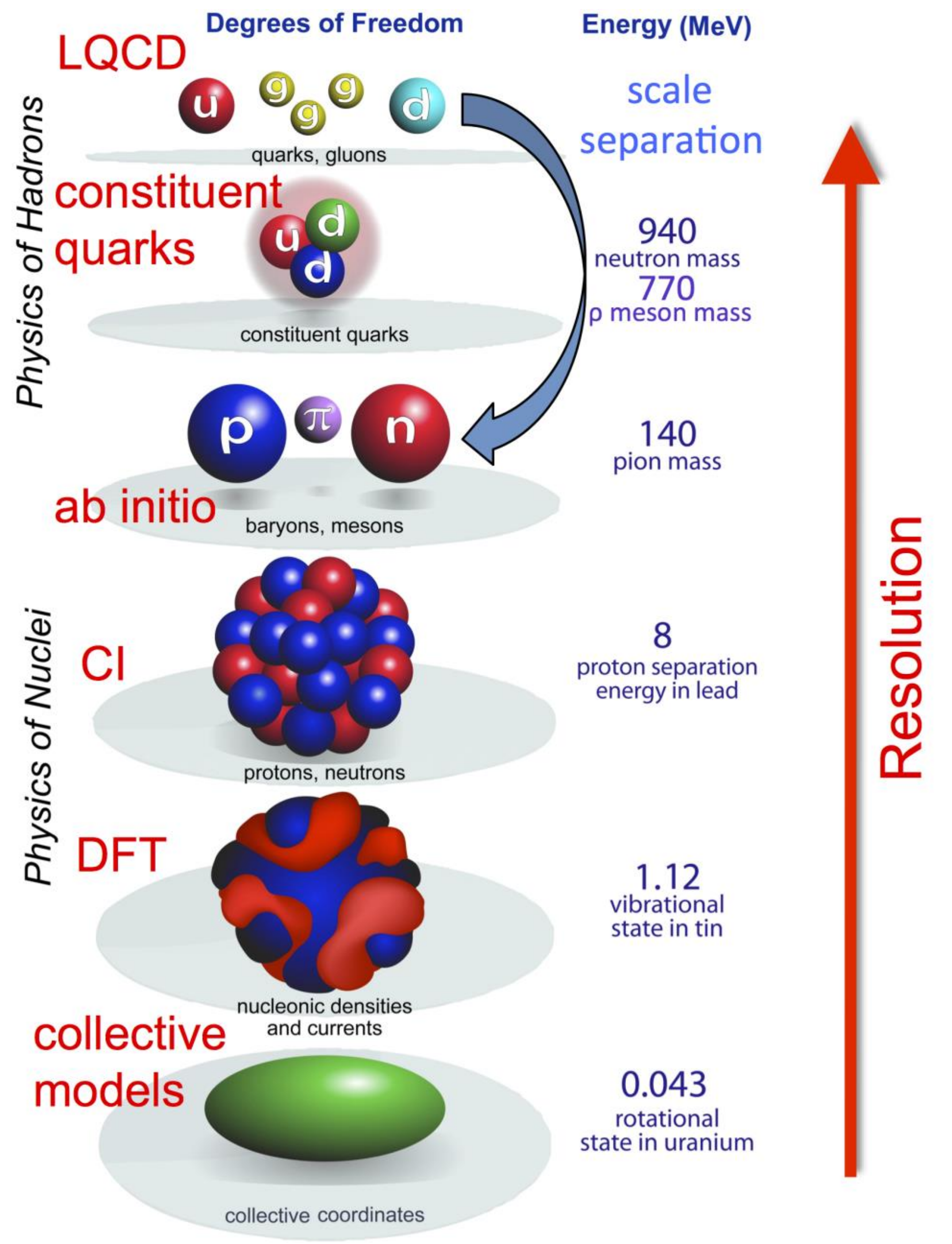


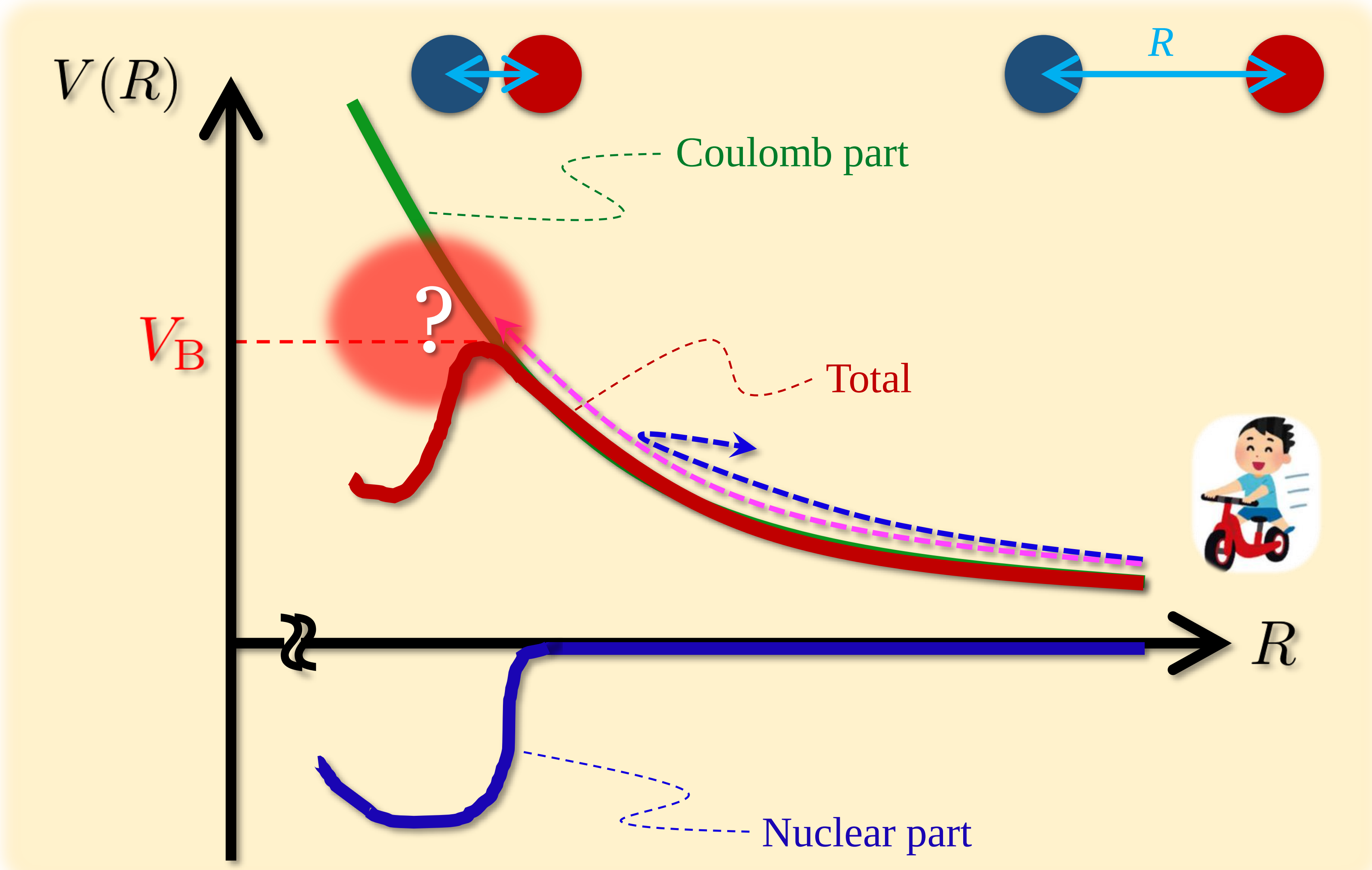
Figure: https://www.asc.ohio-state.edu/physics/ntg/6805/slides/6805_overview_slides.php#Dofs

Not this "high energy" !!

Animation: Brookhaven National Laboratory (http://www.bnl.gov/rhic/); https://youtu.be/Vyq_AYWctSo

We collide two nuclei "gently"

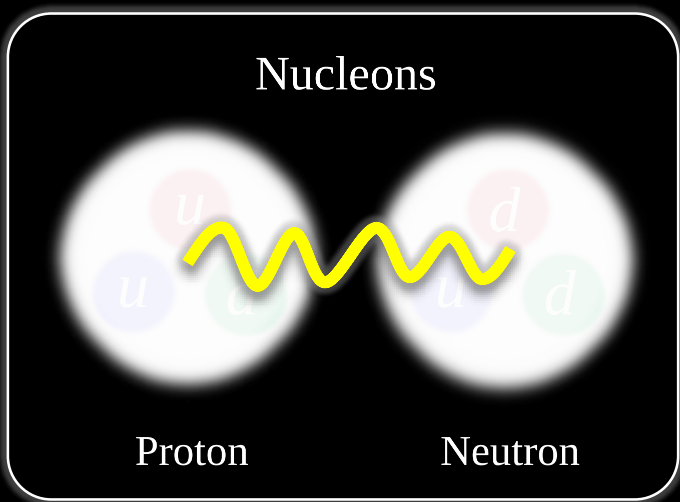
and study quantum many-body dynamics of neutrons and protons

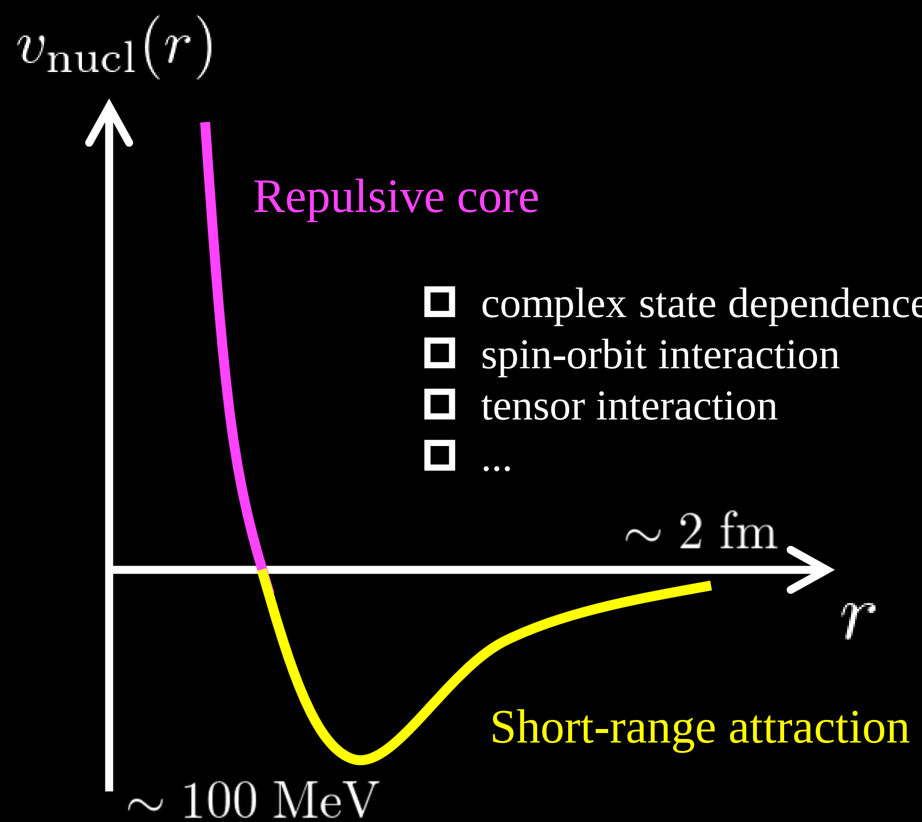


In “low-energy” nuclear physics,
we treat neutrons and protons as building blocks

What we study is:

A quantum many-body problem of fermions interacting through the nuclear force





(TD)DFT in a tiny nutshell

A theory which gives us access to the *exact* solution

Equivalent!
(for a special EDF)

$$\hat{H}\Psi(\boldsymbol{r}_1,\dots,\boldsymbol{r}_N) = E\Psi(\boldsymbol{r}_1,\dots,\boldsymbol{r}_N)$$

Kohn-Sham equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + v_{\rm KS}[\rho(\boldsymbol{r})]\right]\phi_i(\boldsymbol{r}) = \varepsilon_i\,\phi_i(\boldsymbol{r})$$

EDF — This is the key!

$$v_{\rm KS}[\rho(\boldsymbol{r})] = \frac{\delta\mathcal{E}[\rho]}{\delta\rho} \qquad \rho(\boldsymbol{r}) = \sum_{i=1}^{N}|\phi_i(\boldsymbol{r})|^2$$

Existence was proven, but "shape" is unknown :(

Energy can also be written as a functional of density

$$E[\rho] = \langle\Psi[\rho]|\hat{H}|\Psi[\rho]\rangle$$

w.f. is a functional of density

P. Hohenberg and W. Kohn, Phys. Rev. B **136**, 864 (1964)

What is DFT?

A theory which gives us access to the *exact* solution

Equivalent!
(for a special EDF)

$$\hat{H}\Psi(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_N) = E\Psi(\boldsymbol{r}_1,\ldots,\boldsymbol{r}_N)$$

Kohn-Sham equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + v_{\text{KS}}[\rho(\boldsymbol{r})]\right]\phi_i(\boldsymbol{r}) = \varepsilon_i\,\phi_i(\boldsymbol{r})$$

EDF

This is the key!

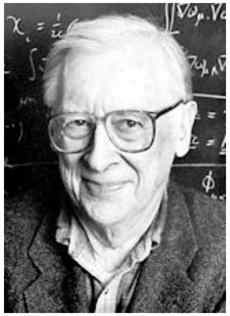
$$v_{\text{KS}}[\rho(\boldsymbol{r})] = \frac{\delta\mathcal{E}[\rho]}{\delta\rho} \qquad \rho(\boldsymbol{r}) = \sum_{i=1}^{N}|\phi_i(\boldsymbol{r})|^2$$

Existence was proven, but "shape" is unknown :(

Great Success of the Density Functional Theory

The Nobel Prize in Chemistry 1998

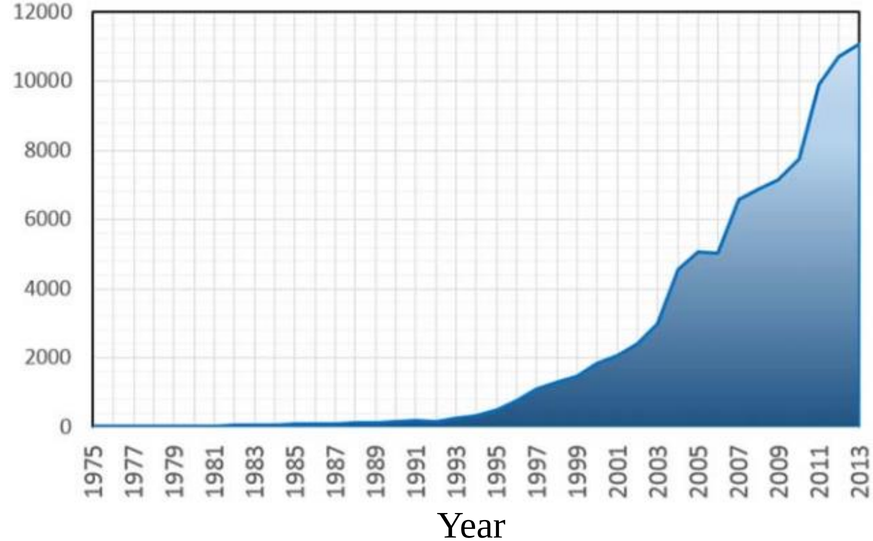
Walter Kohn



John Pople

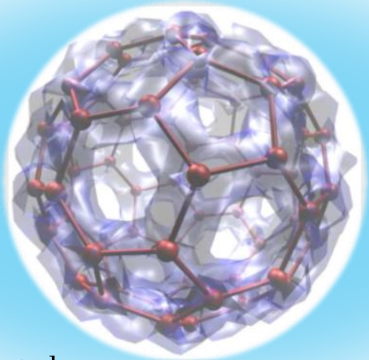
©https://www.nobelprize.org

Number of publications with "DFT" (till 2013)



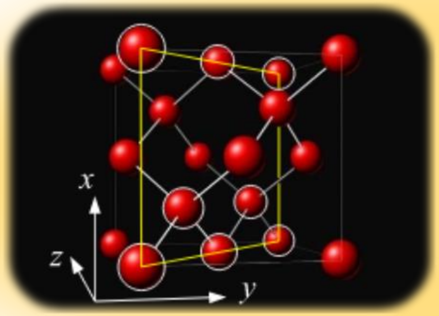
A. Galano and J.R. Alvarez-Idadoy, J. Compt. Chem. **35**, 2019 (2014)

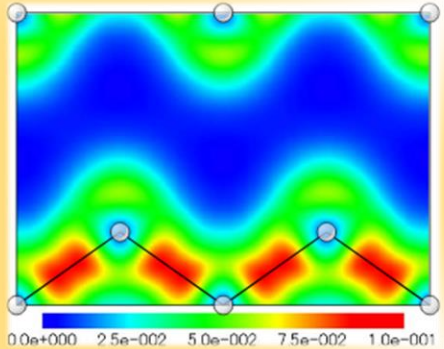
Fullerene: C_{60}



C-Z. Gao et al.,
J. Phys. B: At. Mol. Opt. Phys. **48**, 105102 (2015)

Si crystal



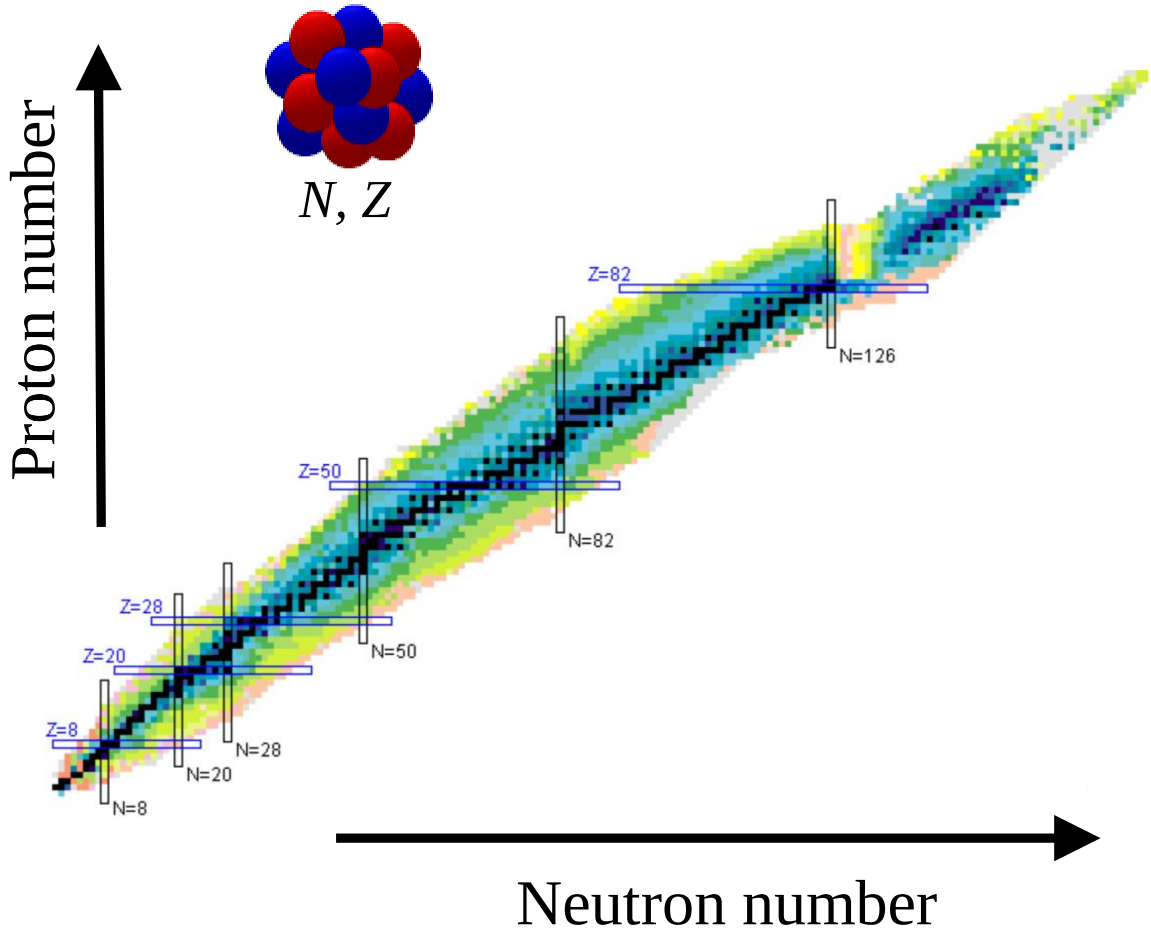


Y. Shinohara, K. Yabana, Y. Kawashita, J.-I. Iwata, T. Otobe, and G. F. Bertsch,
Phys. Rev. B **82**, 155110 (2010)

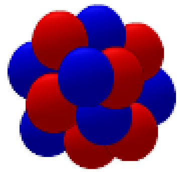
The seminal papers on DFT:
- P. Hohenberg and W. Kohn, Phys. Rev. **136**, B864 (1964) → **19,015 citations!**
- W. Kohn and L.J. Sham, Phys. Rev. **140**, A1133 (1965) → **24,384 citations!**

Then, how it works?

All nuclei can be described with a *single* EDF

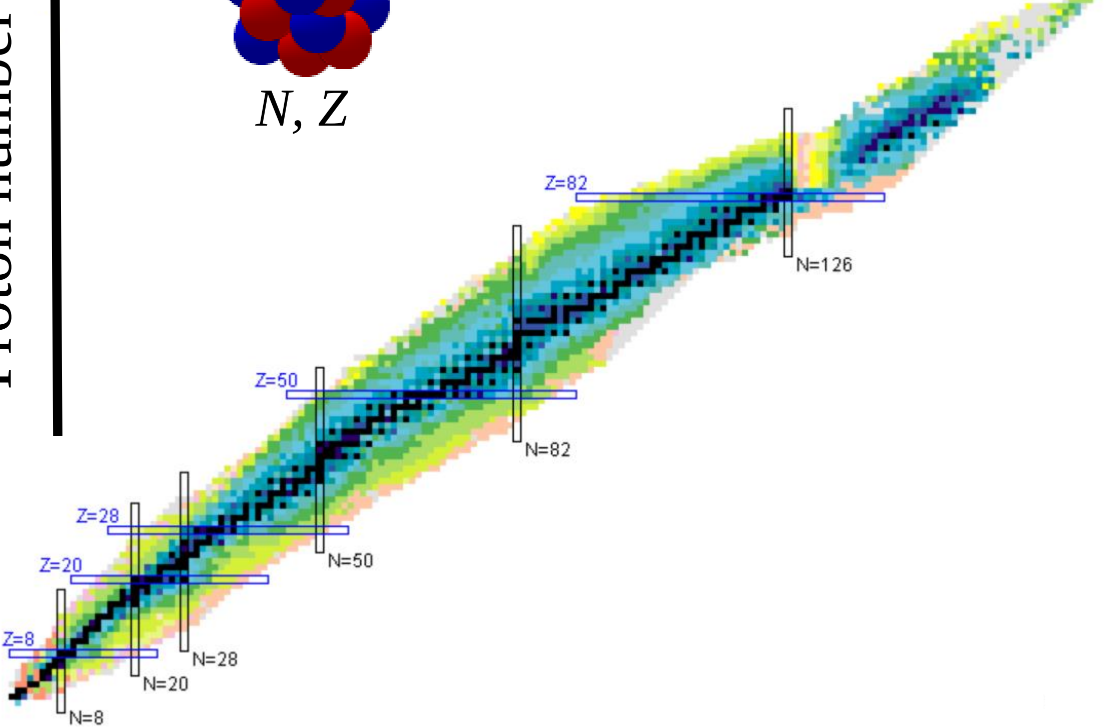


All nuclei can be described with a *single* EDF

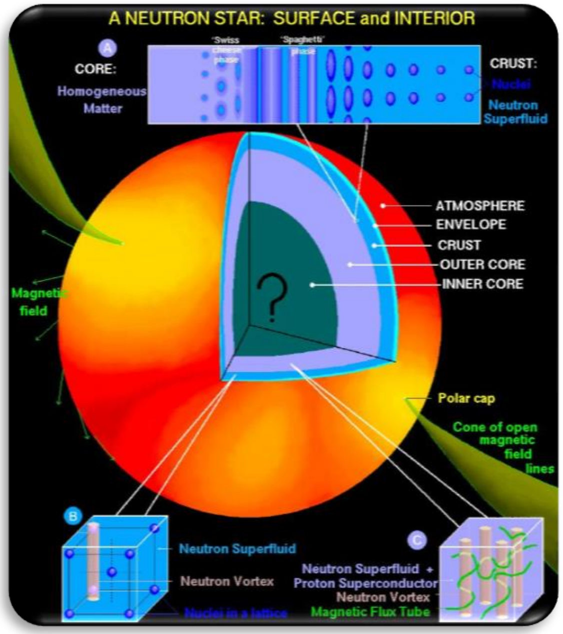


N, Z

Proton number



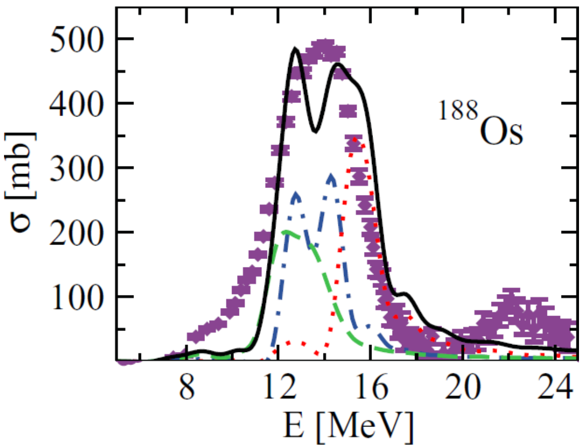
Neutron star



Neutron number

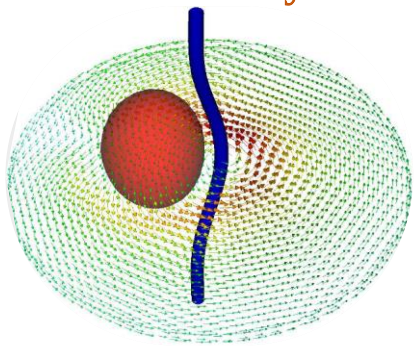
TDDFT is a versatile tool!!

IVGDR



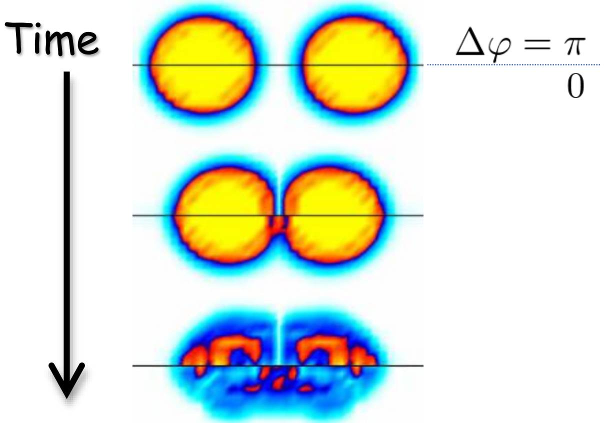
Phys. Rev. C **84**, 051309(R) (2011)
I. Stetcu, A. Bulgac, P. Magierski, and K.J. Roche

Vortex-nucleus dynamics



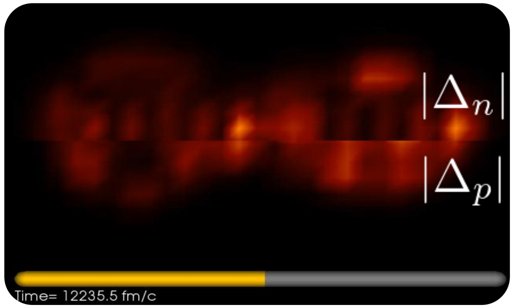
Phys. Rev. Lett. **117**, 232701 (2016)
G. Wlazłowski, K.S., P. Magierski, A. Bulgac, and M.M. Forbes

Low-energy heavy-ion reactions



Phys. Rev. Lett. **119**, 042501 (2017)
P. Magierski, K.S., and G. Wlazłowski

Induced fission of ^{240}Pu



Phys. Rev. Lett. **116**, 122504 (2016)
A. Bulgac, P. Magierski, K.J. Roche, and I. Stetcu

Voyage towards the limit of existence

The continent of stability has been well explored..

The Great Wall of China by Stacy Funderburke @shutterstock

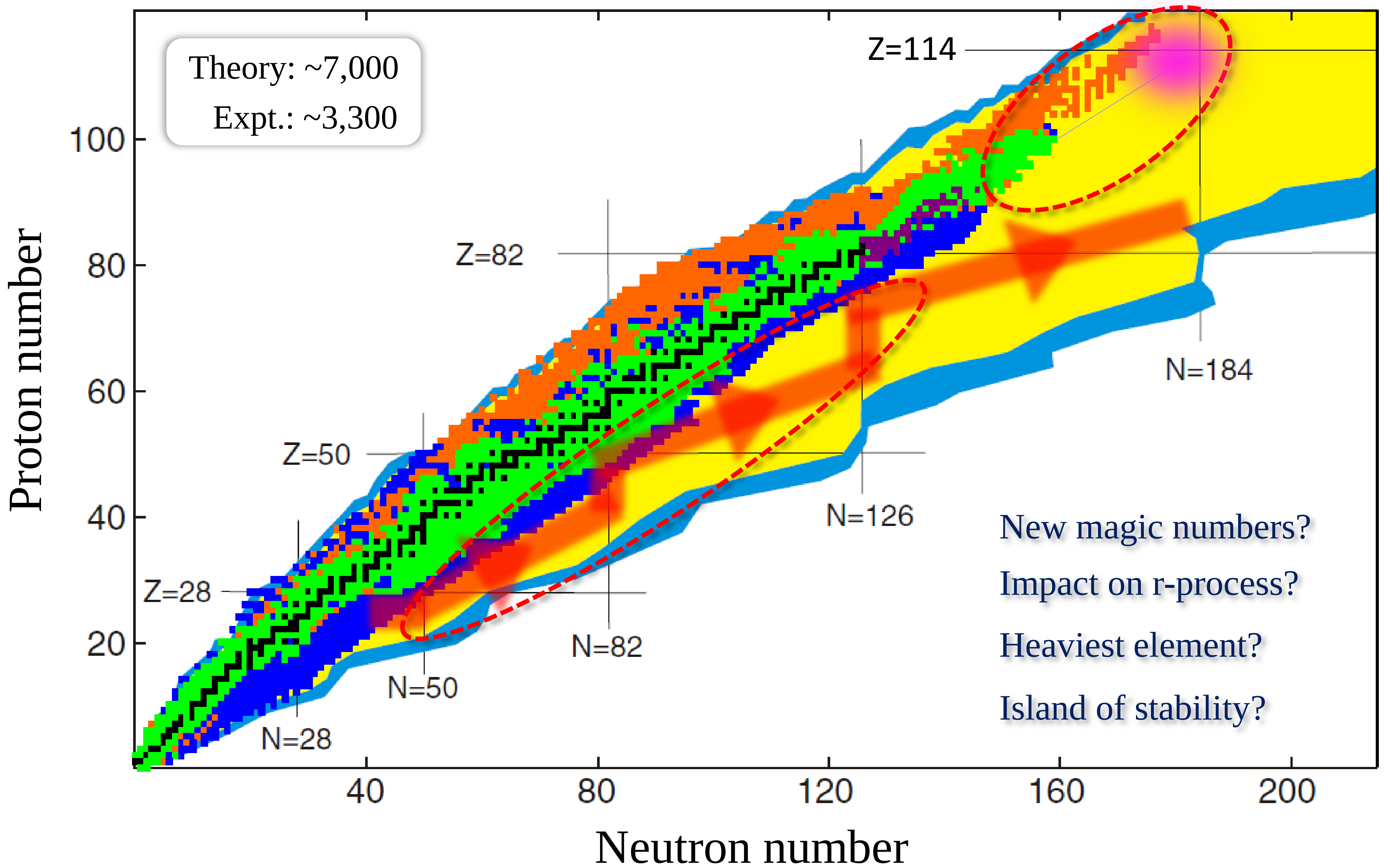
Now we are sailing towards the edge of the nuclear landscape..

Stable nuclei: 288
Experiment: ~3300
Theory: ~7000-10000

- drip lines
- shell structure
- deformation
- skin, halo
- nuclear matter properties
- nucleosynthesis
- ...

"Flat Earth" by iStock; "Fanciful view of ship sailing over edge of Earth" by Georgia Studies Images

How can we create yet-unknown neutron-rich nuclei?



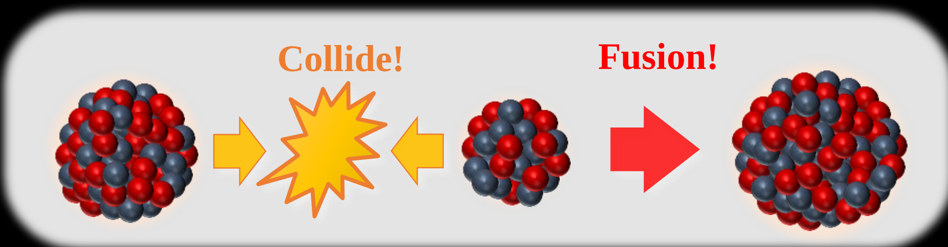
M. Thoennessen, Rep. Prog. Phys. **76**, 056301 (2013)

At the frontiers in nuclear physics:

Synthesis of superheavy elements

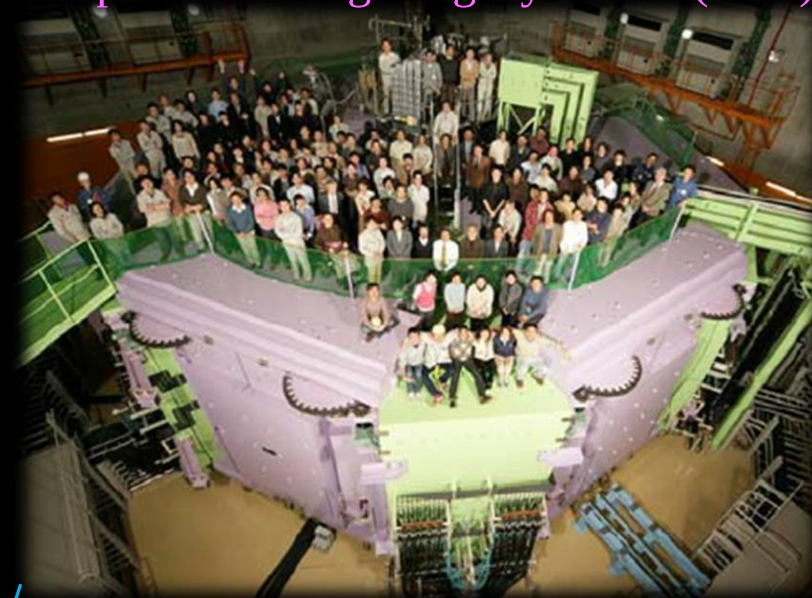
→ **also relevant to chemistry**

Nucleosynthesis at accelerator facilities

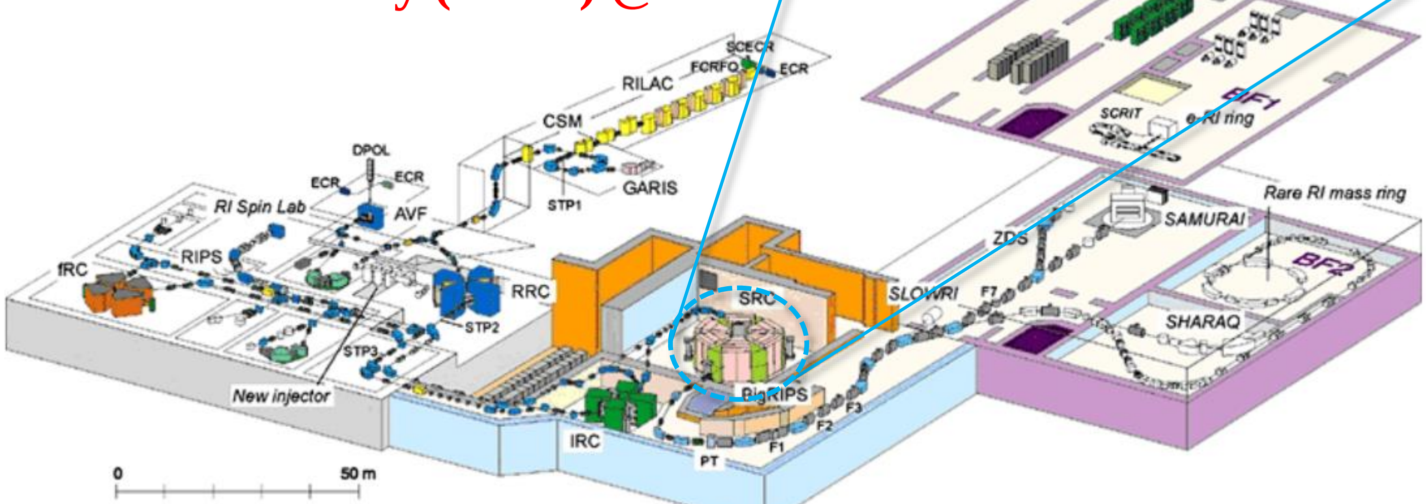


The world-leading factory of unstable nuclei!

Superconducting Ring Cyclotron (SRC)

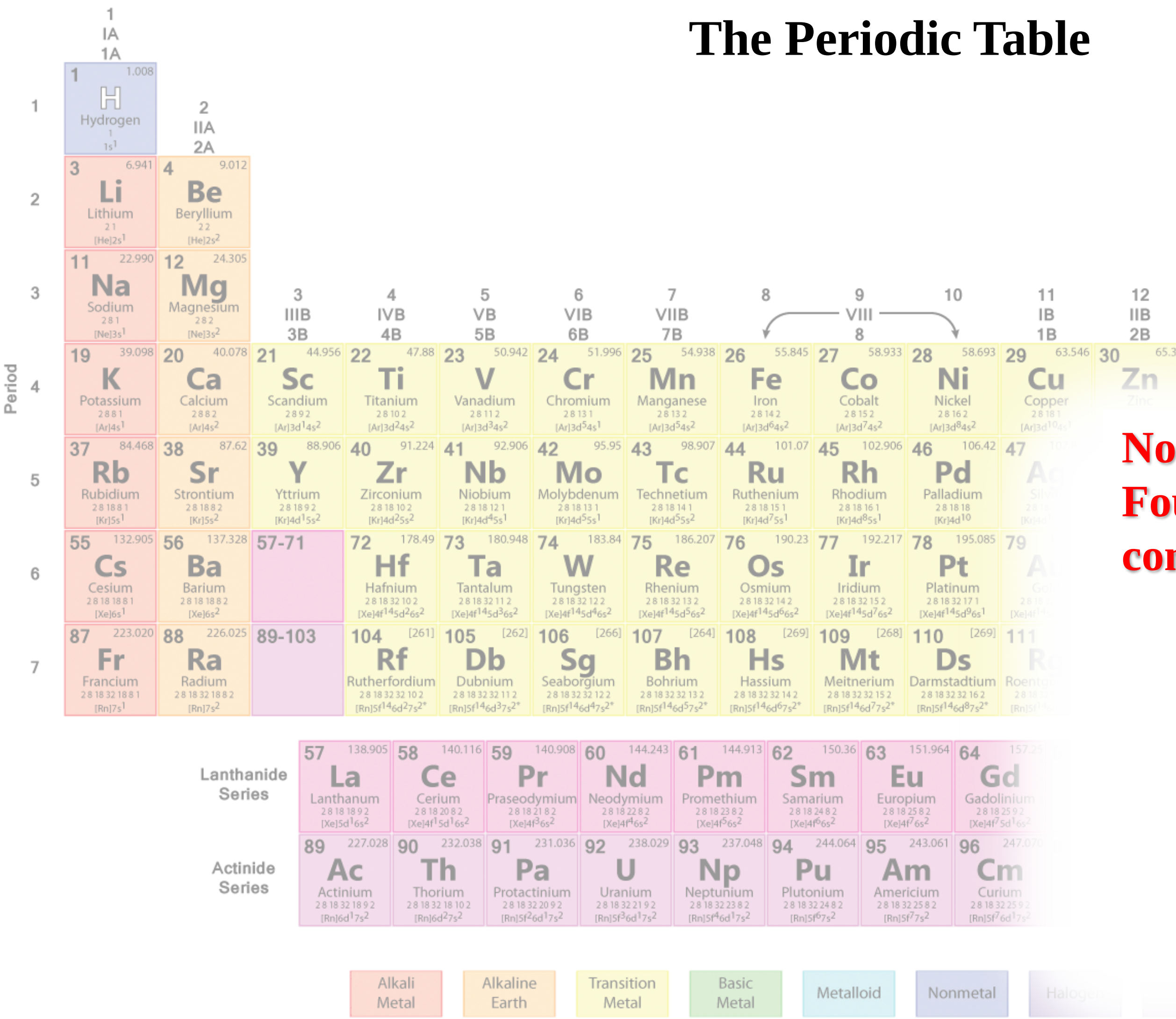


RI Beam Factory (RIBF) @RIKEN



The periodic table is growing!

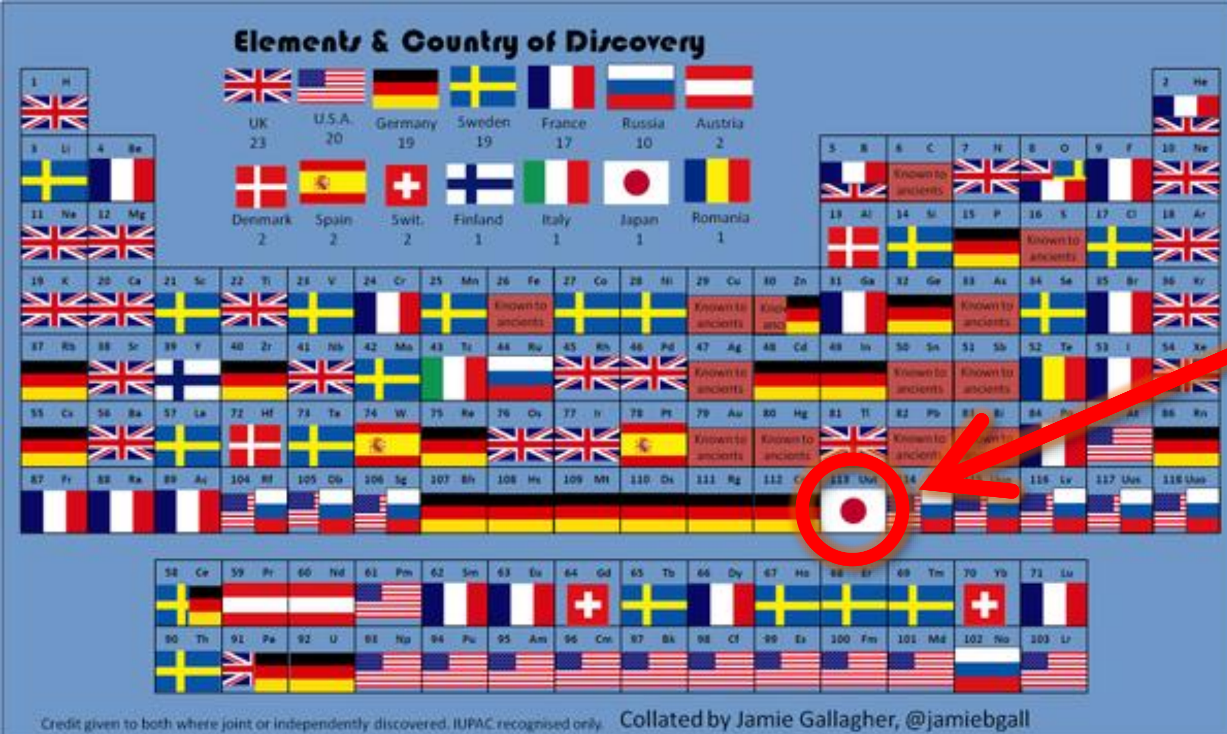
Synthesis of “superheavy element” is the most challenging subject in nuclear physics



November 2016
Four new elements were added, completing the seventh row!!

Nh	Mc	Ts	Og

113: Nh (Nihonium)
115: Ms (Moscovium)
117: Ts (Tennessine)
118: Og (Oganesson)



Nihonium is the only one discovered by Asian country:

The element 113

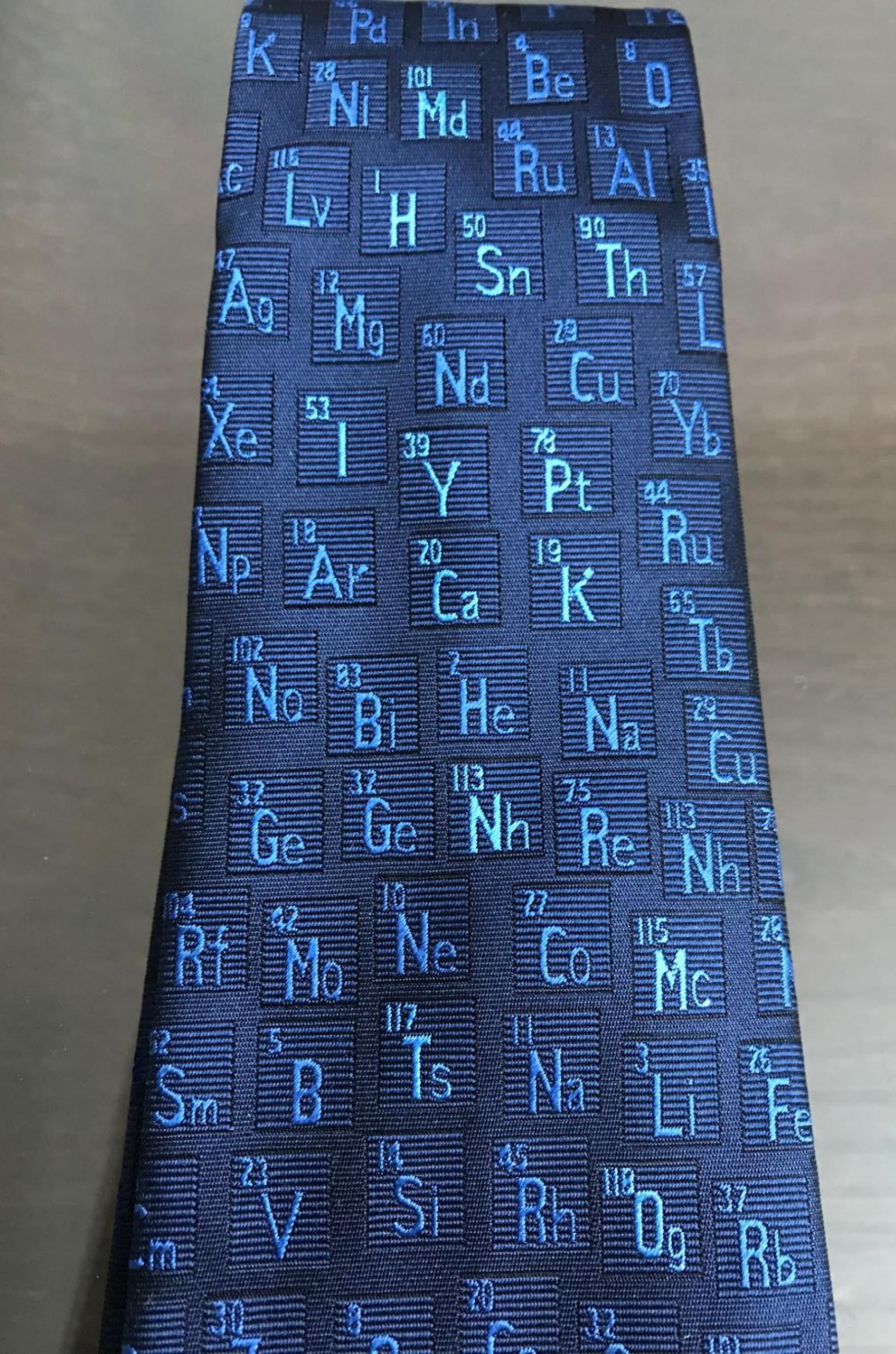
Nov. 30, 2016: Name confirmed

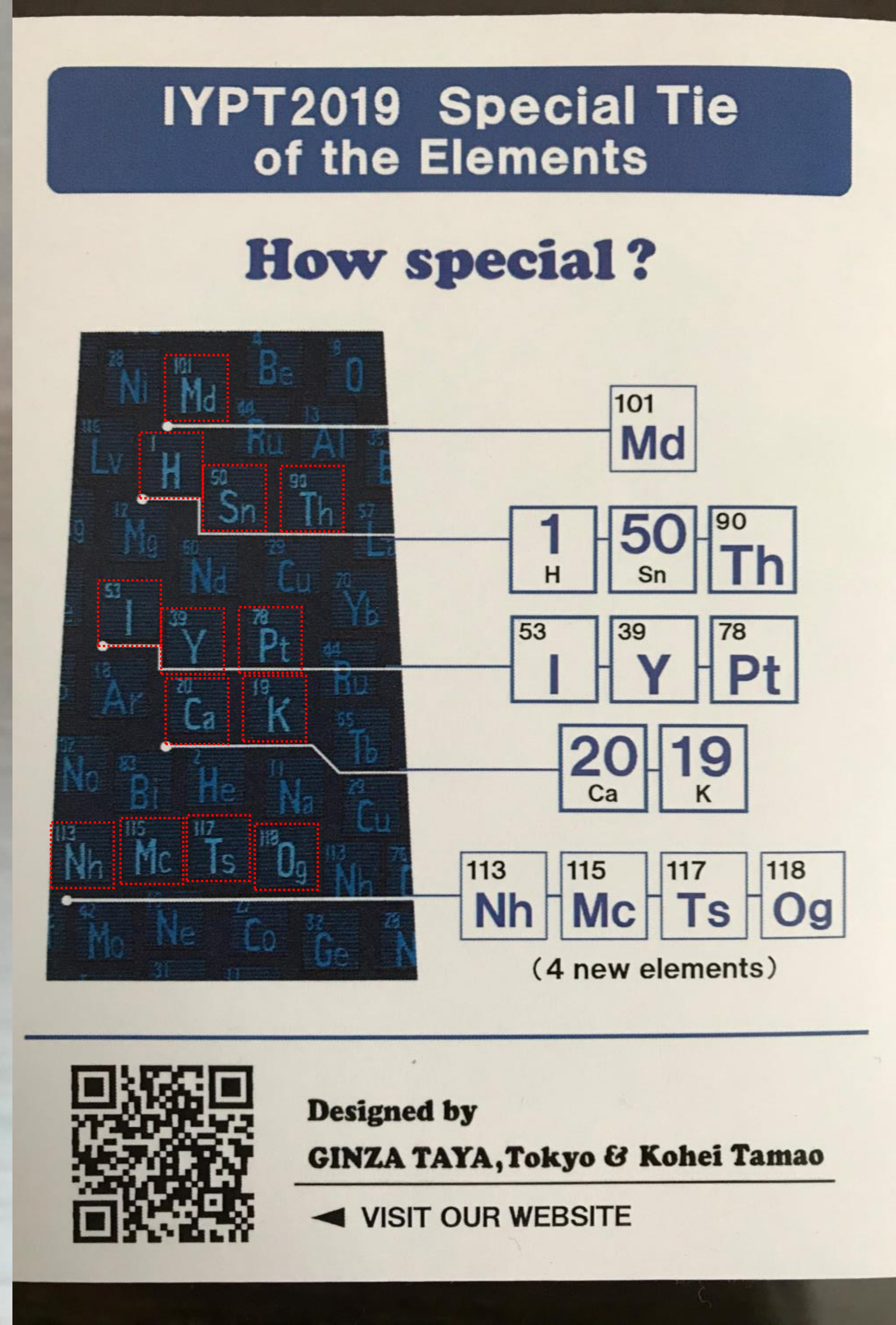
June 8, 2016: Name announced

Dec. 31, 2015: Naming rights given

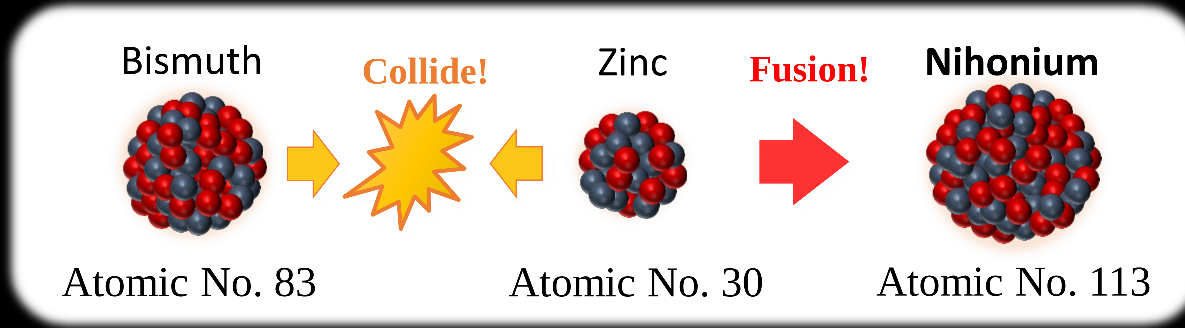
2019 was the 150th anniversary of the Periodic Table!

Dec. 5, 2019@Tokyo Prince Hotel




IYPT2019 Special Tie
of the Elements
How special ?
101
Md
1
H
50
Sn
90
Th
53
I
39
Y
78
Pt
20
Ca
19
K
113
Nh
115
Mc
117
Ts
118
Og
(4 new elements)
Designed by
GINZA TAYA, Tokyo & Kohei Tamao
◀ VISIT OUR WEBSITE

A long way to synthesize Nihonium



Sep. 2003 expt. started --> ~100 days irradiation --> ~100 days irradiation --> ~360 days irradiation

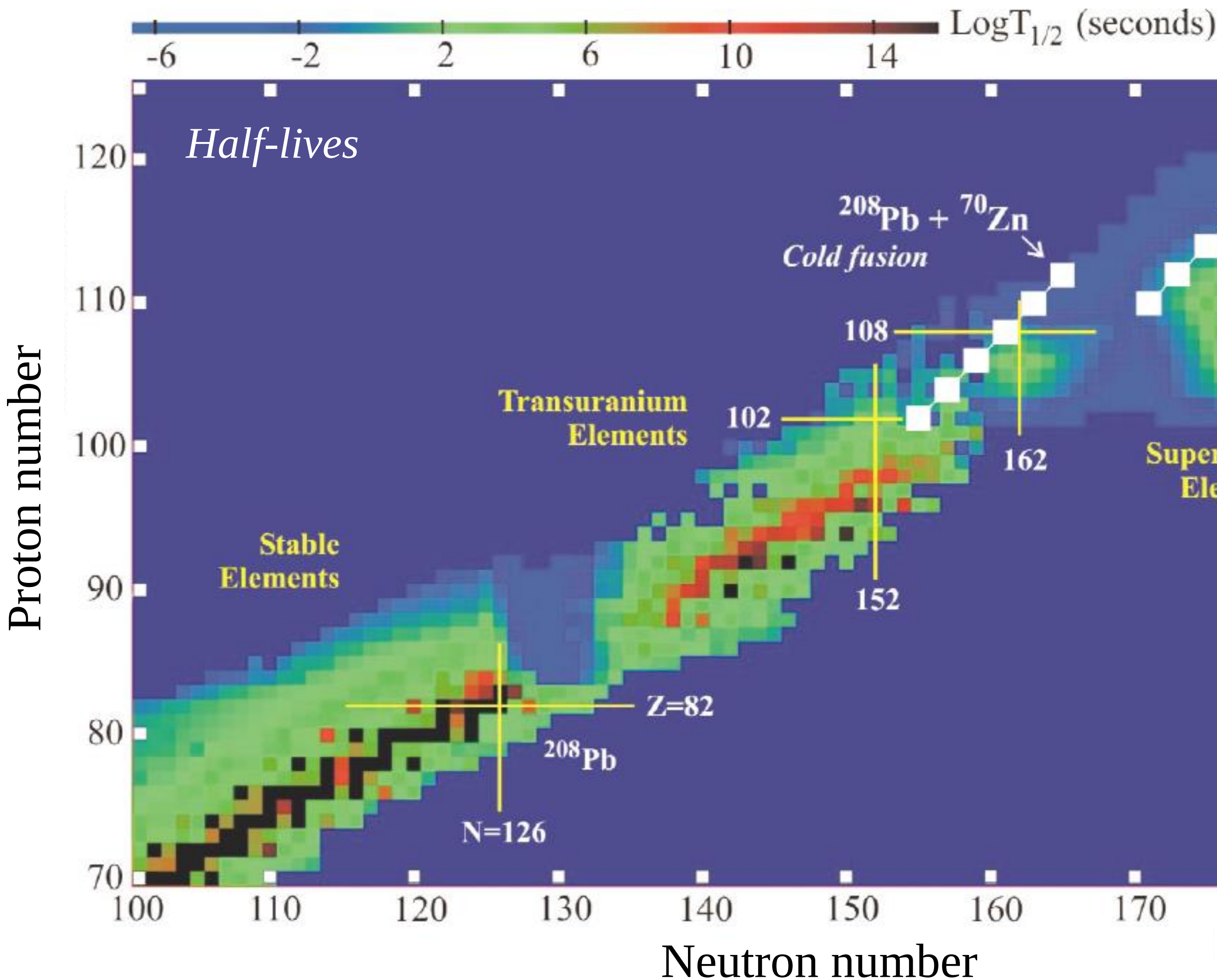
July 23, 2004 | April 2, 2005 | Aug. 12, 2012

~ 2.4×10^{12} nuclei per second

Not confident enough as Bh ⟶ Db α-decay was measured only once in the past

1/3 ⟶ fission
2/3 ⟶ α-decay

The north-east part of the nuclear map

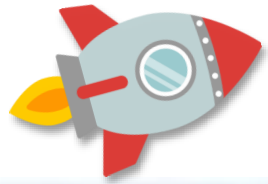


*10^8 sec ~ 3 years

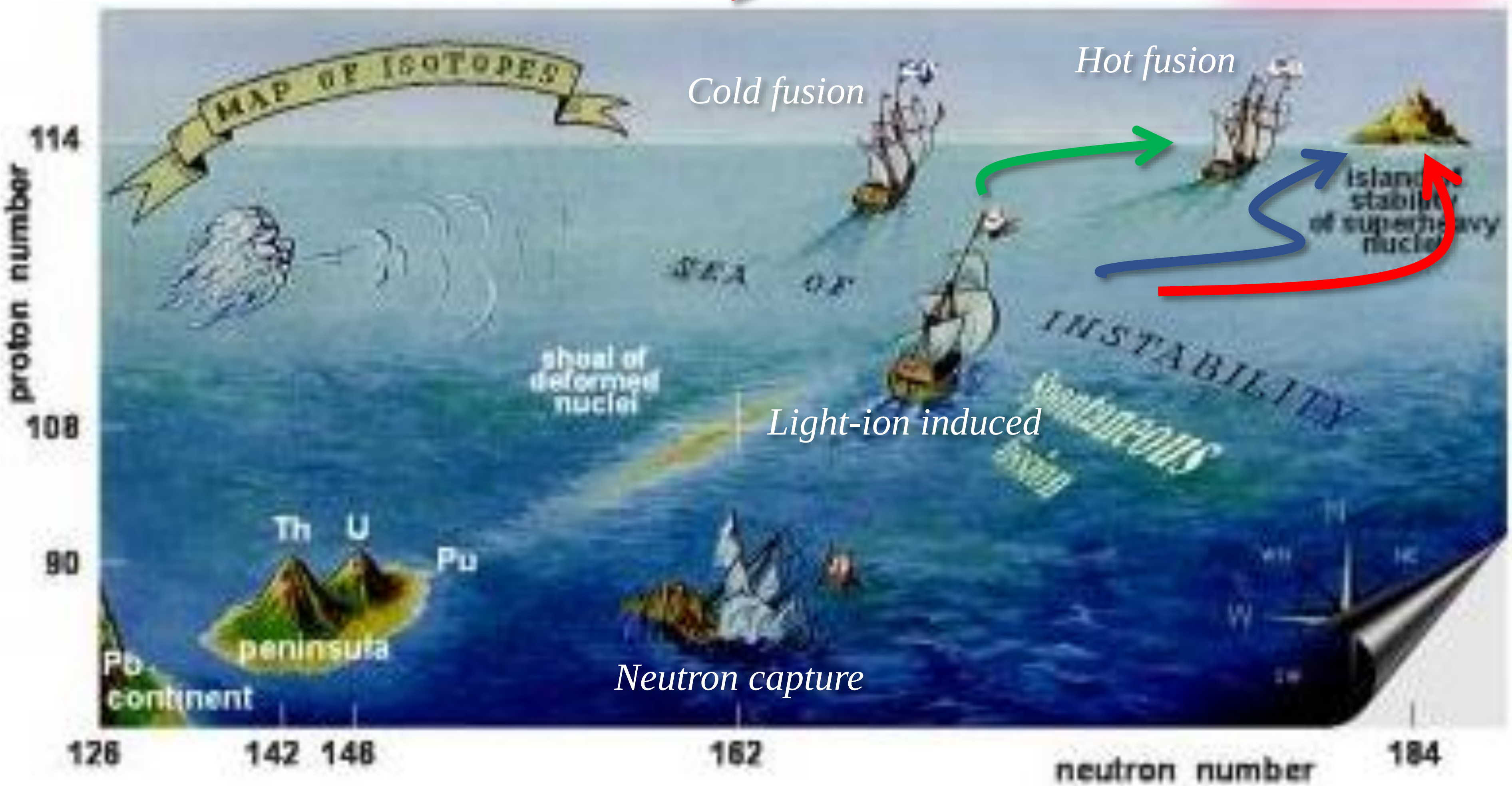
Yuri Oganessian, Pure Appl. Chem. **78**, 889 (2006)

The north-east part of the nuclear map

We do need theoretical predictions!

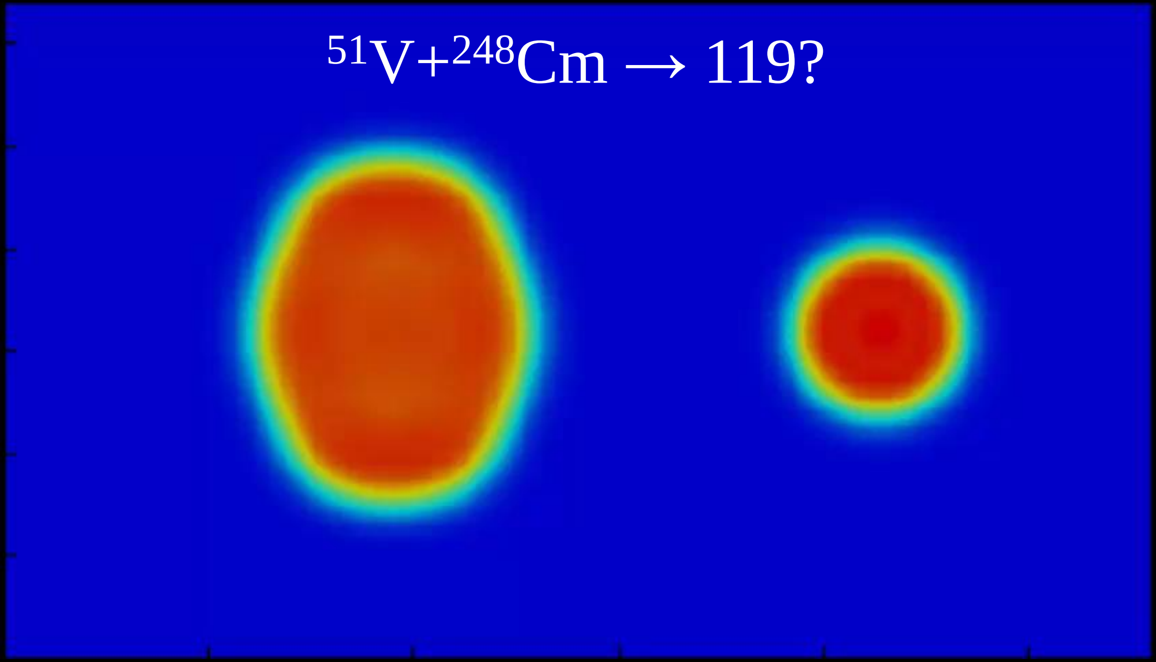


Z = 119, 120, ...



The next is: 119 or 120

TDDFT simulation for SHE synthesis on a supercomputer



Collaborative works are in progress with Kyoto U., Kindai U., and Kyushu U.
cf. K.S. and K. Hagino, Phys. Rev. C **99**, 051602(R) (2019)

Now we are sailing towards the edge of the nuclear landscape..

Let’s leave the planet
of finite nuclei!

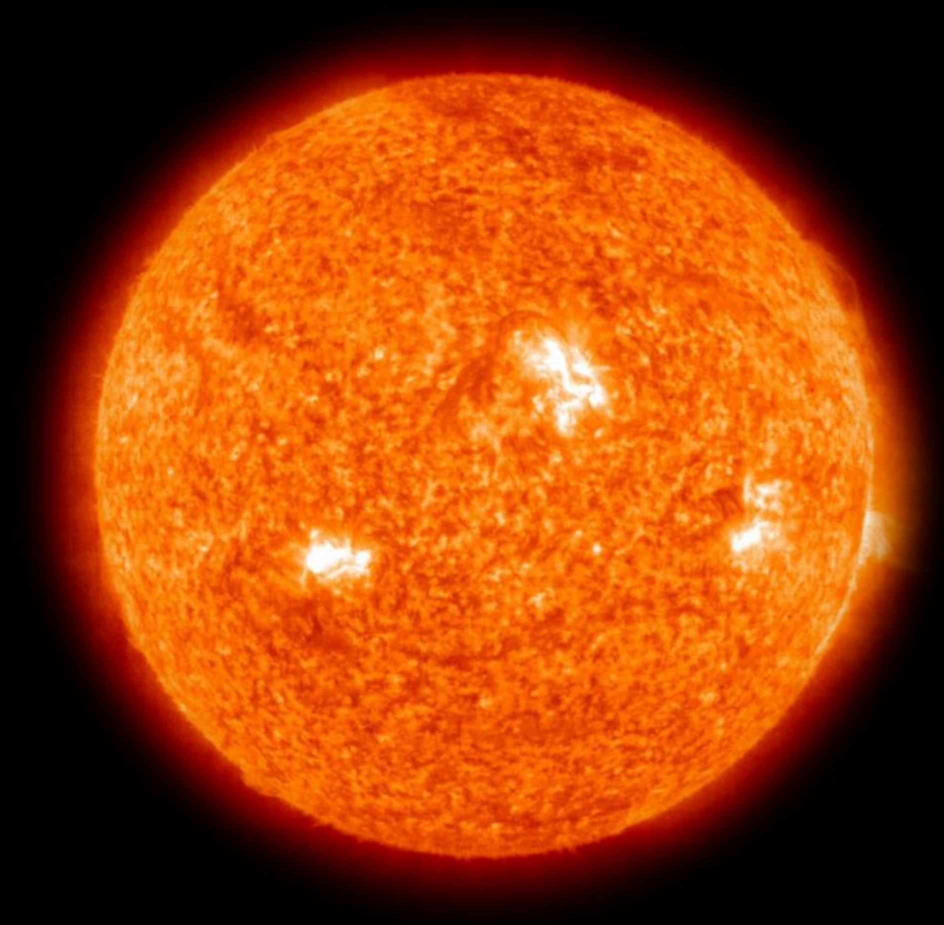
“Flat Earth” by iStock; “Star Wars Spaceship” by Valérian Pierret @Artstation

From nuclei to **neutron stars**

“3d hyperspace background with warp tunnel effect Free Photo” @freepik

The Sun

- Radius: ~7×10^8m (~109 times bigger than Earth)
- Mass: ~2×10^{30}kg (~330 thousands times heavier than Earth)
- Central temp.: ~10 million ℃
- Surface temp.: ~5000 ℃

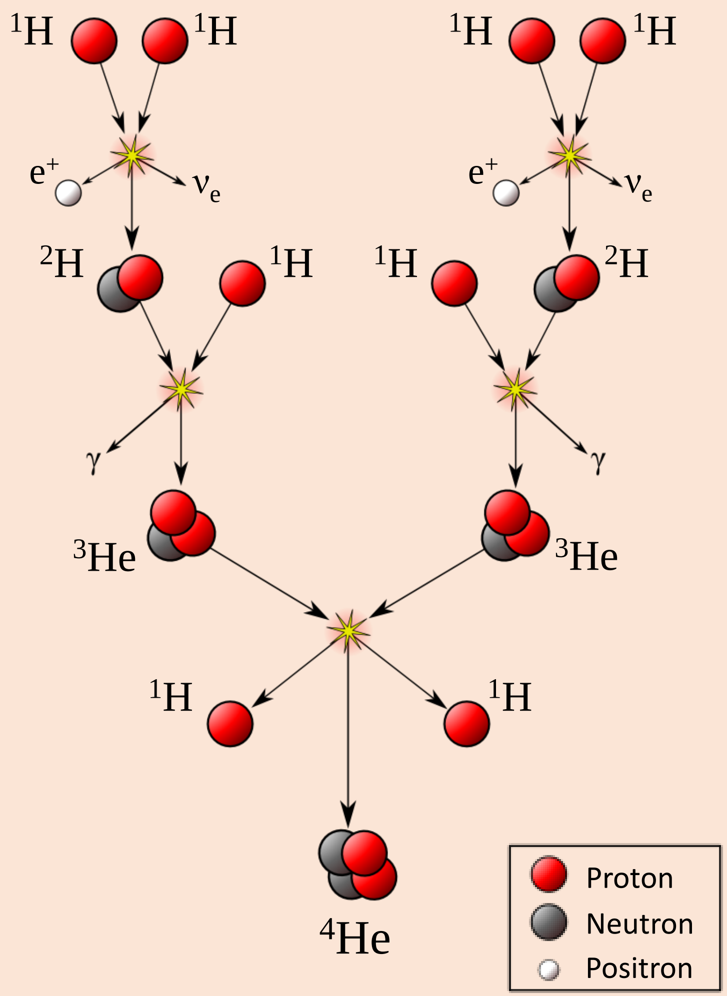


Stars shine due to nuclear fusion reactions

Picture: https://en.wikipedia.org/wiki/Sun

Energy source of the Sun: Nuclear fusion

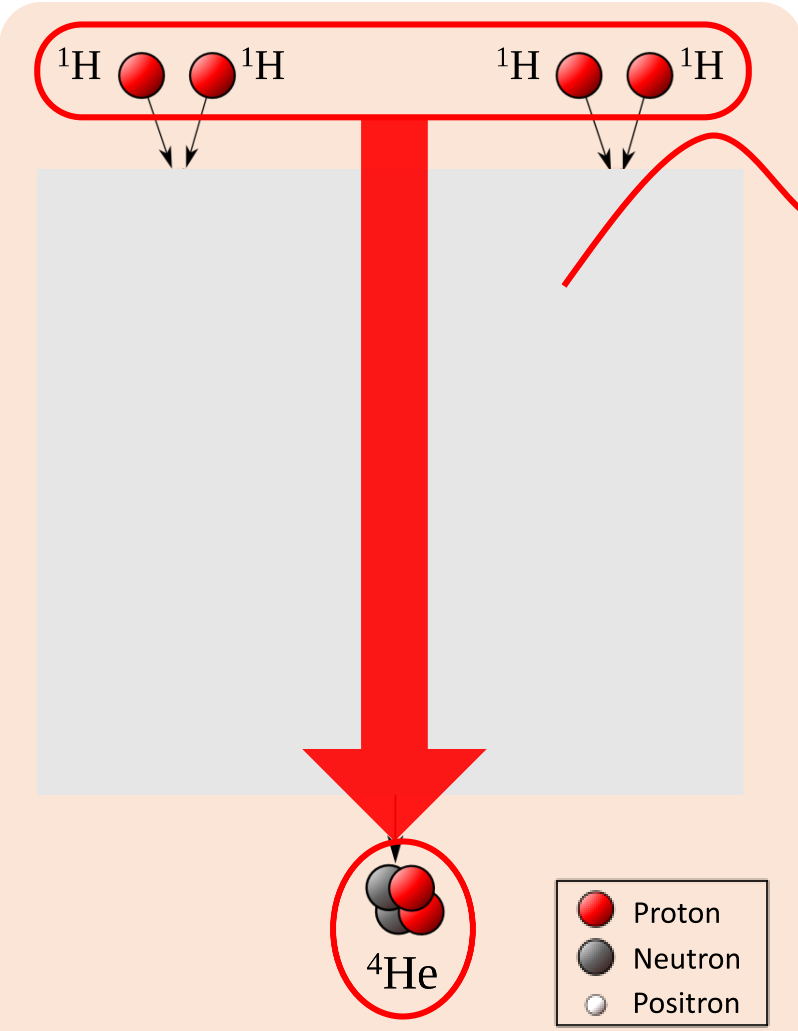
Proton-proton (p-p) chain



$$4p + 2e^- \rightarrow {}^4He + 6\,\gamma + 2\nu_e + 26.65\,MeV$$

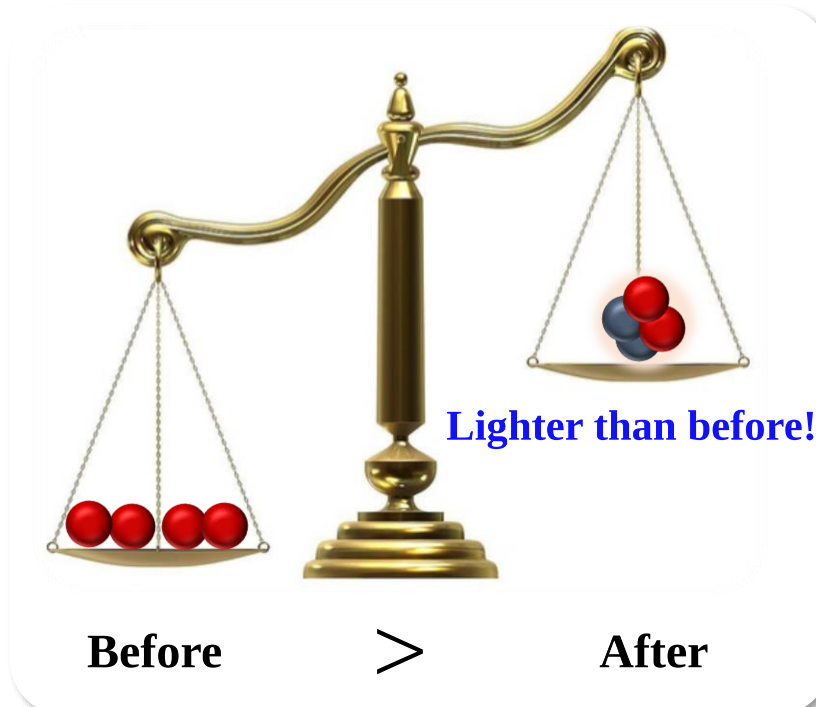
Energy source of the Sun: Nuclear fusion

Proton-proton (p-p) chain



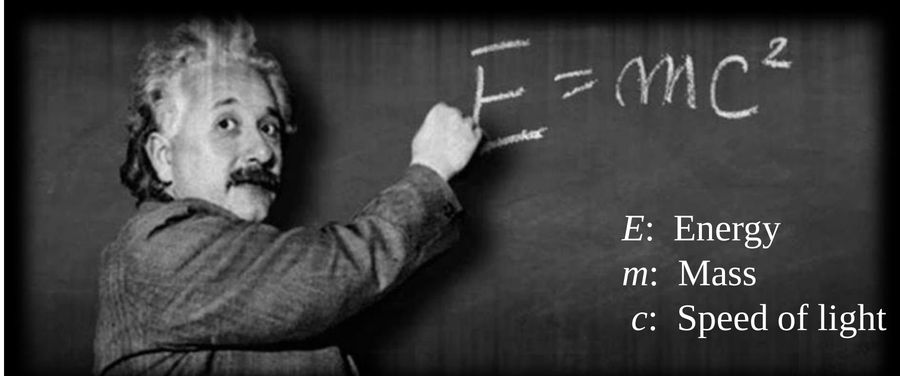
$$4\mathrm{p} + 2\mathrm{e}^- \rightarrow {}^4\mathrm{He} + 6\,\gamma + 2\nu_\mathrm{e} + 26.65\,\mathrm{MeV}$$

What happened:
Four protons became a helium nucleus!



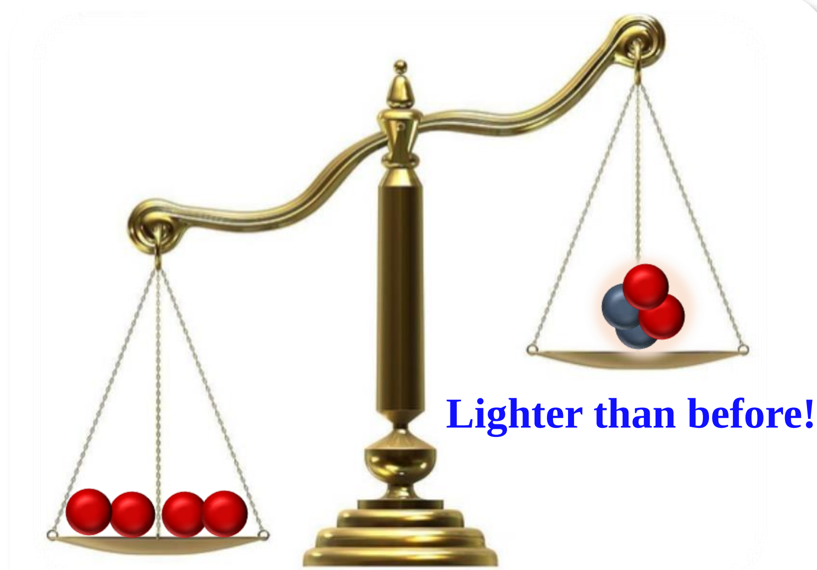
Energy source of the Sun: Nuclear fusion

Equivalence between mass and energy



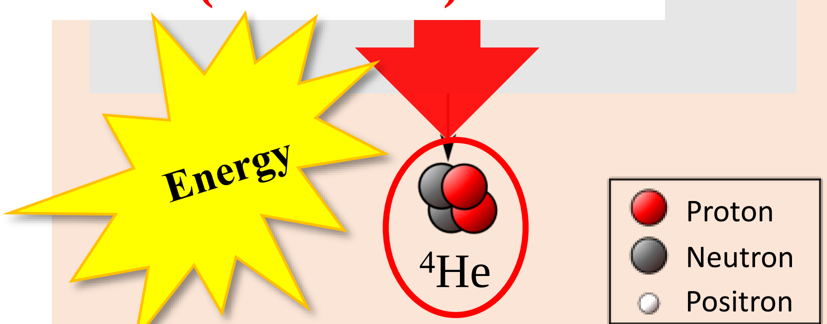
Albert Einstein (1879-1955)

What happened:
Four protons became a helium nucleus!



Before > After

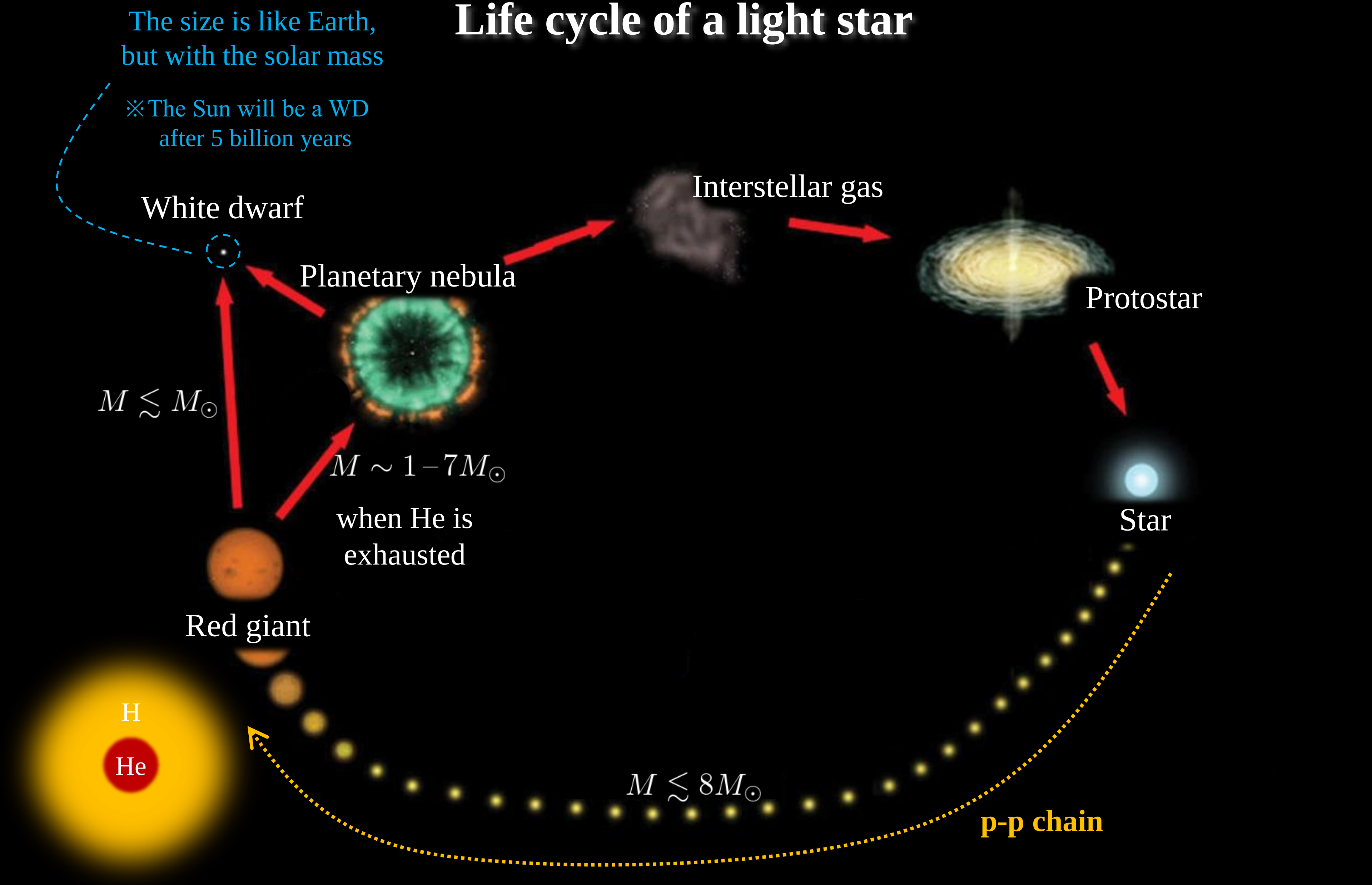
This reaction generates energy (exothermic)



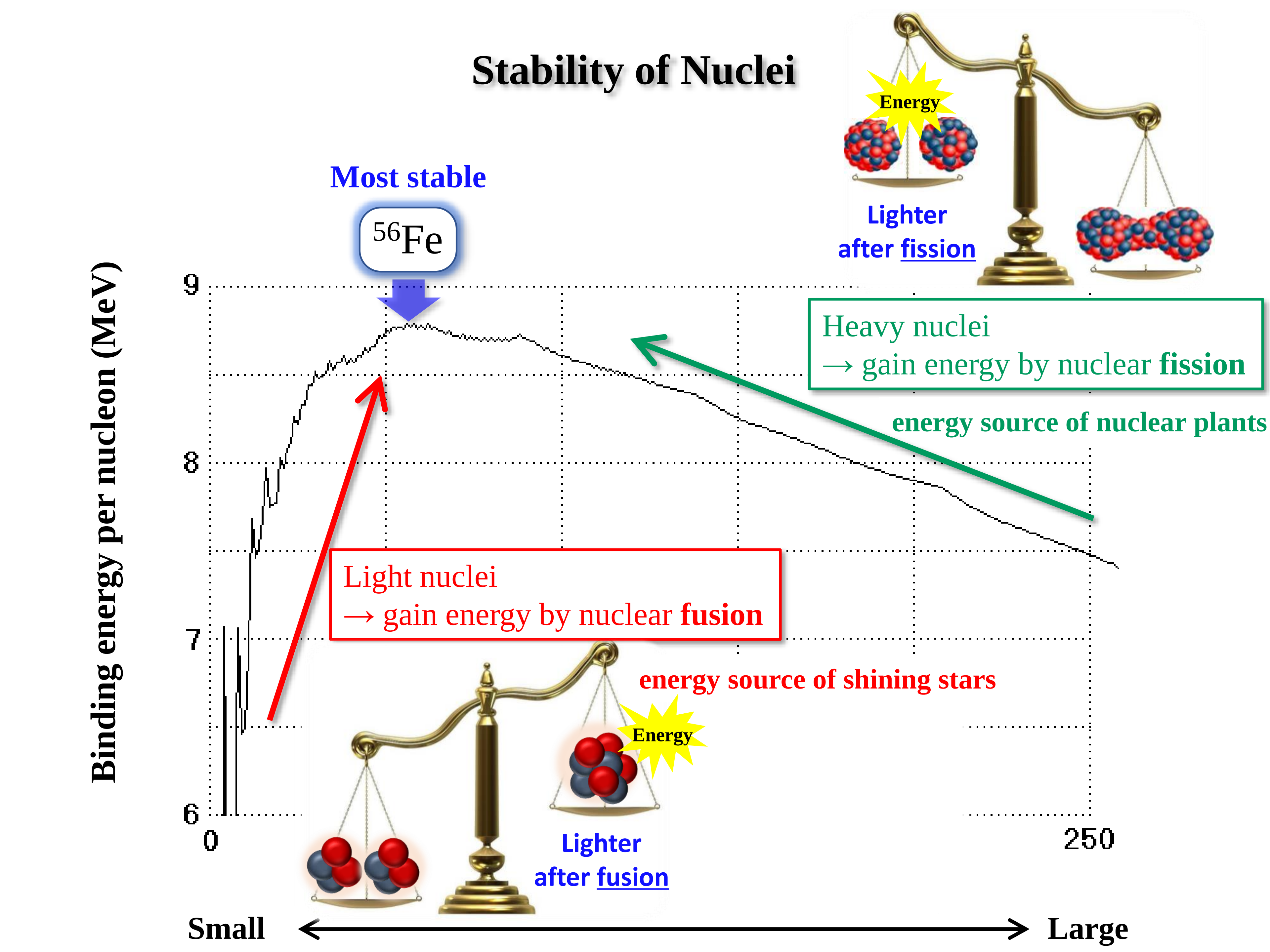
$$4\mathrm{p} + 2\mathrm{e}^- \rightarrow {}^4\mathrm{He} + 6\,\gamma + 2\nu_\mathrm{e} + 26.65\,\mathrm{MeV}$$

Quantum tunneling allows for overcoming the Coulomb barrier!

Life cycle of a light star

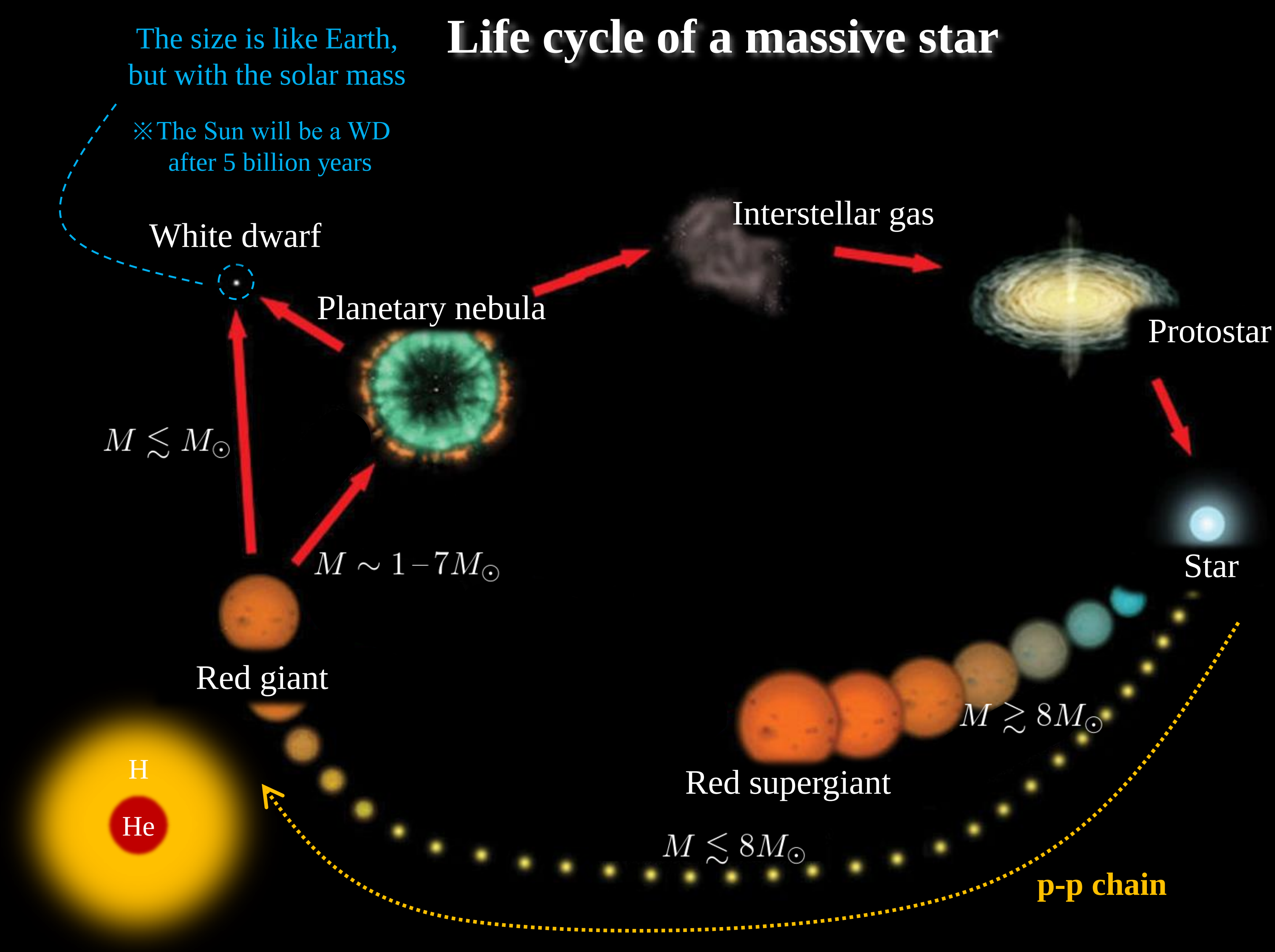


Picture: https://www.astroarts.co.jp/products/stlnav7/spec/plane_list-j.shtml



Stability of Nuclei
Most stable
56Fe
Binding energy per nucleon (MeV)
9
8
7
6
0
250
Small
Large
Heavy nuclei
→ gain energy by nuclear fission
energy source of nuclear plants
Light nuclei
→ gain energy by nuclear fusion
energy source of shining stars
Energy
Lighter
after fission
Energy
Lighter
after fusion

Life cycle of a massive star



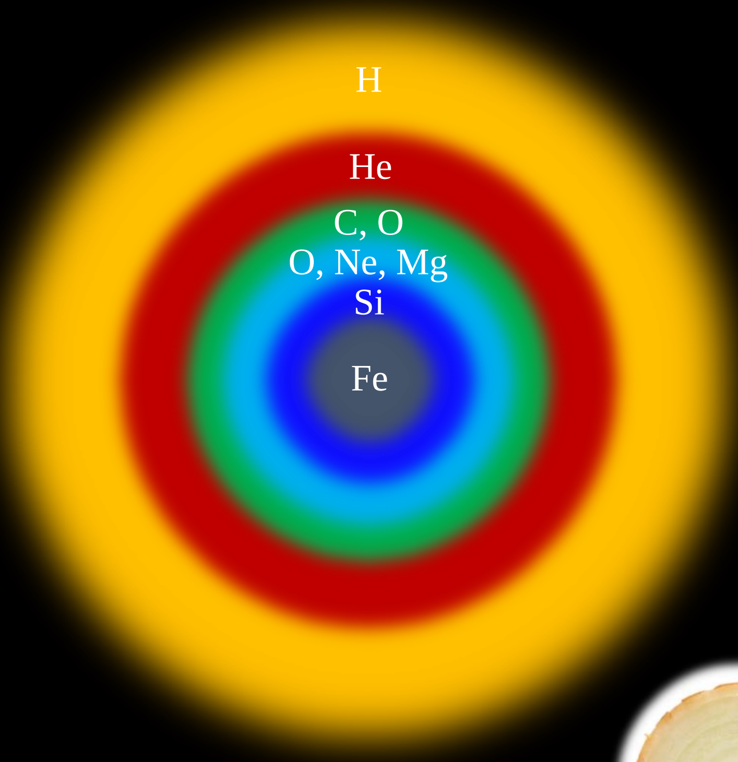
Picture: https://www.astroarts.co.jp/products/stlnav7/spec/plane_list-j.shtml

The fate of a massive star

Nuclear reactions:

$^{1}H \rightarrow {}^{4}He \rightarrow {}^{12}C \rightarrow {}^{16}O \rightarrow {}^{20}Ne \rightarrow {}^{24}Mg \rightarrow {}^{28}Si \rightarrow \ldots \rightarrow {}^{56}Fe$

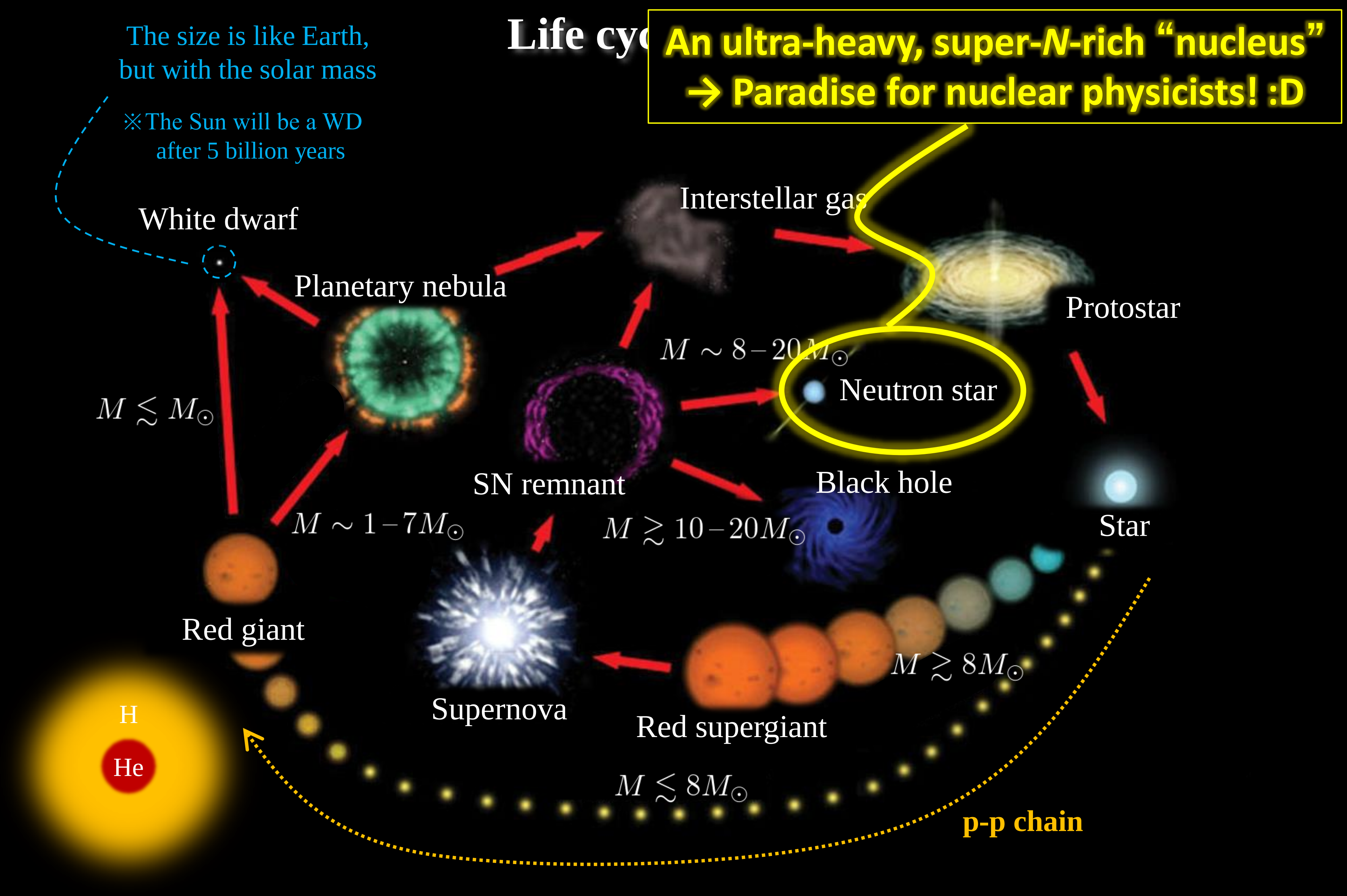
“Onion structure”



After forming the iron core…
→ no more fuel
→ gravitational collapse
→ supernova explosion

The Crab Nebula
Remnant of the SN in 1054

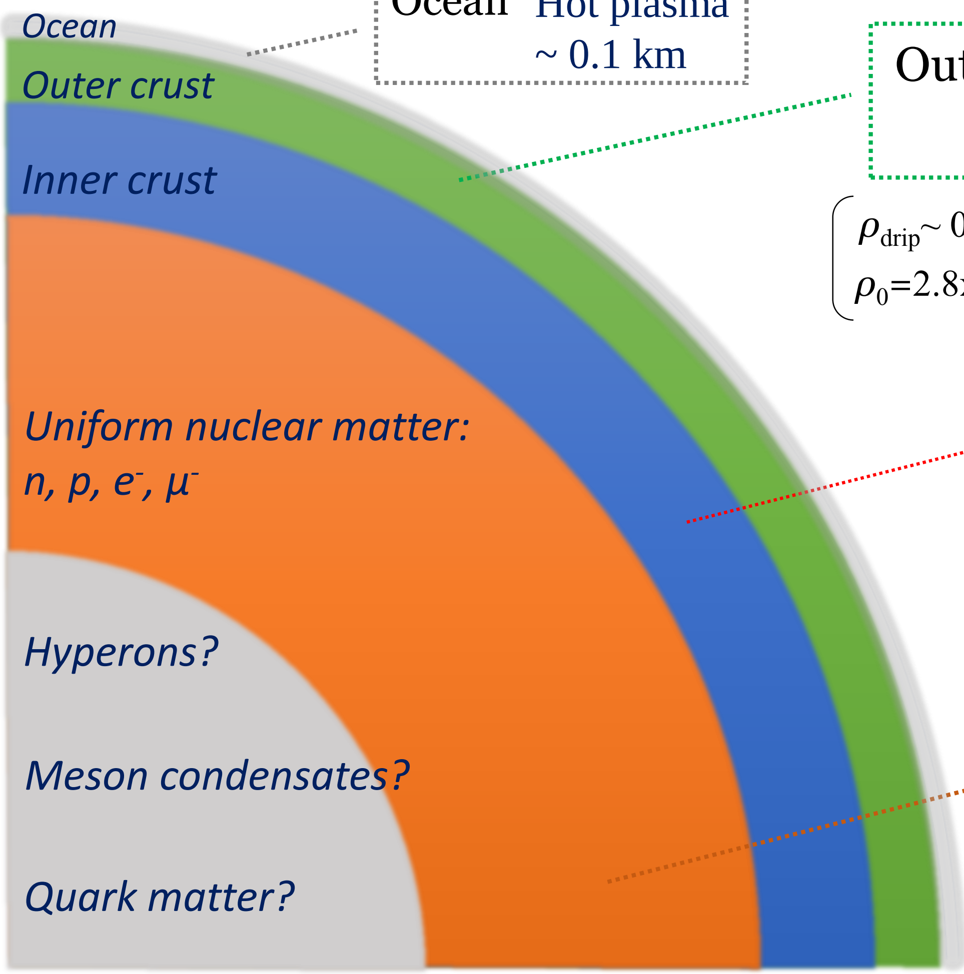
Picture: https://en.wikipedia.org/wiki/Crab_Nebula



Picture: https://www.astroarts.co.jp/products/stlnav7/spec/plane_list-j.shtml

Structure of a neutron star

Neutron star is a great playground for nuclear physicists



Ocean Hot plasma ~ 0.1 km

Outer crust Nuclei (Fe), electrons

0.3-0.6 km $\rho < \rho_{\rm drip}$

$\rho_{\rm drip} \sim 0.0014\rho_0$; Above $\rho_{\rm drip}$ unbound neutrons exist outside nuclei

$\rho_0 = 2.8 \times 10^{14}$ g/cm^3 $= 0.16$ fm^{-3} ; Nuclear saturation density

Inner crust Nuclei, electrons, "dripped" neutrons

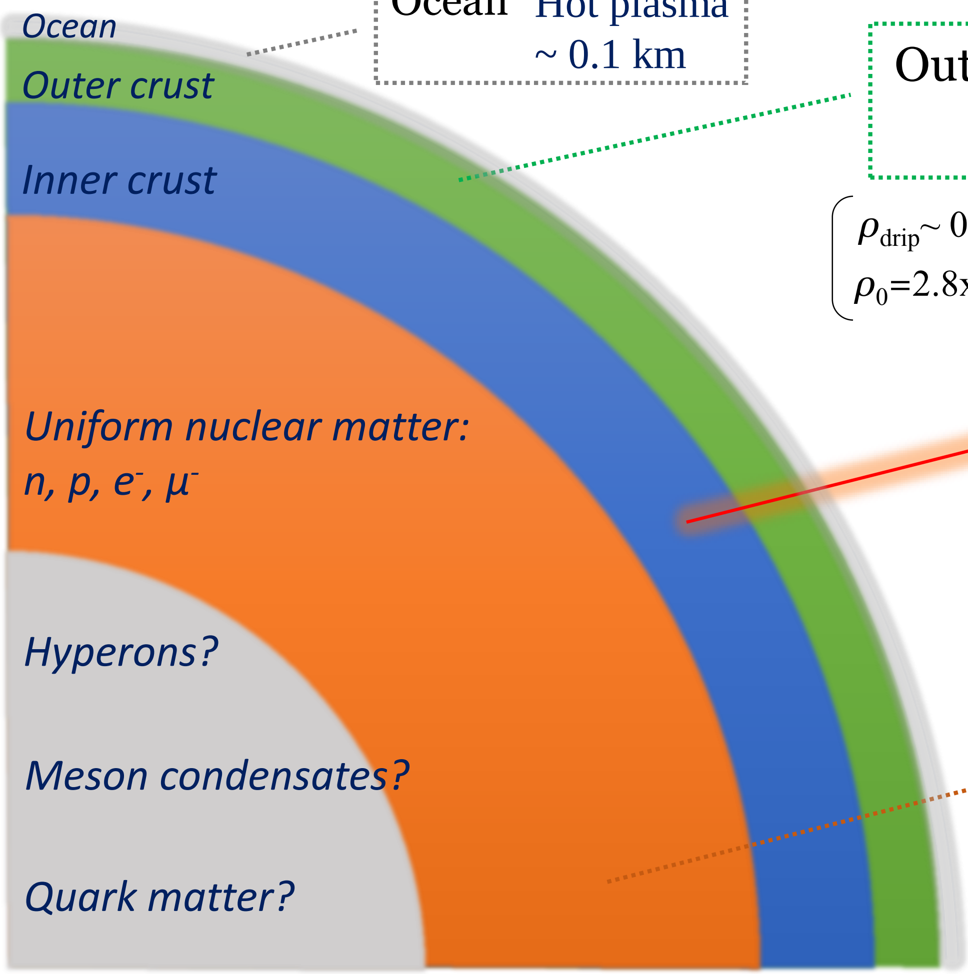
0.5-0.8 km $\rho_{\rm drip} \leq \rho \leq 0.6\rho_0$

Outer/inner core

9-12 km $0.6\rho_0 < \rho \leq 3\text{-}5\rho_0$

Structure of a neutron star

Neutron star is a great playground for nuclear physicists



Ocean Hot plasma
~ 0.1 km

Outer crust Nuclei (Fe), electrons
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$\rho_{drip} \sim 0.0014\rho_0$; Above ρ_{drip} unbound neutrons exist outside nuclei
$\rho_0 = 2.8\text{x}10^{14}$ g/cm^3=0.16 fm^{-3} ; Nuclear saturation density

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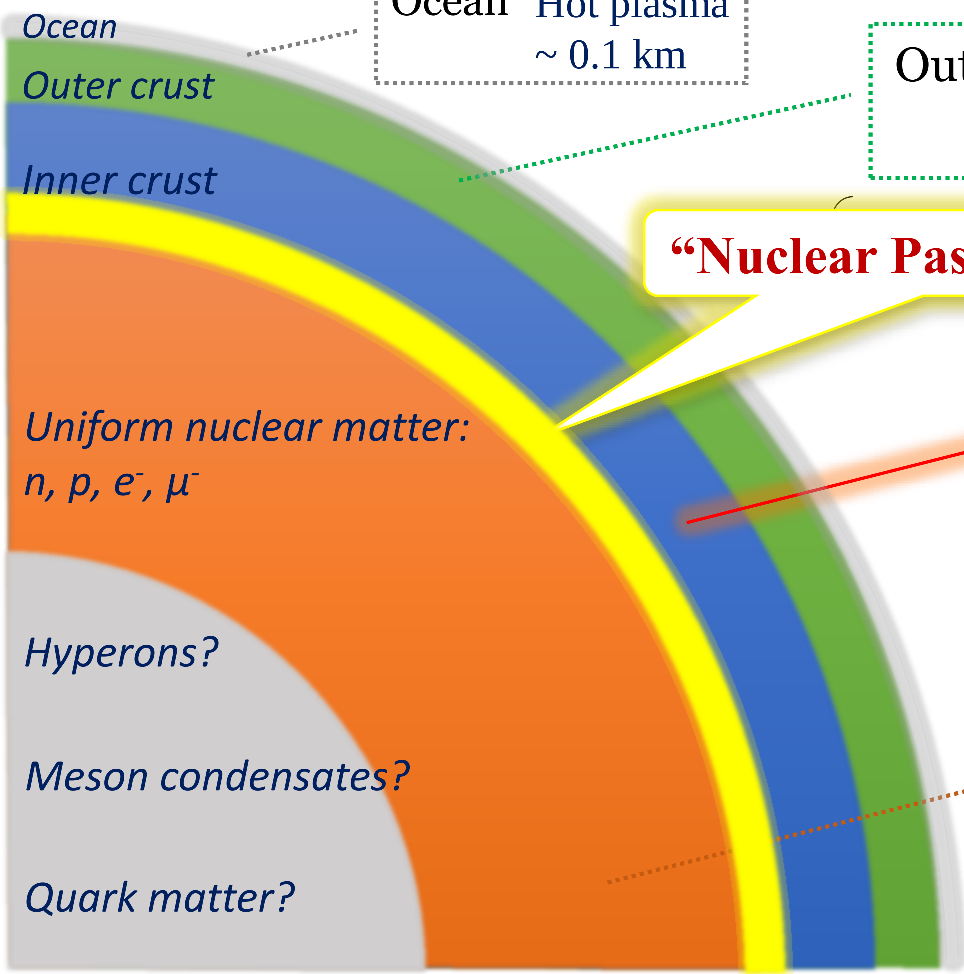
Neutron star is a great playground for nuclear physicists



Ocean Hot plasma
~ 0.1 km

Outer crust Nuclei (Fe), electrons
0.3-0.6 km $\rho < \rho_{drip}$

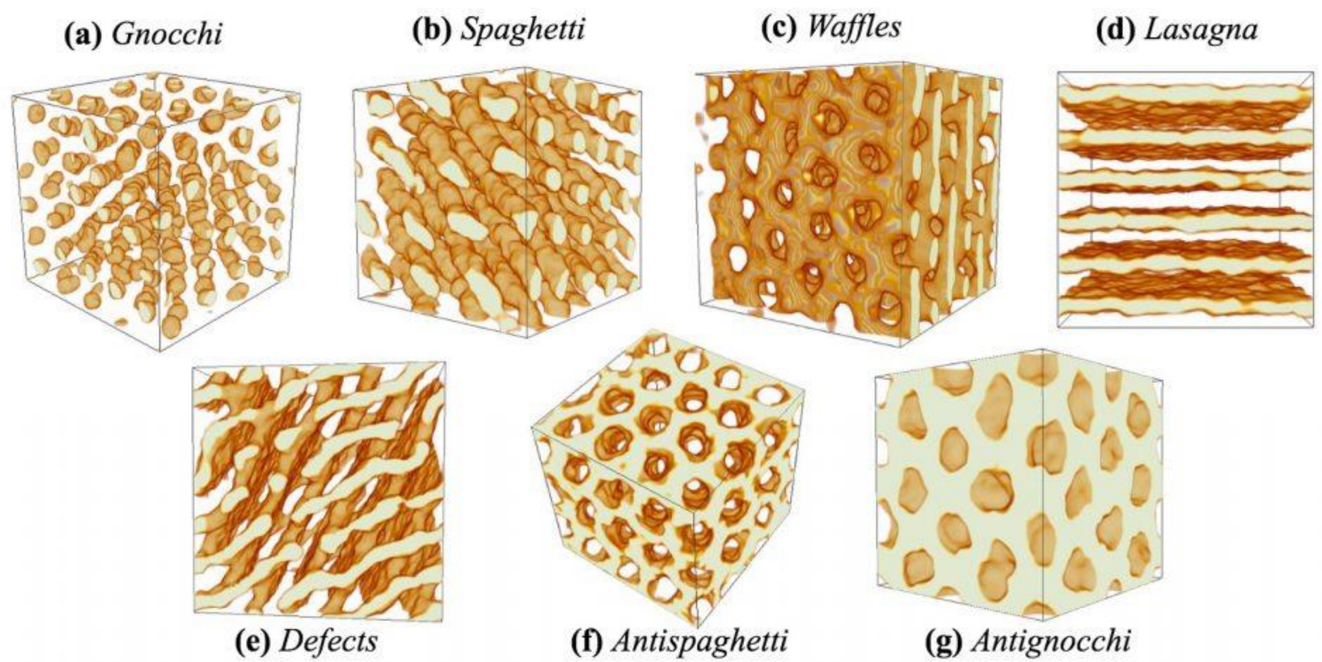
…ound neutrons exist outside nuclei
; Nuclear saturation density

"Nuclear Pasta" may exist!

Inner crust Nuclei, electrons, "dripped" neutrons
0.5-0.8 km $\rho_{drip} \leq \rho \leq 0.6\rho_0$

Outer/inner core
9-12 km $0.6\rho_0 < \rho \leq 3\text{-}5\rho_0$

What is Nuclear Pasta?



M. E. Caplan and C. J. Horowitz, Rev. Mod. Phys. **89**, 041002 (2017)

What is Nuclear Pasta?

We have developed

a time-dependent band theory based on TDDFT

for nuclear dynamics in crystalline pasta phases

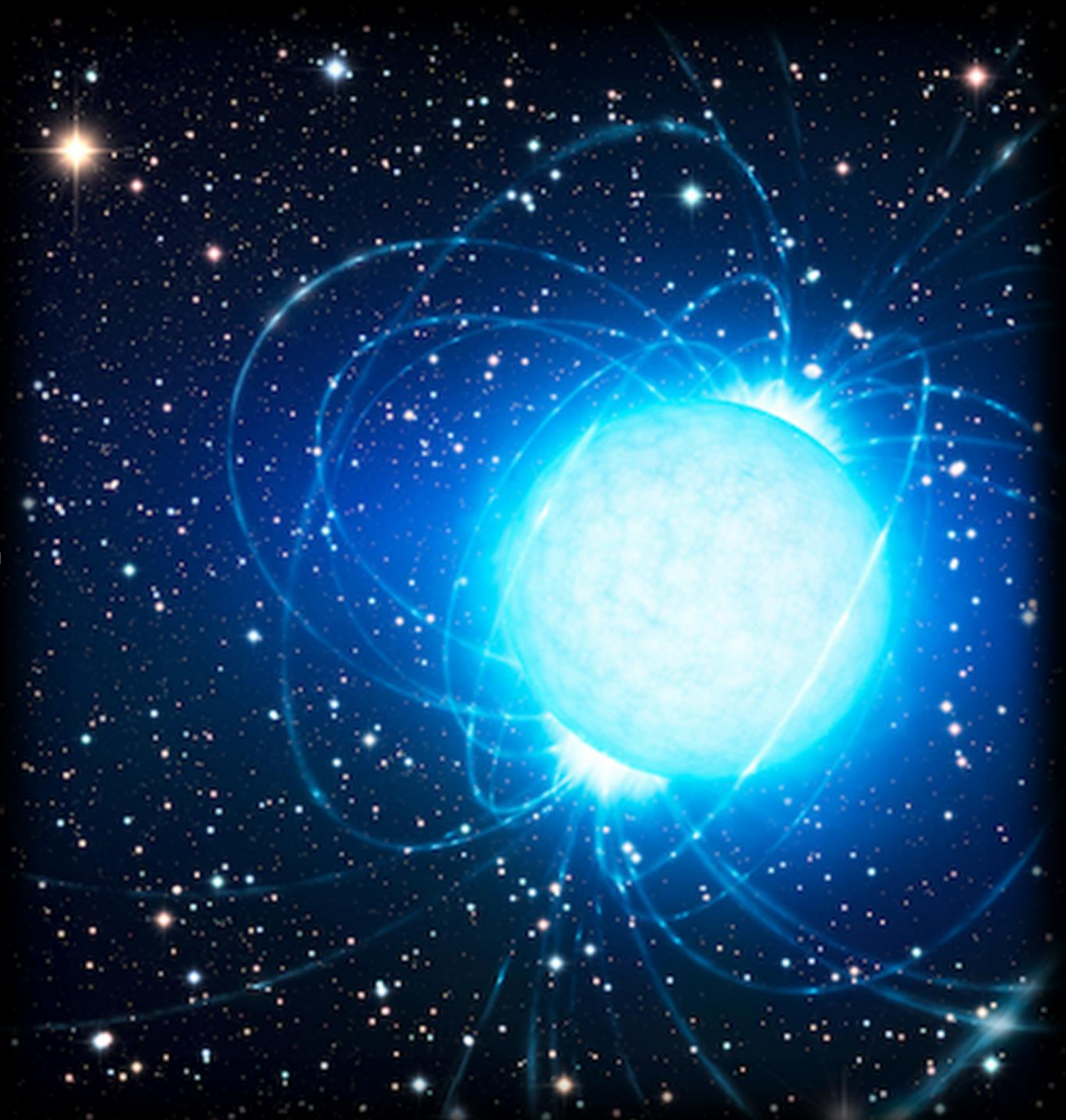
→ relevant to solid-state physics

K.S., S. Kobayashi, and M. Matsuo, in preparation.

(e) *Defects* **(f)** *Antispaghetti* **(g)** *Antignocchi*

M. E. Caplan and C. J. Horowitz, Rev. Mod. Phys. **89**, 041002 (2017)

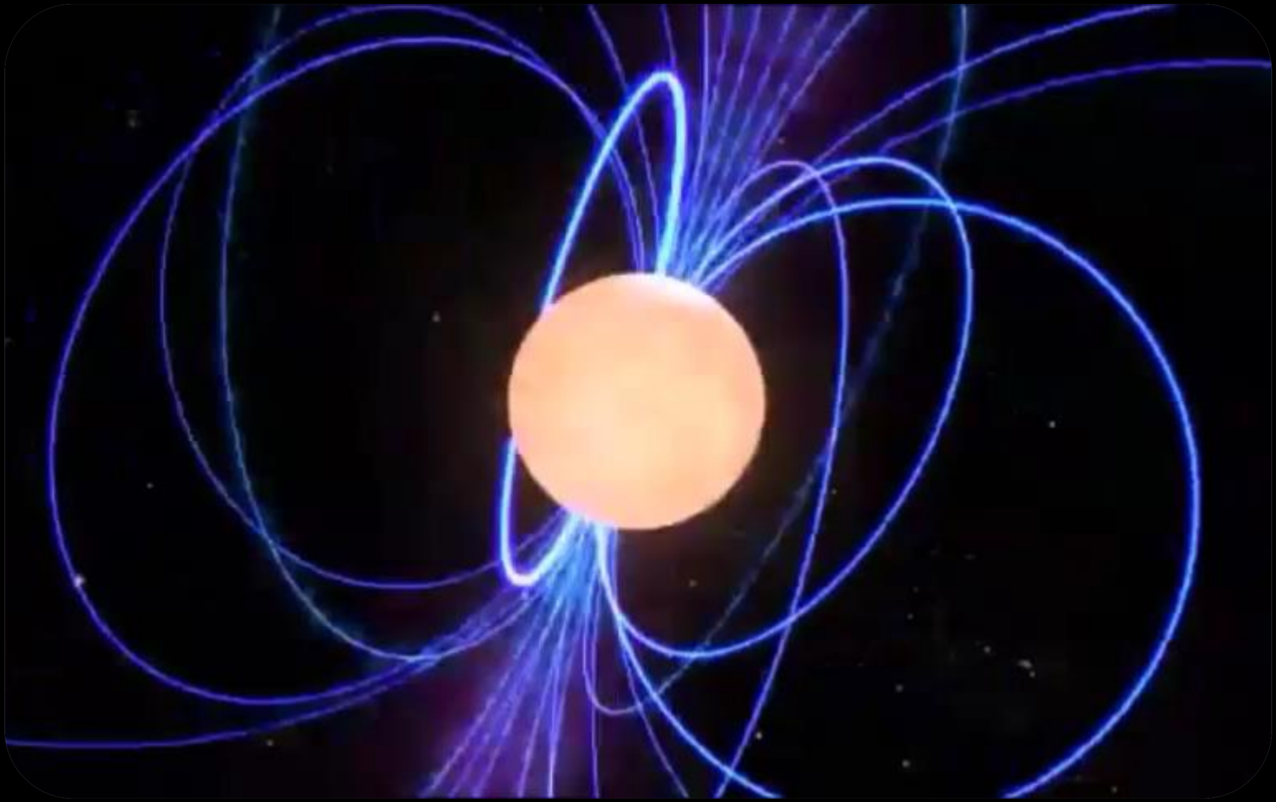
Neutron-star "glitch"



Picture: https://astronomy.com/magazine/ask-astro/2017/12/stellar-magnets

Pulsar - a rotating neutron star

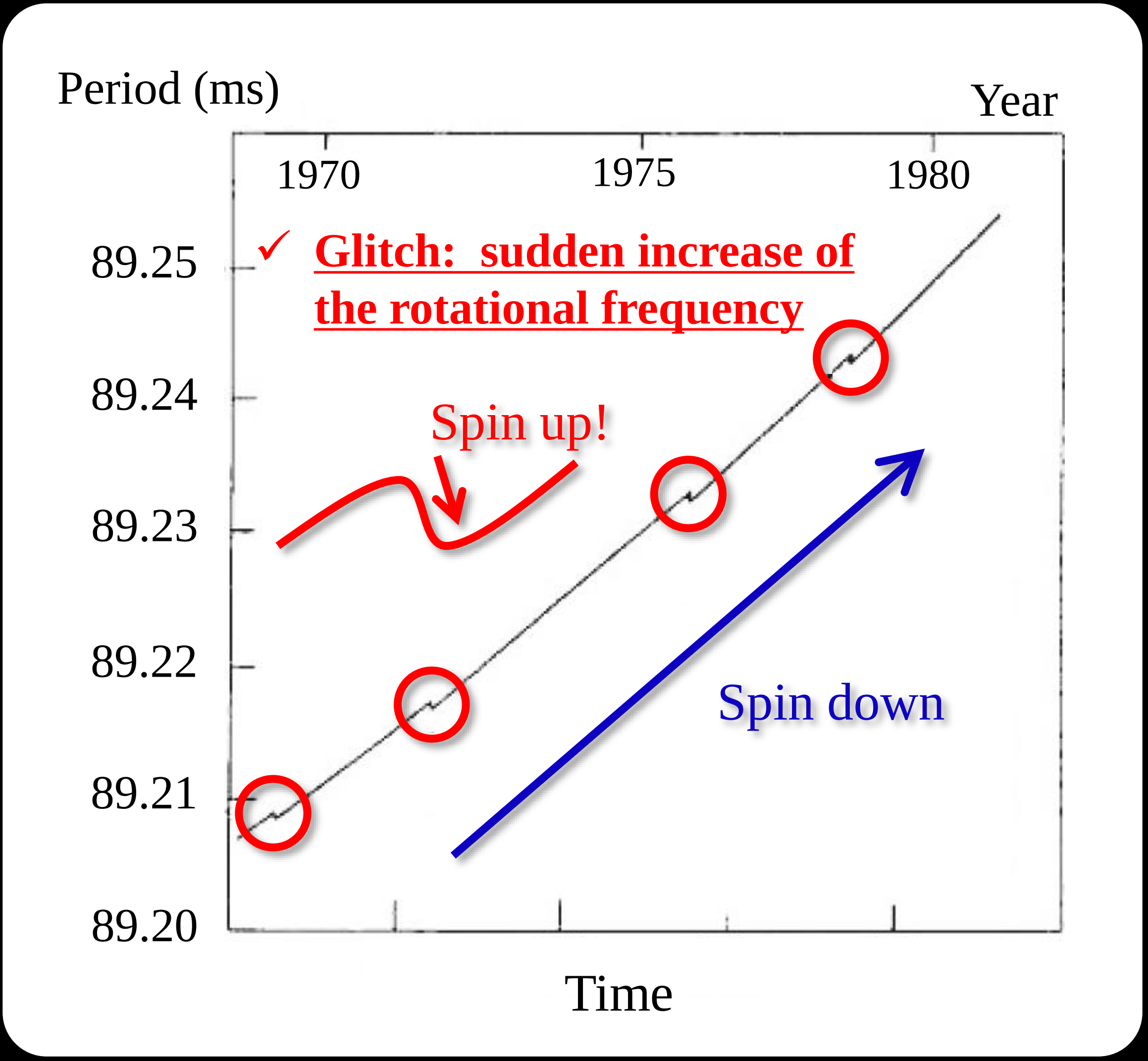
- ✓ First discovery in August 1967 → "Little Green Man" LGM-1 → PSR B1919+21
- ✓ Since then, more than 2650 pulsars have been observed
- ✓ It gradually spins down due to the EM radiation



What is the glitch?

Typical example: the Vela pulsar

- *Irregularity* has been observed from continuous monitoring of the pulsation period

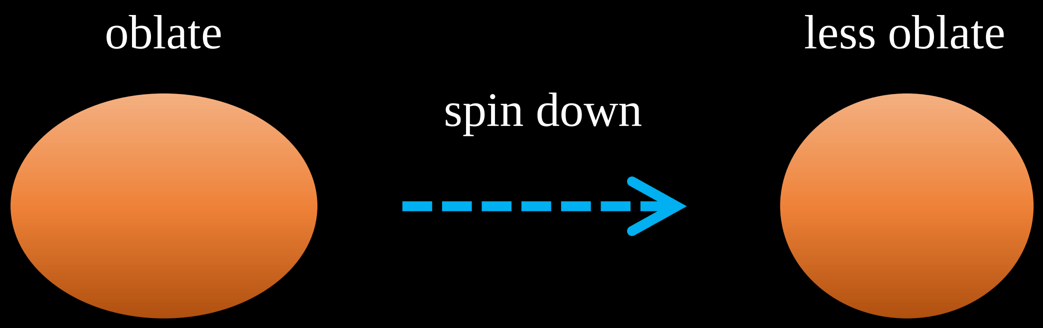


What happened?

Something must happen inside the neutron star!

✓ "Starquake" model [G. Baym et al., Nature **224**, 872 (1969)]

Preferable shape



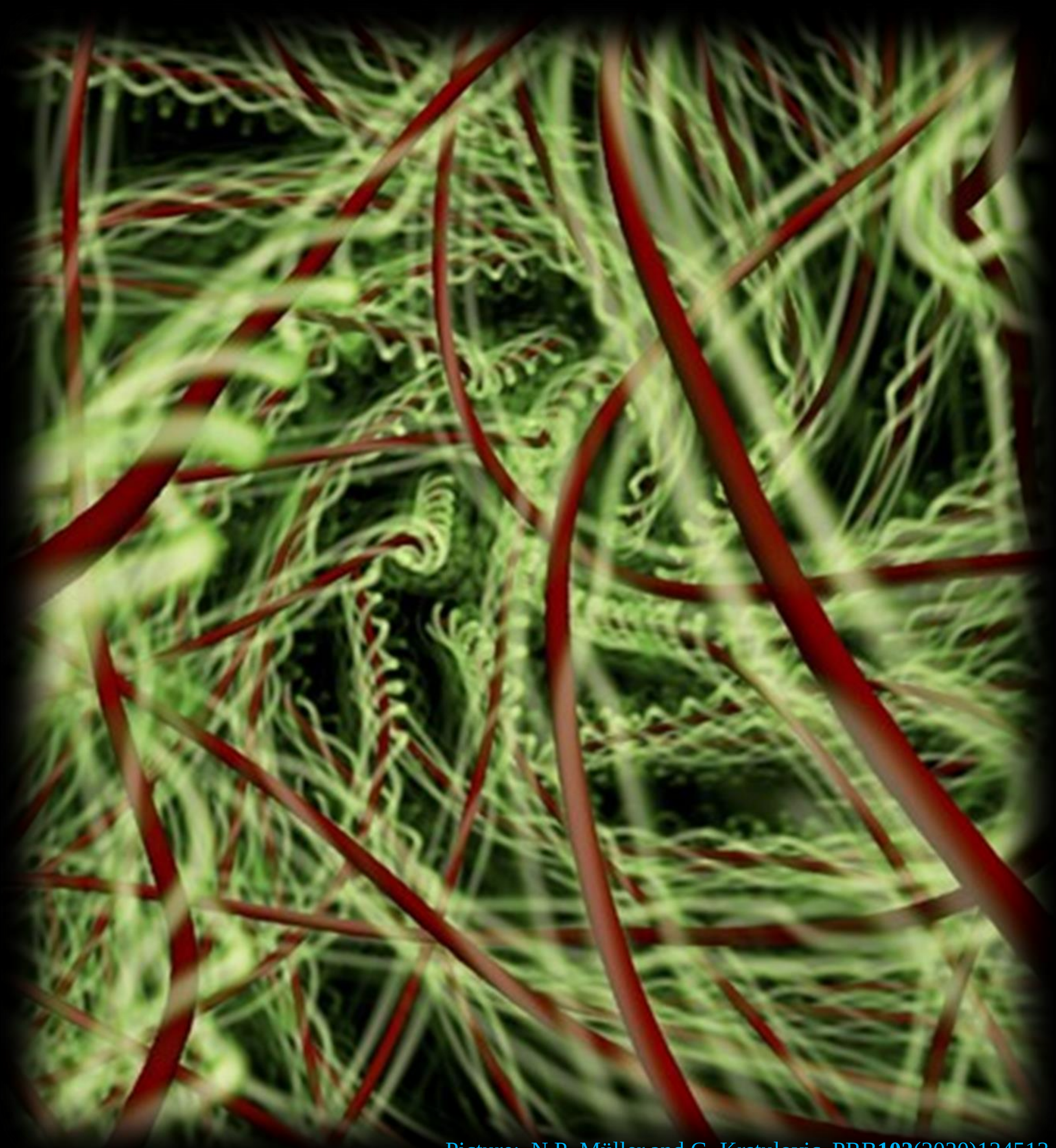
✓ Sudden change of moment of inertia → Glitch?

It requires <u>hundreds of years</u> to have a next glitch..

✓ Vortex mediated glitch [P.W. Anderson and N. Itoh, Nature **256**, 25 (1975)]

➢ <u>Dynamics of superfluid *"quantized vortices"* play a key role!</u> **!?**

Quantum vortices



Picture: N.P. Müller and G. Krstulovic, PRB**102**(2020)134513

What is a quantum vortex?

In superfluid, vortices are quantized!

Superfluid order parameter:

$$\Delta(\boldsymbol{r},t) = |\Delta(\boldsymbol{r},t)|e^{i\phi(\boldsymbol{r},t)}$$

Superfluid velocity:

$$\boldsymbol{v}_s(\boldsymbol{r},t) = \frac{\hbar}{m}\nabla\phi(\boldsymbol{r},t)$$

Vorticity:

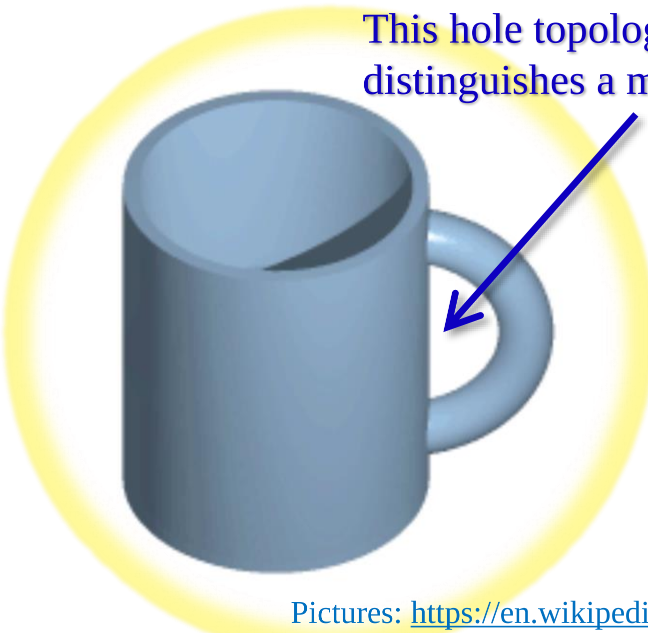
$$\boldsymbol{\omega} = \nabla\times\boldsymbol{v}_s = 0$$

superfluid is irrotational

Circulation:

$$\kappa = \int_S (\nabla\times\boldsymbol{v}_s)\cdot d\boldsymbol{S} = 0$$

*Unless, there is no topological defect



This hole topologically distinguishes a mag from a cow

Pictures: https://en.wikipedia.org/wiki/Topology

In superfluid, vortices are quantized!

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$$\Delta(\boldsymbol{r},t) = |\Delta(\boldsymbol{r},t)|e^{i\phi(\boldsymbol{r},t)}$$

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Circulation:

$$\kappa = \int_S (\nabla \times \boldsymbol{v}_s)\cdot d\boldsymbol{S} = 0$$

*Unless, there is no topological defect

If there is a defect:

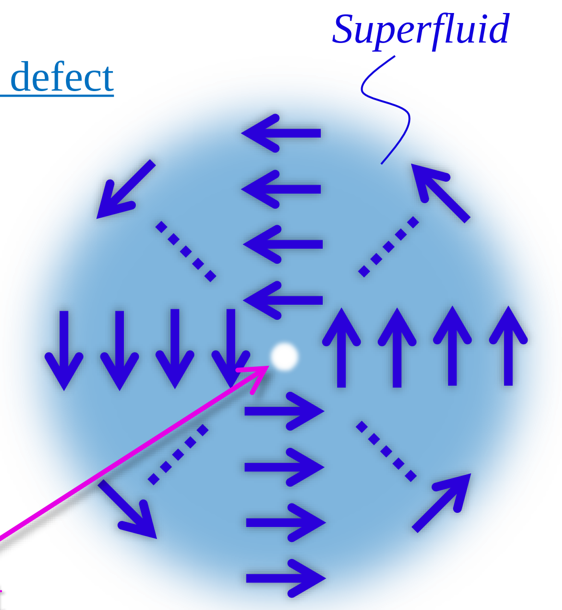
$$\kappa = \oint_C \boldsymbol{v}_s \cdot d\boldsymbol{l} = \frac{\hbar}{m}\oint_C \nabla\phi\cdot d\boldsymbol{l} = \frac{\hbar}{m}2\pi n$$

Quantization of circulation

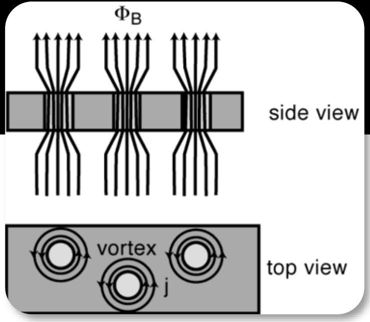
*the phase $\phi(\boldsymbol{r})$ at the same point must be equivalent!

Flow velocity of rotation shall be quantized!

A hole at which superfluidity is lost



Quantum vortex



In superconductor, magnetic flux is quantized!

Magnetic flux:

Meissner effect

$$\Phi = \int_S \boldsymbol{B} \cdot d\boldsymbol{S} = \int_S (\nabla \times \boldsymbol{A}) \cdot d\boldsymbol{S} = 0$$

$$\boldsymbol{j}_s = -\frac{n_s e_s^2}{m_s}\boldsymbol{A} + \frac{n_s e_s \hbar}{m_s}\nabla\phi$$: the London equation

If there is a defect:

$$\Phi = \oint_C \boldsymbol{A} \cdot d\boldsymbol{l} \approx \frac{\hbar}{e_s}\oint_C \nabla\phi \cdot d\boldsymbol{l} = \frac{\hbar}{e_s} 2\pi n$$

n_s, m_s, e_s: density, mass, and charge of a carrier (Cooper pair)

Quantization of magnetic flux (fluxtube, fluxoid, or fluxon)

Circulation:

$$\kappa = \int_S (\nabla \times \boldsymbol{v}_s) \cdot d\boldsymbol{S} = 0$$

*Unless, there is no topological defect

If there is a defect:

$$\kappa = \oint_C \boldsymbol{v}_s \cdot d\boldsymbol{l} = \frac{\hbar}{m}\oint_C \nabla\phi \cdot d\boldsymbol{l} = \frac{\hbar}{m} 2\pi n$$

Quantization of circulation

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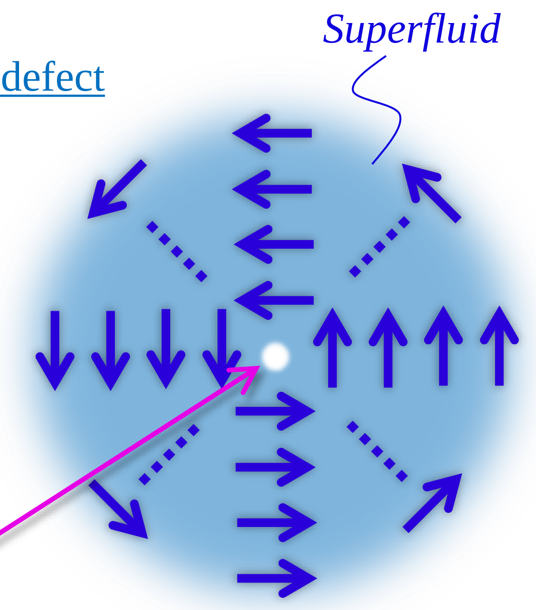
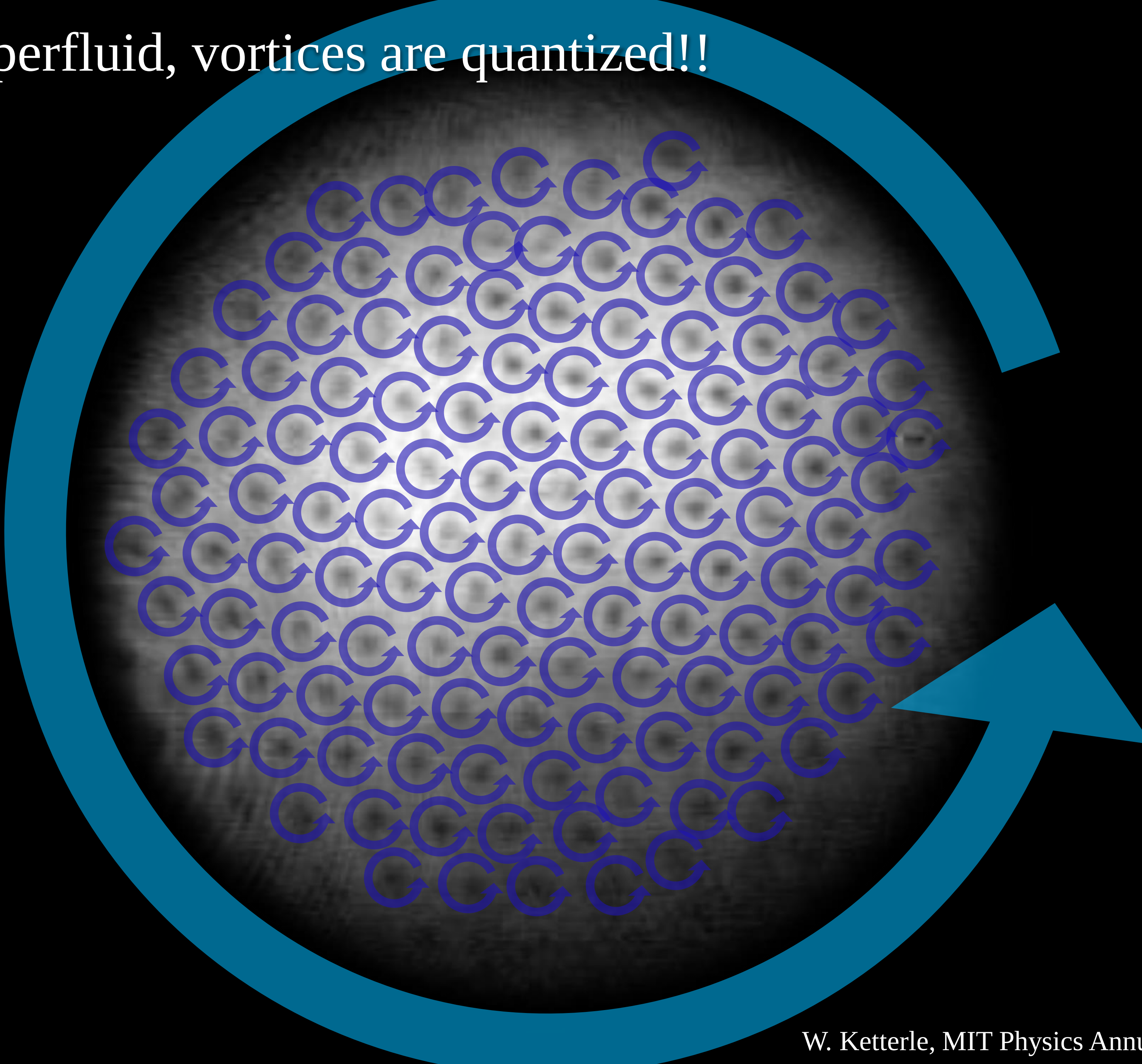


Quantum vortex

In daily life, a vortex is continuous..

In superfluid, vortices are quantized!!



W. Ketterle, MIT Physics Annual. 2001

G.P. Bewley, D.P. Lathrop, and K.R. Sreenivasan, Nature **441**, 588 (2006)

A movie from a talk by W. Guo (available from https://youtu.be/P2ckefSAN20) at
INT Program 19-1a "Quantum Turbulence: Cold Atoms, Heavy Ions, and Neutron Stars"
March 18 - April 19, 2019

Direct visualization of quantized vortices

Hydrogen particles were trapped in the vortex core, then worked as a tracer

A lattice of neutron-rich nuclei are immersed in a neutron superfluid

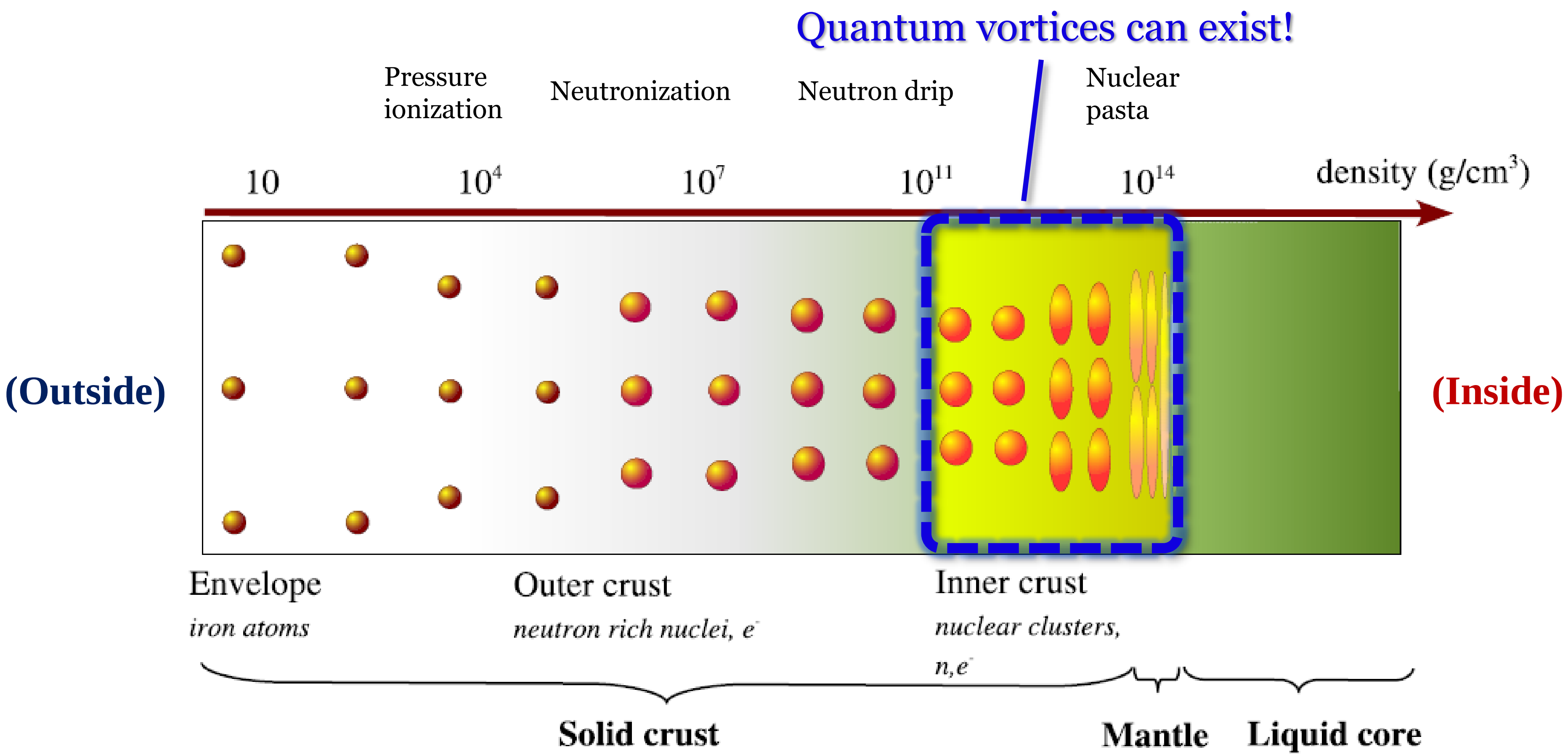


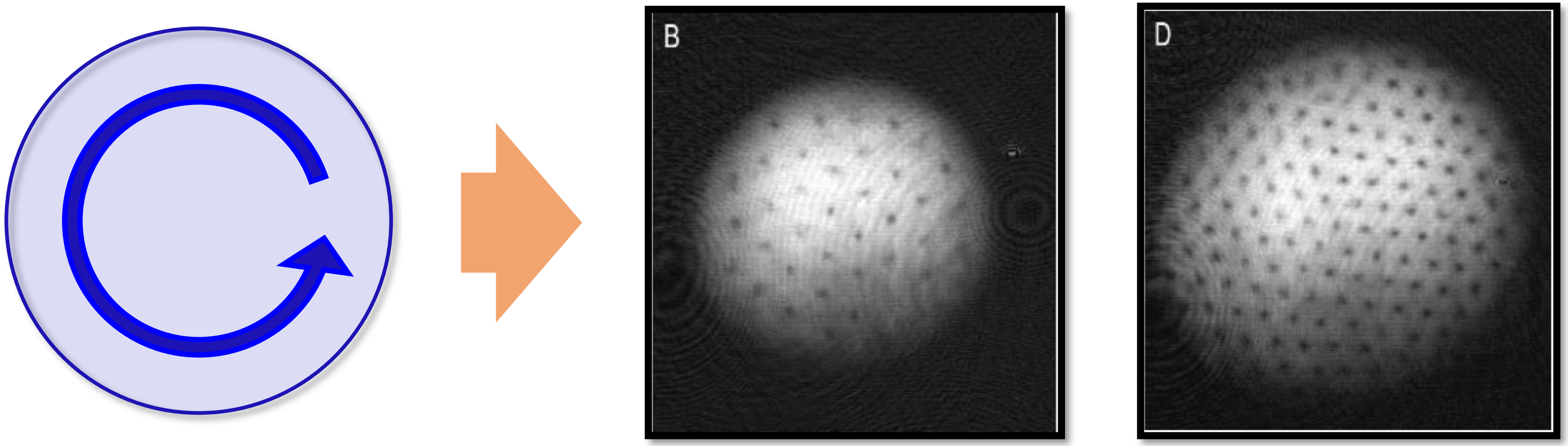
Fig.4 in N. Chamel and P. Haensel, Living Rev. Relativity **11**, 10 (2008)

In rotating superfluid, an array of quantum vortices is generated

- Observation in ultra-cold atomic gases

Vortex-number density $\propto$ Rotation speed

Low ⟷ High



W. Ketterle, MIT Physics Annual. 2001

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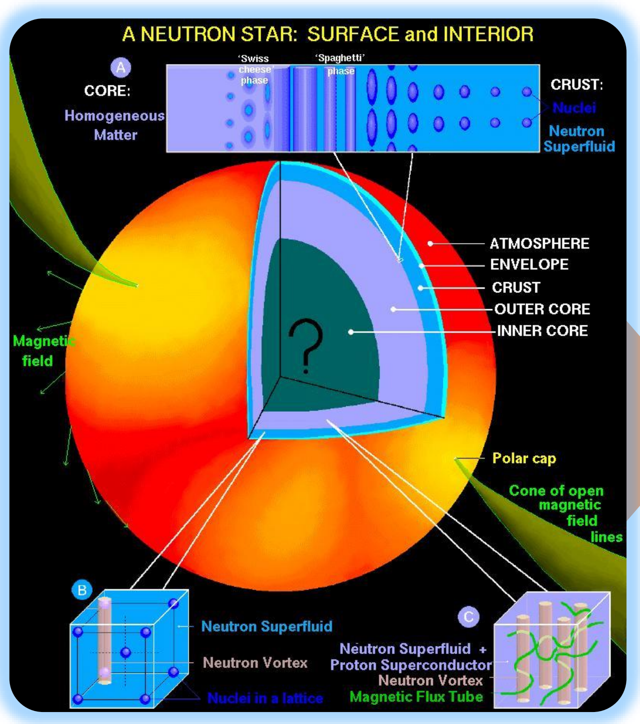
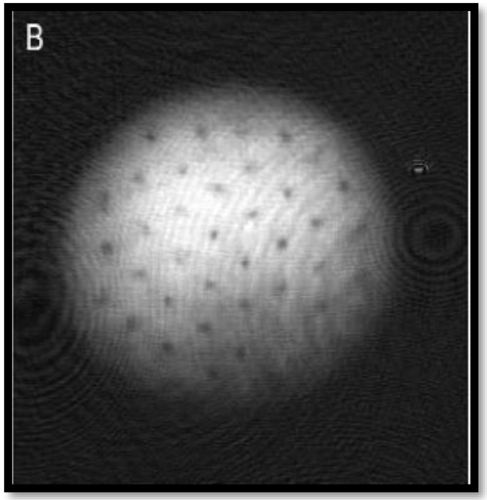
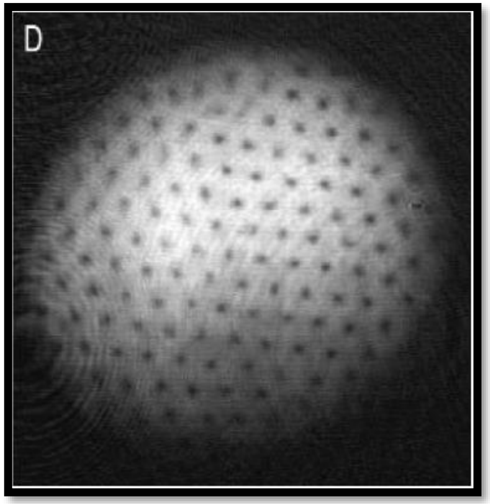


Vortex-number density
$\propto$
Rotation speed

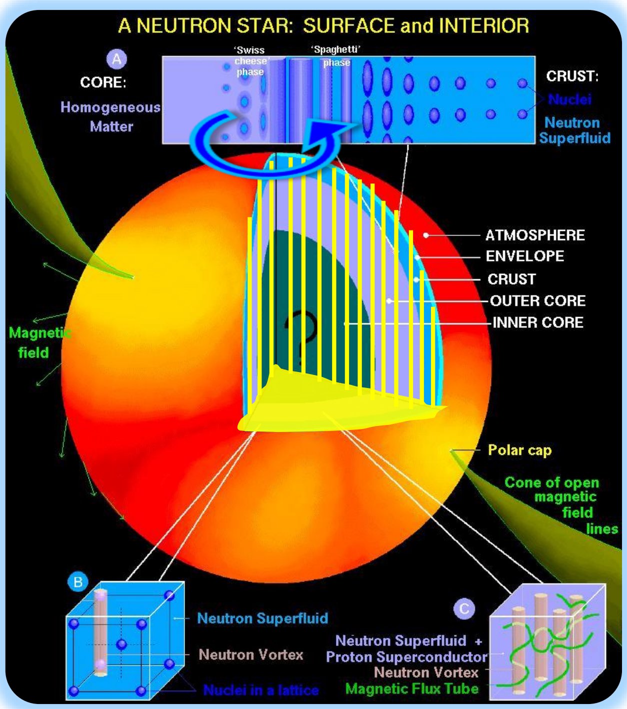
Low ⟵⟶ High



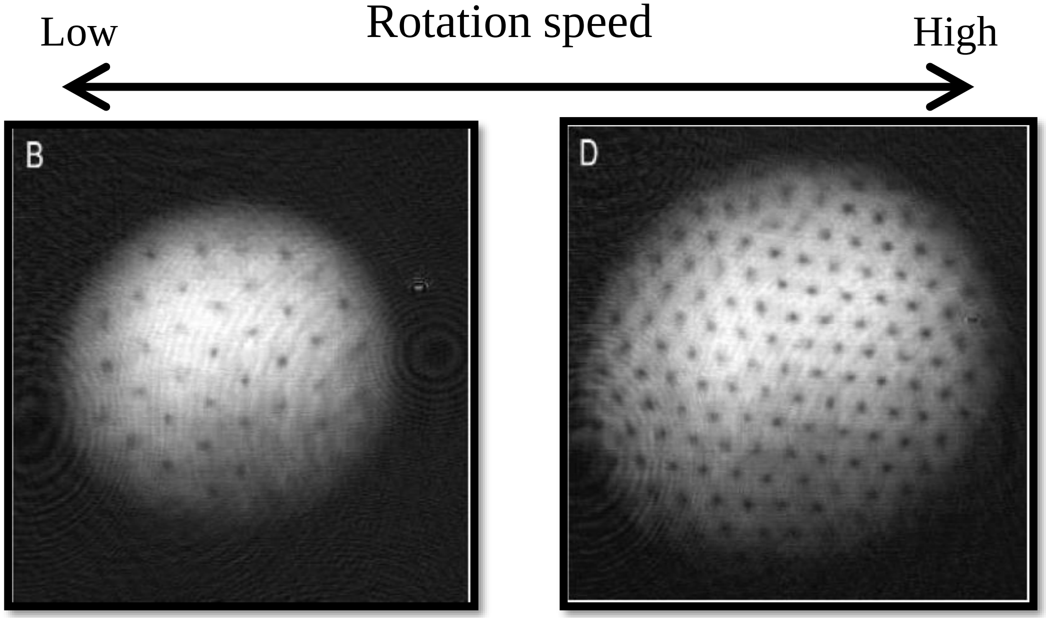


W. Ketterle, MIT Physics Annual. 2001

There must be a huge number (~10^{18}) of vortices inside a neutron star!!

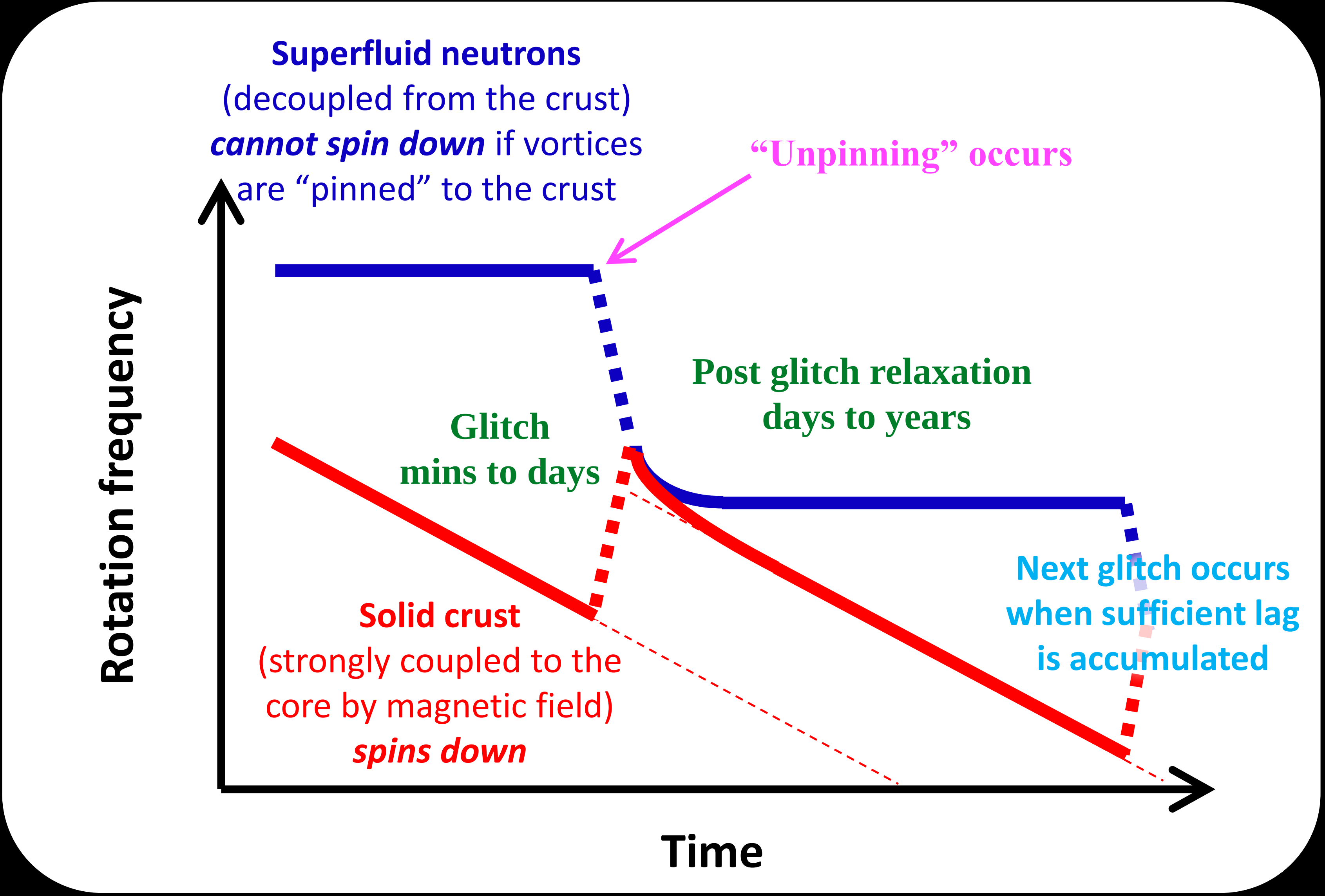


Vortex-number density
$\propto$
Rotation speed



W. Ketterle, MIT Physics Annual. 2001

The vortex mediated glitch: Naive picture



Recent progress:

inclusion of superfluidity/superconductivity in (TD)DFT

We can now study dynamics of quantum vortices microscopically from nucleonic degrees of freedom!

TDSLDA: TDDFT with local treatment of pairing

Kohn-Sham scheme is extended for non-interacting quasiparticles

➢ TDSLDA equations (formally equivalent to TDHFB or TD-BdG equations)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}u_{k,\uparrow}(\boldsymbol{r},t)\\u_{k,\downarrow}(\boldsymbol{r},t)\\v_{k,\uparrow}(\boldsymbol{r},t)\\v_{k,\downarrow}(\boldsymbol{r},t)\end{pmatrix}=\begin{pmatrix}h_{\uparrow\uparrow}(\boldsymbol{r},t) & h_{\uparrow\downarrow}(\boldsymbol{r},t) & 0 & \Delta(\boldsymbol{r},t)\\h_{\downarrow\uparrow}(\boldsymbol{r},t) & h_{\downarrow\downarrow}(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0\\0 & -\Delta^*(\boldsymbol{r},t) & -h^*_{\uparrow\uparrow}(\boldsymbol{r},t) & -h^*_{\uparrow\downarrow}(\boldsymbol{r},t)\\\Delta^*(\boldsymbol{r},t) & 0 & -h^*_{\downarrow\uparrow}(\boldsymbol{r},t) & -h^*_{\downarrow\downarrow}(\boldsymbol{r},t)\end{pmatrix}\begin{pmatrix}u_{k,\uparrow}(\boldsymbol{r},t)\\u_{k,\downarrow}(\boldsymbol{r},t)\\v_{k,\uparrow}(\boldsymbol{r},t)\\v_{k,\downarrow}(\boldsymbol{r},t)\end{pmatrix}$$

$$h_\sigma=\frac{\delta E}{\delta n_\sigma}\ :\ \text{s.p. Hamiltonian}$$

$$\Delta=-\frac{\delta E}{\delta \nu^*}\ :\ \text{pairing field}$$

$$n_\sigma(\boldsymbol{r},t)=\sum_{E_k<E_c}|v_{k,\sigma}(\boldsymbol{r},t)|^2\ :\ \text{number density}$$

$$\nu(\boldsymbol{r},t)=\sum_{E_k<E_c}u_{k,\uparrow}(\boldsymbol{r},t)v^*_{k,\downarrow}(\boldsymbol{r},t)\ :\ \text{anomalous density}$$

$$\boldsymbol{j}_\sigma(\boldsymbol{r},t)=\hbar\sum_{E_k<E_c}\mathrm{Im}[v^*_{k,\sigma}(\boldsymbol{r},t)\boldsymbol{\nabla}v_{k,\sigma}(\boldsymbol{r},t)]\ :\ \text{current}$$

A large number (10^4-10^6) of 3D coupled non-linear PDEs have to be solved!!

# of qp orbitals ~ # of grid points

TDSLDA: TDDFT with local treatment of pairing

Kohn-Sham scheme is extended for non-interacting quasiparticles

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$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix} = \begin{pmatrix} h_{\uparrow\uparrow}(\boldsymbol{r},t) & h_{\uparrow\downarrow}(\boldsymbol{r},t) & 0 & \Delta(\boldsymbol{r},t) \\ h_{\downarrow\uparrow}(\boldsymbol{r},t) & h_{\downarrow\downarrow}(\boldsymbol{r},t) & -\Delta(\boldsymbol{r},t) & 0 \\ 0 & -\Delta^*(\boldsymbol{r},t) & \text{[illegible]} & -h^*_{\uparrow\downarrow}(\boldsymbol{r},t) \\ \Delta^*(\boldsymbol{r},t) & \text{[illegible]} & \text{[illegible]} & -h^*_{\downarrow\downarrow}(\boldsymbol{r},t) \end{pmatrix}\begin{pmatrix} u_{k,\uparrow}(\boldsymbol{r},t) \\ u_{k,\downarrow}(\boldsymbol{r},t) \\ v_{k,\uparrow}(\boldsymbol{r},t) \\ v_{k,\downarrow}(\boldsymbol{r},t) \end{pmatrix}$$

Supercomputing!!

$$h_\sigma = \frac{\delta E}{\delta n_\sigma}$$: s.p. Hamiltonian

$$\Delta = -\frac{\delta E}{\delta \nu^*}$$: pairing field

$$n_\sigma(\boldsymbol{r},t) = \sum_{E_k<E_c} |v_{k,\sigma}(\boldsymbol{r},t)|^2$$: number density

$$\nu(\boldsymbol{r},t) = \sum_{E_k<E_c} u_{k,\uparrow}(\boldsymbol{r},t)v^*_{k,\downarrow}(\boldsymbol{r},t)$$: anomalous density

$$\boldsymbol{j}_\sigma(\boldsymbol{r},t) = \hbar\sum_{E_k<E_c} \mathrm{Im}[v^*_{k,\sigma}(\boldsymbol{r},t)\boldsymbol{\nabla} v_{k,\sigma}(\boldsymbol{r},t)]$$: current

A large number (10^4-10^6) of 3D coupled non-linear PDEs have to be solved!!

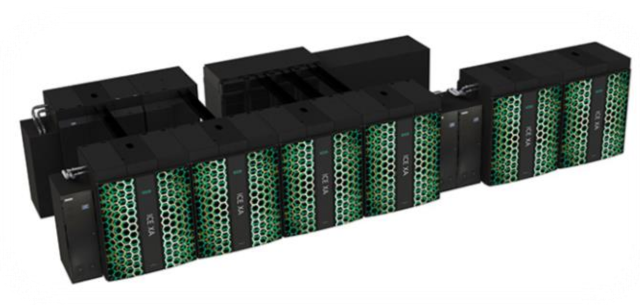
# of qp orbitals ~ # of grid points

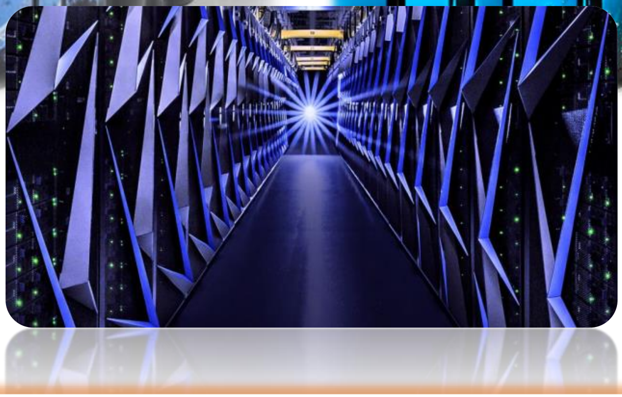
*The number indicates the rank according to the TOP500 list (November 2021)

Piz Daint, CSCS, Switzerland (No. 20)

TITAN, ORNL, USA

TSUBAME3.0, Japan (No. 59)

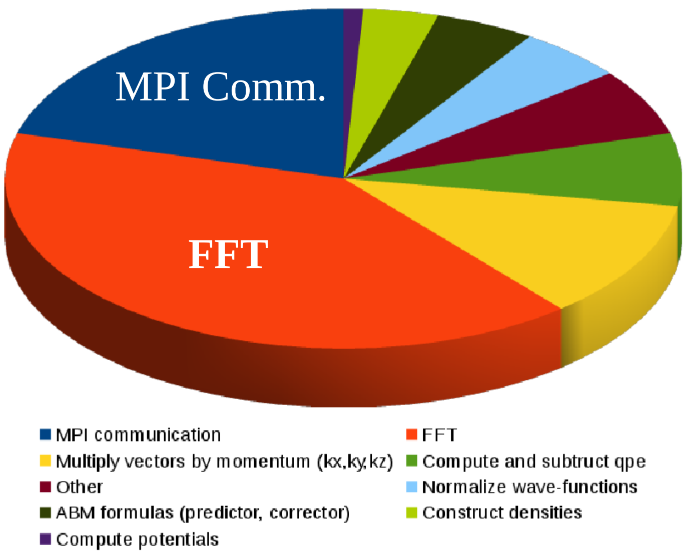




Summit, ORNL, USA (No. 2)
GPU, 200 PFlops/s

Present computing capabilities:

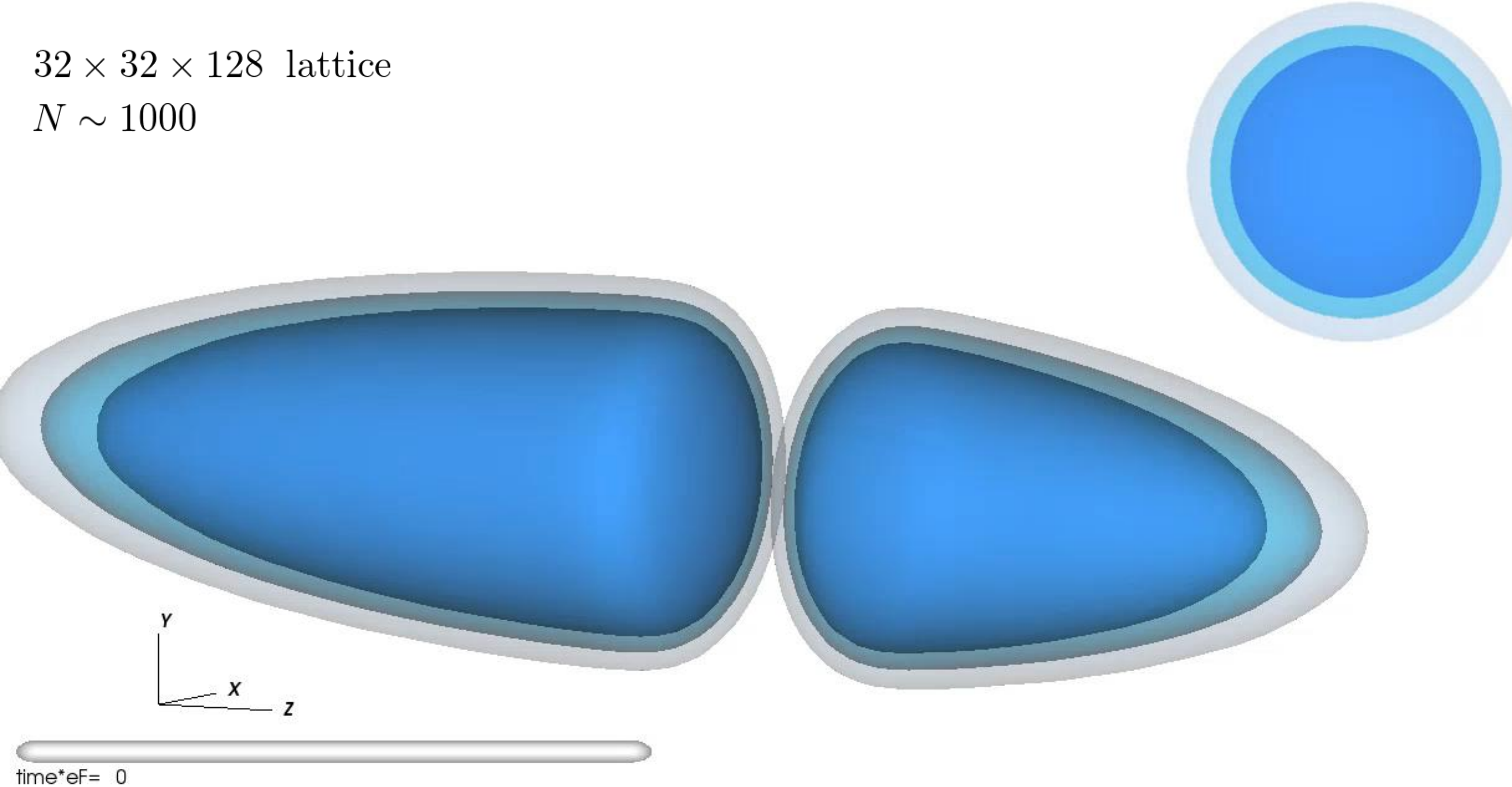
- ✓ Full 3D (w/o symmetry restrictions)
- ✓ Volume as large as 100^3 lattice points
- ✓ Evolution up to 10^6 time steps (as long as 10^{-19} sec)



Result of TDSLDA simulation:

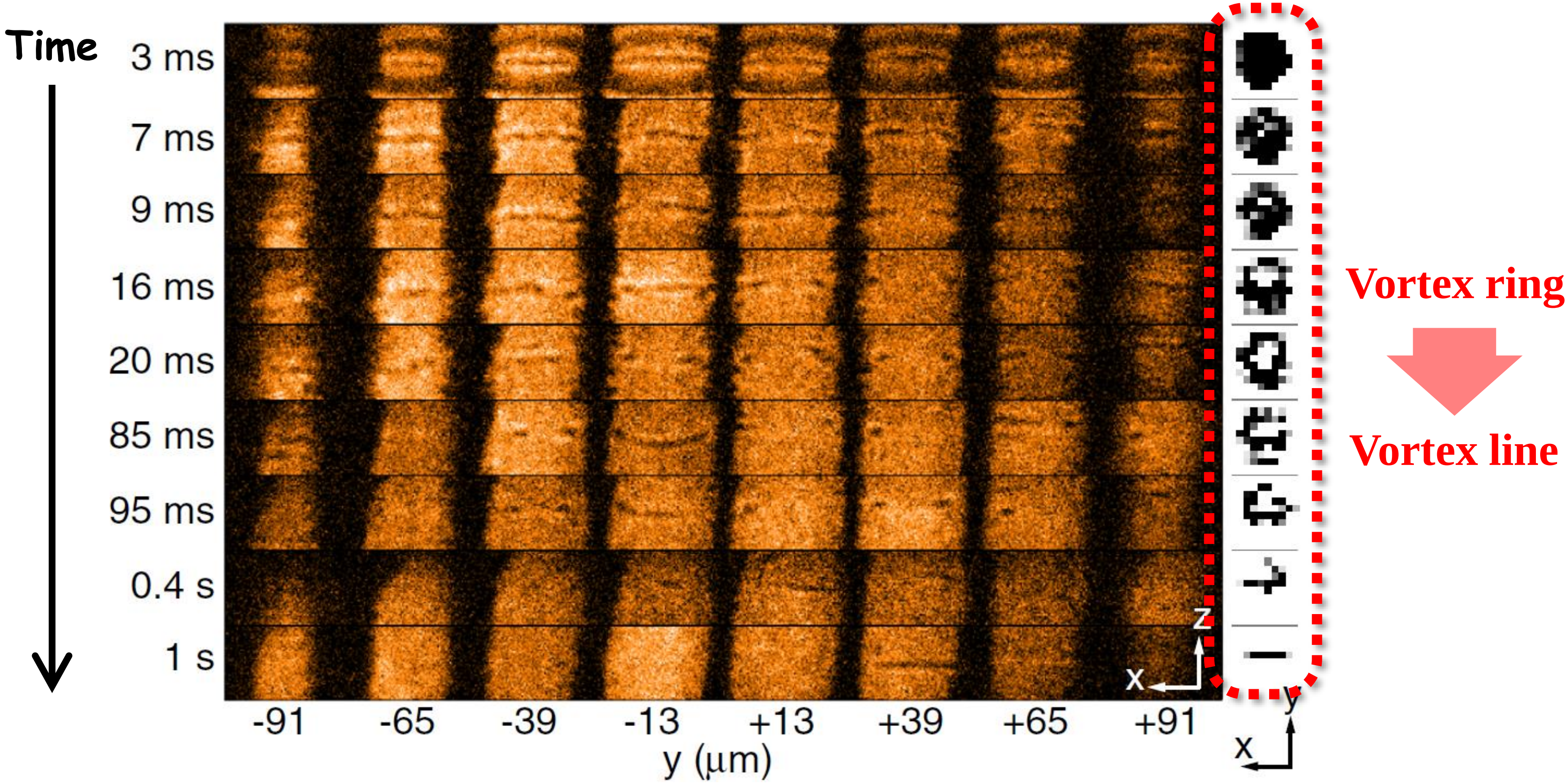
Phase discontinuity creates a vortex ring which decays into a vortex line

$32 \times 32 \times 128$ lattice
$N \sim 1000$



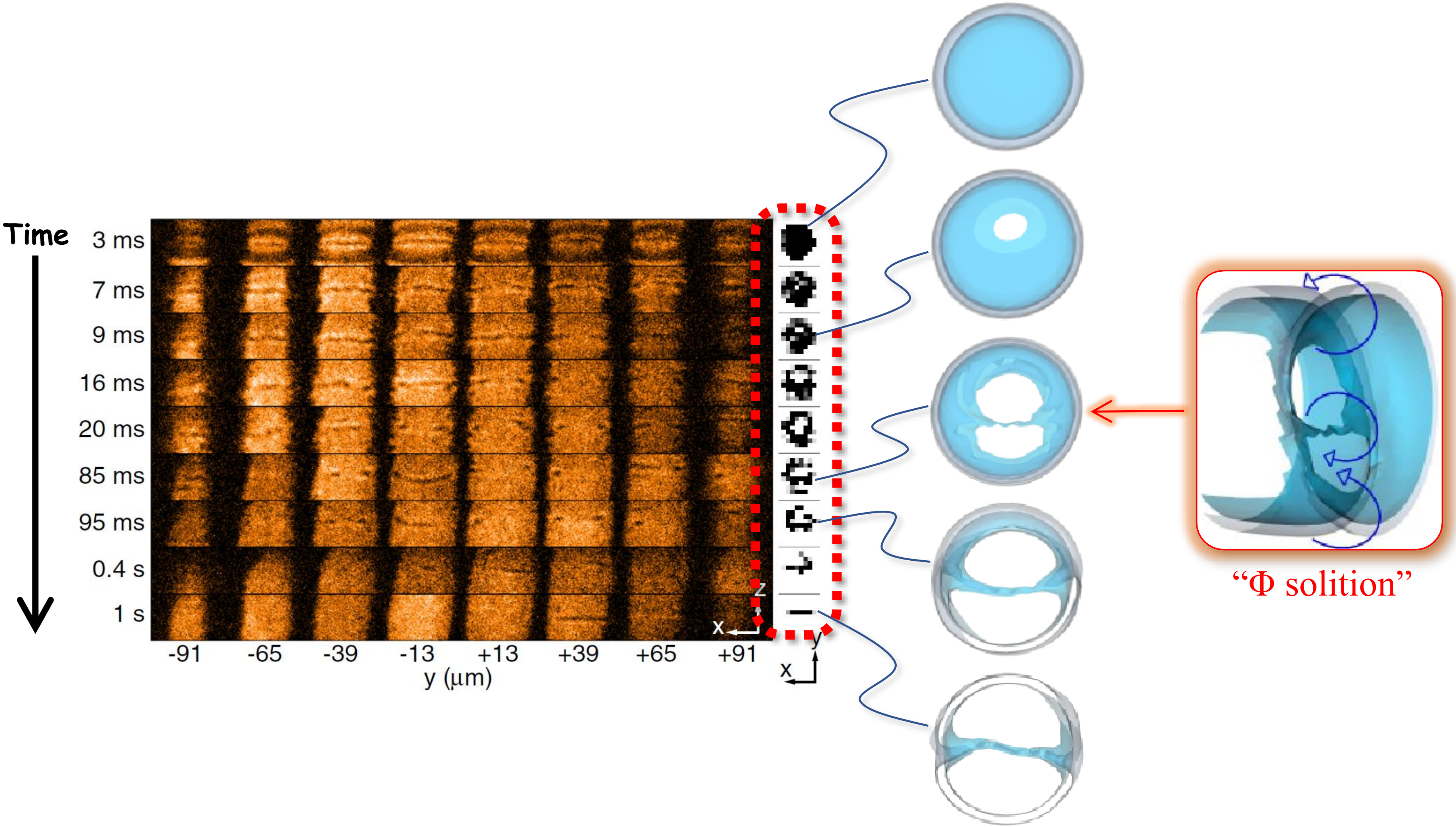
G. Wlazłowski, A. Bulgac, M.M. Forbes, and K.J. Roche, Phys. Rev. A **91**, 031602(R) (2015)

The cascades of solitonic excitations have been identified experimentally



M.J.H. Ku, B. Mukherjee, T. Yefsah, and M.W. Zwierlein, Phys. Rev. Lett. **116**, 045304 (2016)

Each stage of solitonic cascade could be reproduced with TDSLDA!



G. Wlazłowski, K.S. M. Marchwiany, and P. Magierski, Phys. Rev. Lett. **120**, 253002 (2018)

To fully understand the glitches, we need to clarify:

Glitch dynamics

How do vortices move?

and, of course,
details of NS matter..

Pinning mechanism

How are vortices pinned?

Trigger mechanism

How are vortices unpinned?

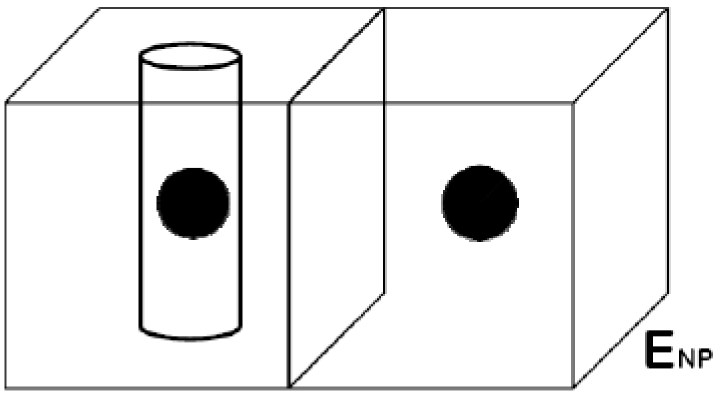
**We attacked this problem using
the state-of-the-art microscopic nuclear theory**

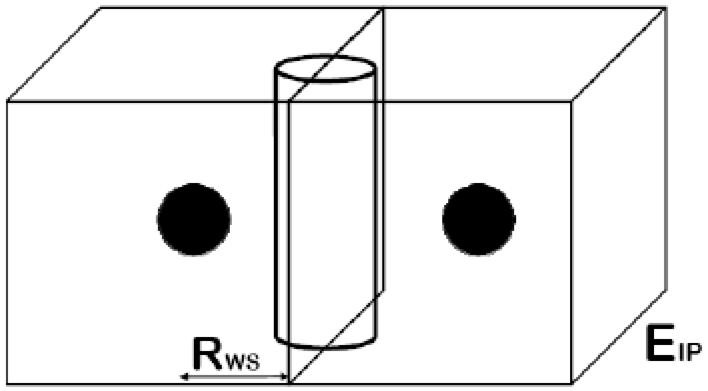
A key to understand the glitches is:

Vortex pinning mechanism in the inner crust of neutron stars

Q. Is the vortex-nucleus interaction

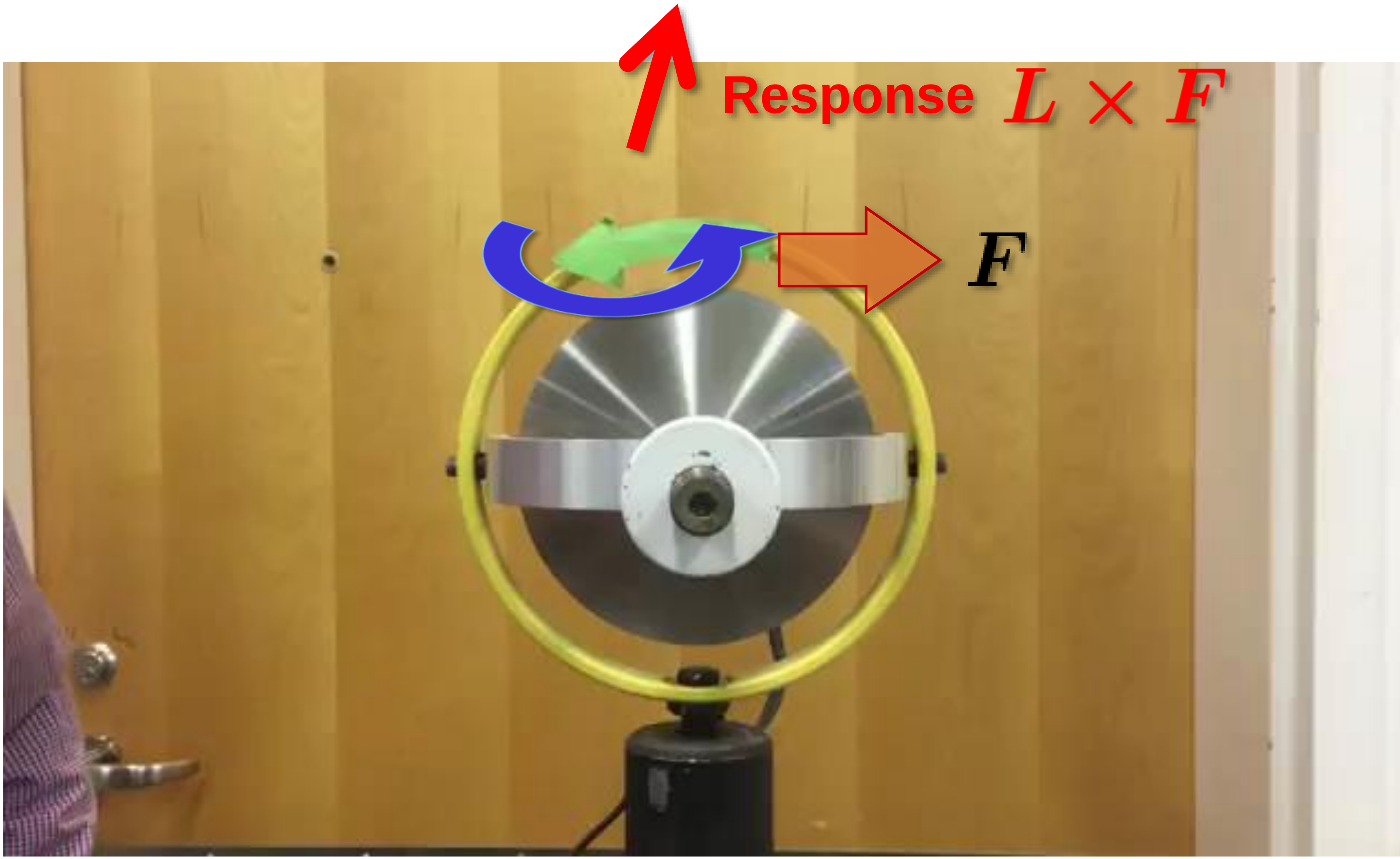
Attractive? or Repulsive?





"Nuclear pinning" "Interstitial pinning"

Response of a spinning gyroscope when pushed



We performed 3D, dynamical simulations by TDDFT with superfluidity

TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}u_i(\mathbf{r})\\ v_i(\mathbf{r})\end{pmatrix} = \begin{pmatrix}h(\mathbf{r}) & \Delta(\mathbf{r})\\ \Delta^*(\mathbf{r}) & -h(\mathbf{r})\end{pmatrix}\begin{pmatrix}u_i(\mathbf{r})\\ v_i(\mathbf{r})\end{pmatrix}$$

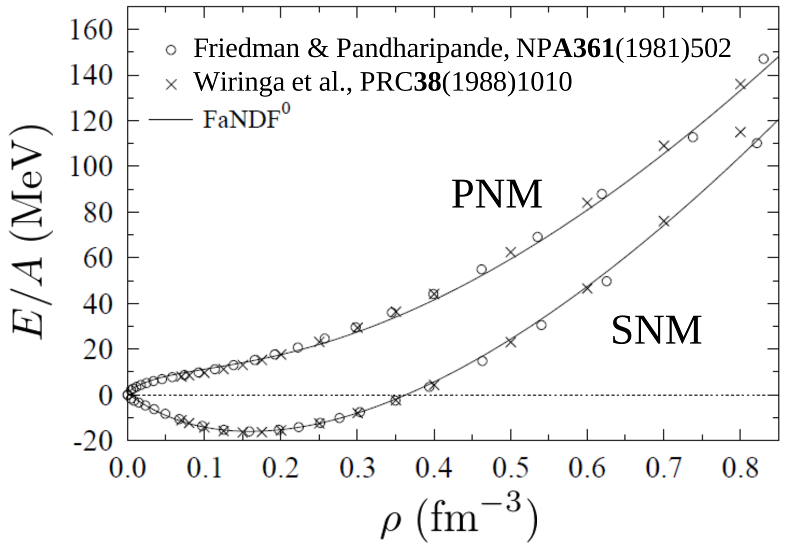
Energy density functional (EDF)

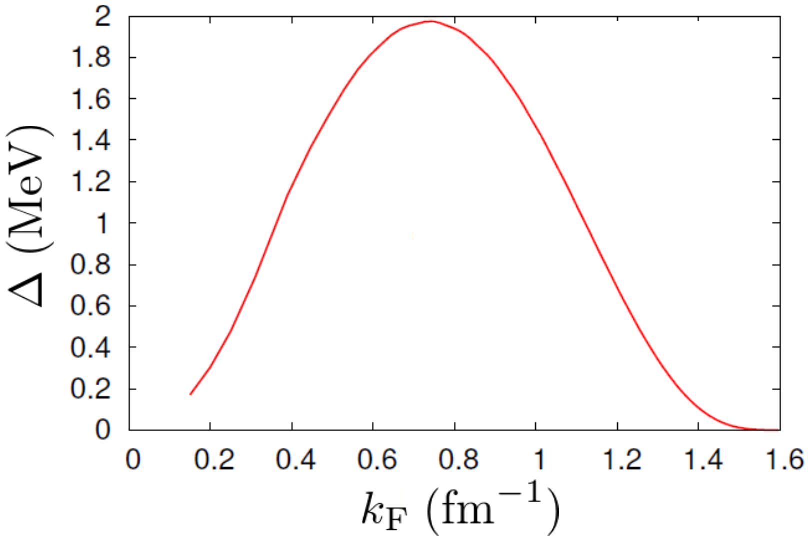
$$\mathcal{E}(\mathbf{r}) = \mathcal{E}_0(\mathbf{r}) + \mathcal{E}_{\rm pair}(\mathbf{r})$$

$\mathcal{E}_0(\mathbf{r})$: Fayans EDF (FaNDF^0) w/o LS

$$\mathcal{E}(\boldsymbol{r}) = \sum_{q=n,p} g[\rho_q(\boldsymbol{r})]|\nu_q(\boldsymbol{r})|^2$$

S.A. Fayans, JETP Lett. **68**, 169 (1998)





We performed 3D, dynamical simulations by TDDFT with superfluidity

TDSLDA equations (or TDHFB, TD-BdG)

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} h(\mathbf{r}) & \Delta(\mathbf{r}) \\ \Delta^*(\mathbf{r}) & -h(\mathbf{r}) \end{pmatrix} \begin{pmatrix} u_i(\mathbf{r}) \\ v_i(\mathbf{r}) \end{pmatrix}$$

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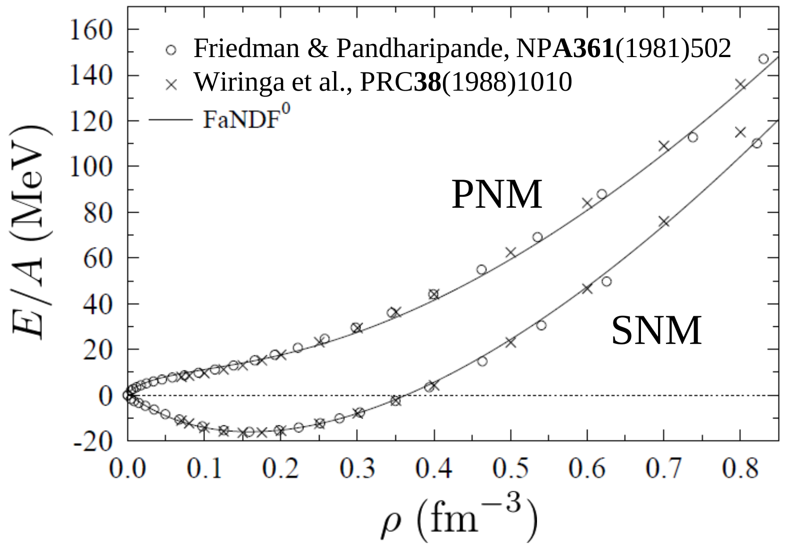

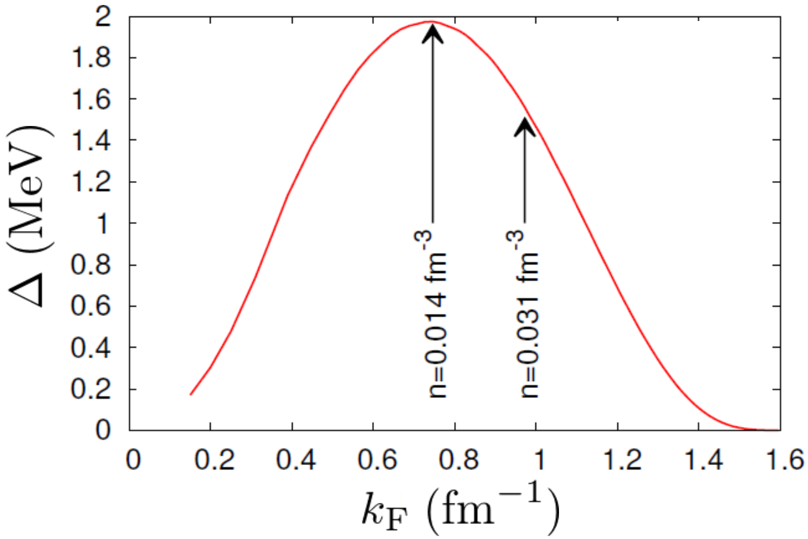

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Computational details

$75\text{ fm}\times 75\text{ fm}\times 60\text{ fm}$

$(50\times 50\times 40,\ \Delta x = 1.5\text{ fm})$

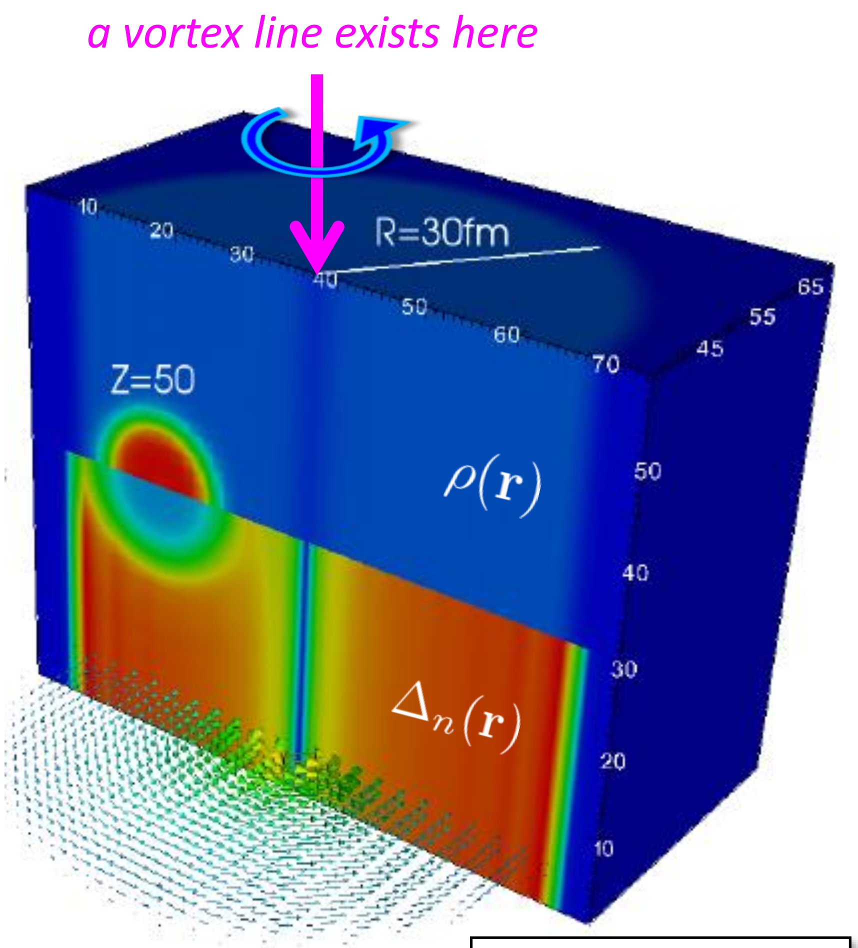
$k_c = \pi/\Delta x > k_F \qquad k_F = (3\pi^2\rho_n)^{1/3}$

Nuclear impurity: $Z = 50$

$\rho_n \simeq 0.014\text{ fm}^{-3}$ $(N \simeq 2{,}530)$

$\rho_n \simeq 0.031\text{ fm}^{-3}$ $(N \simeq 5{,}714)$

# of quasi-particle w.f. $\approx 100,000$



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MPI+GPU
→ 48h w/ 200GPUs
for 10,000 fm/c

TITAN, Oak Ridge

NERSC Edison, Berkeley

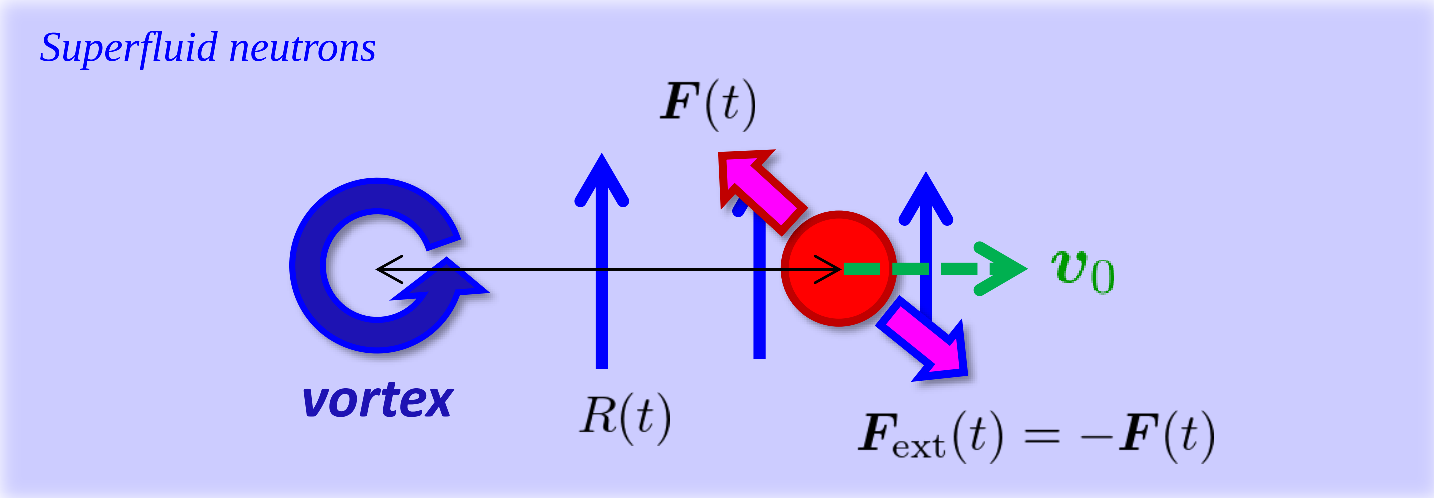
HA-PACS, Tsukuba

We directly measure the force $\boldsymbol{F}(R)$ in dynamical simulation

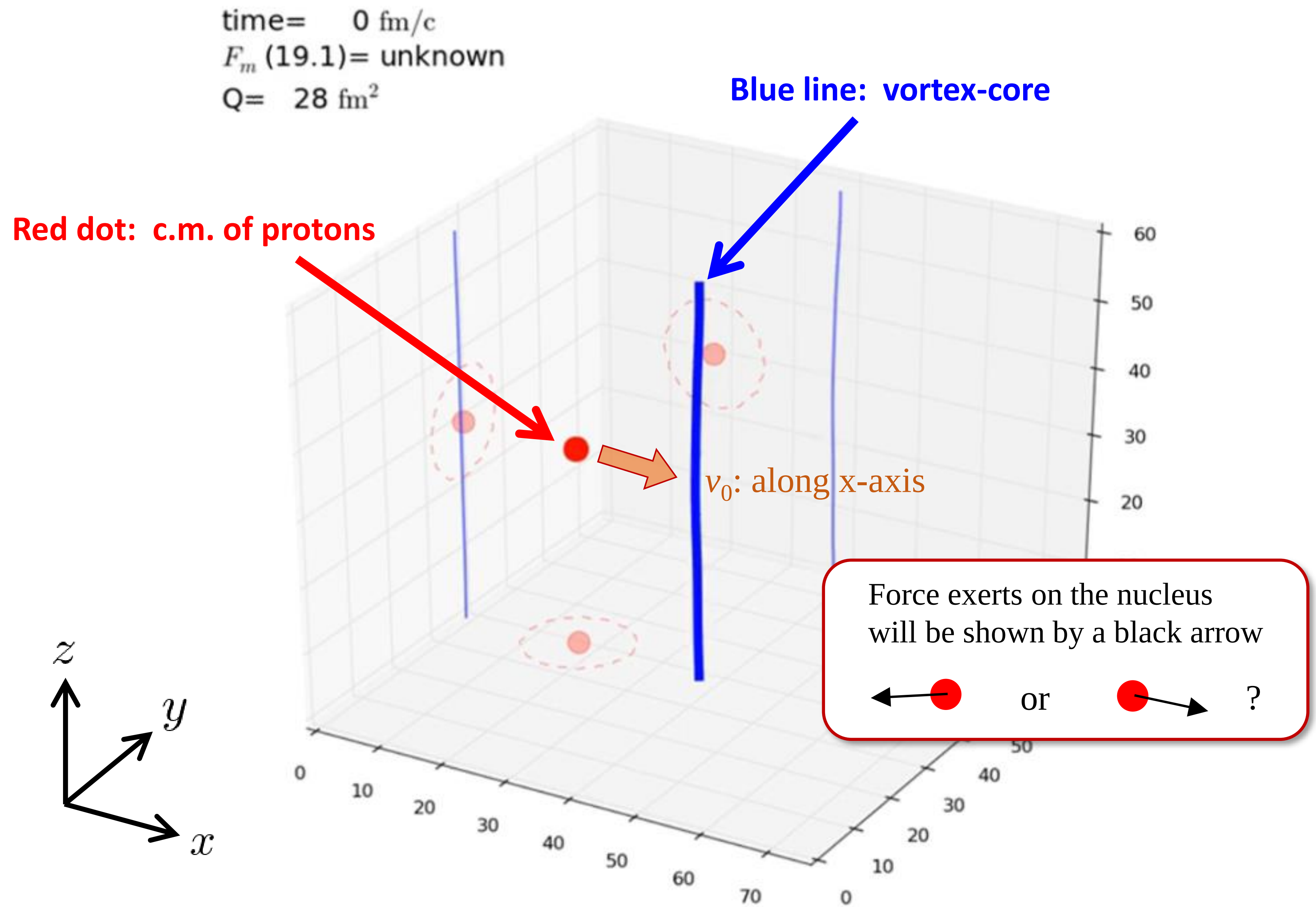
- Newton's law

$$\boldsymbol{F} = M\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} \quad \Rightarrow \quad \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = 0 \quad \text{if} \quad \boldsymbol{F} = 0$$

- We keep a nuclear motion in a constant velocity v_0 $(\ll v_{\mathrm{crit}})$



Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$



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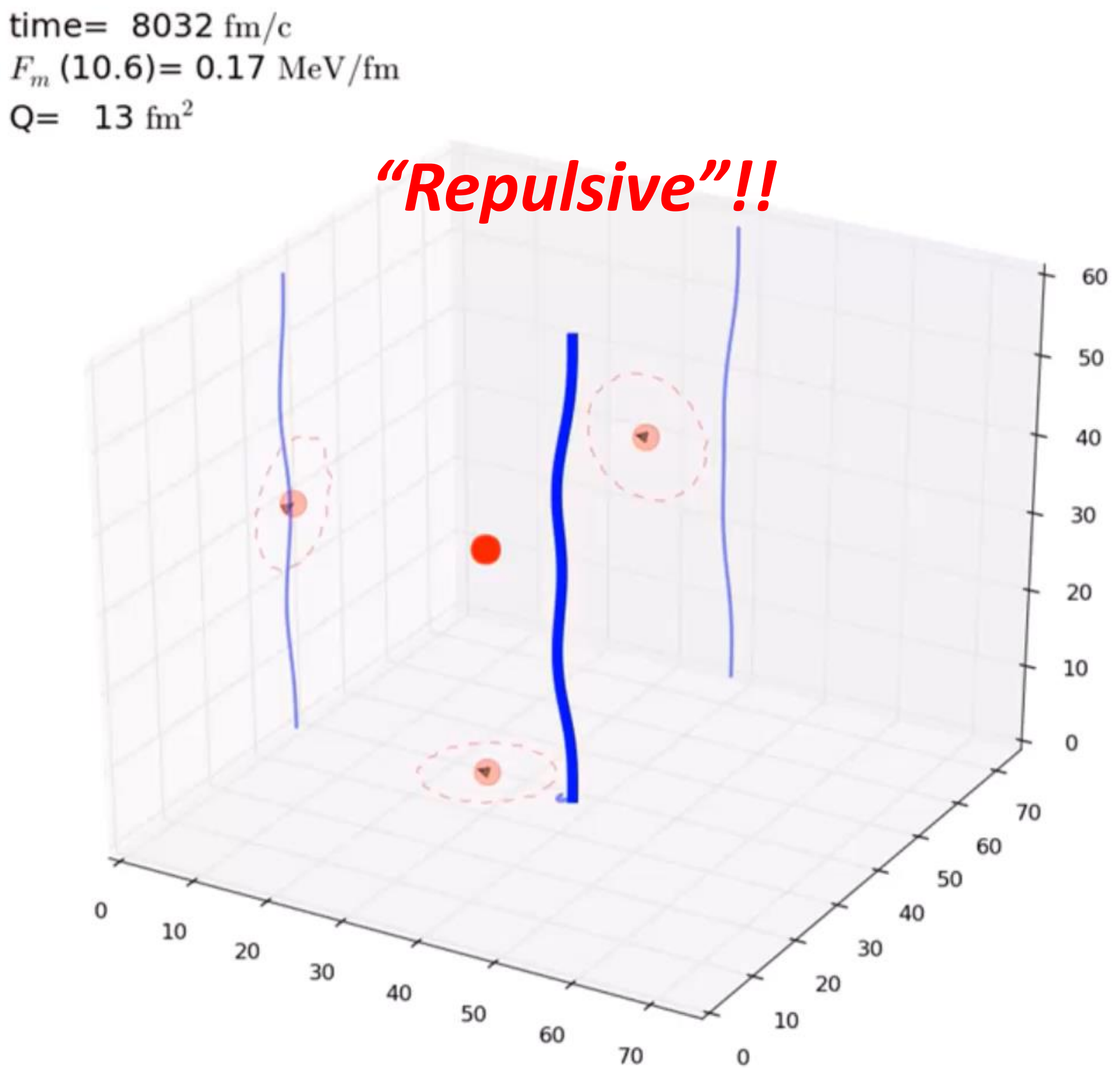
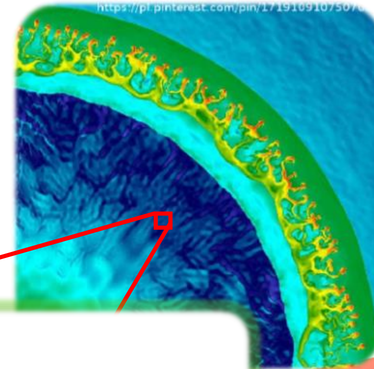
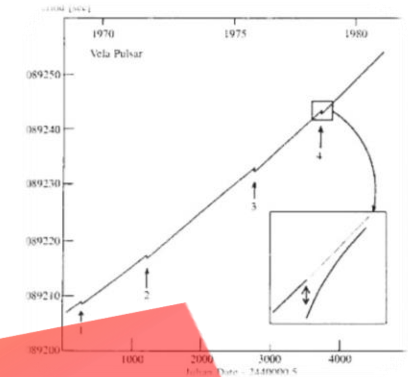


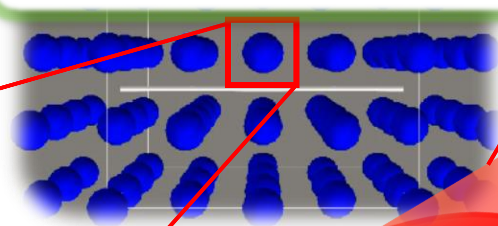
Our goal and strategy

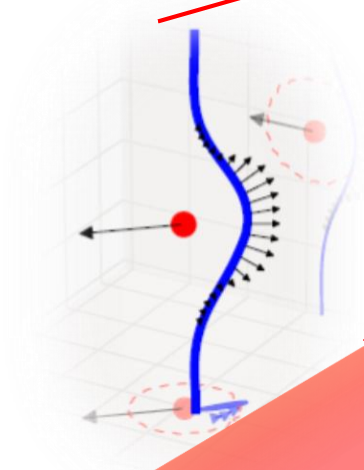
Goal: Unveil the mechanism of glitches

New collaboration started:
Nicolaus Copernicus Astronomical Centre
B. Haskell et al.









10^{4}m

Macroscopic

- observations
- hydrodynamics

~10^{-10}m

Mesoscopic

- dynamics of *vortices* in a lattice of *nuclei* (e.g. filament model)

Provide model ingredients

10^{-15}-10^{-13}m

Microscopic

Nuclear Physics!!

- vortex-nucleus dynamics from *neutrons and protons*

Fe is the most stable!

Well, then how were elements heavier than iron produced!?

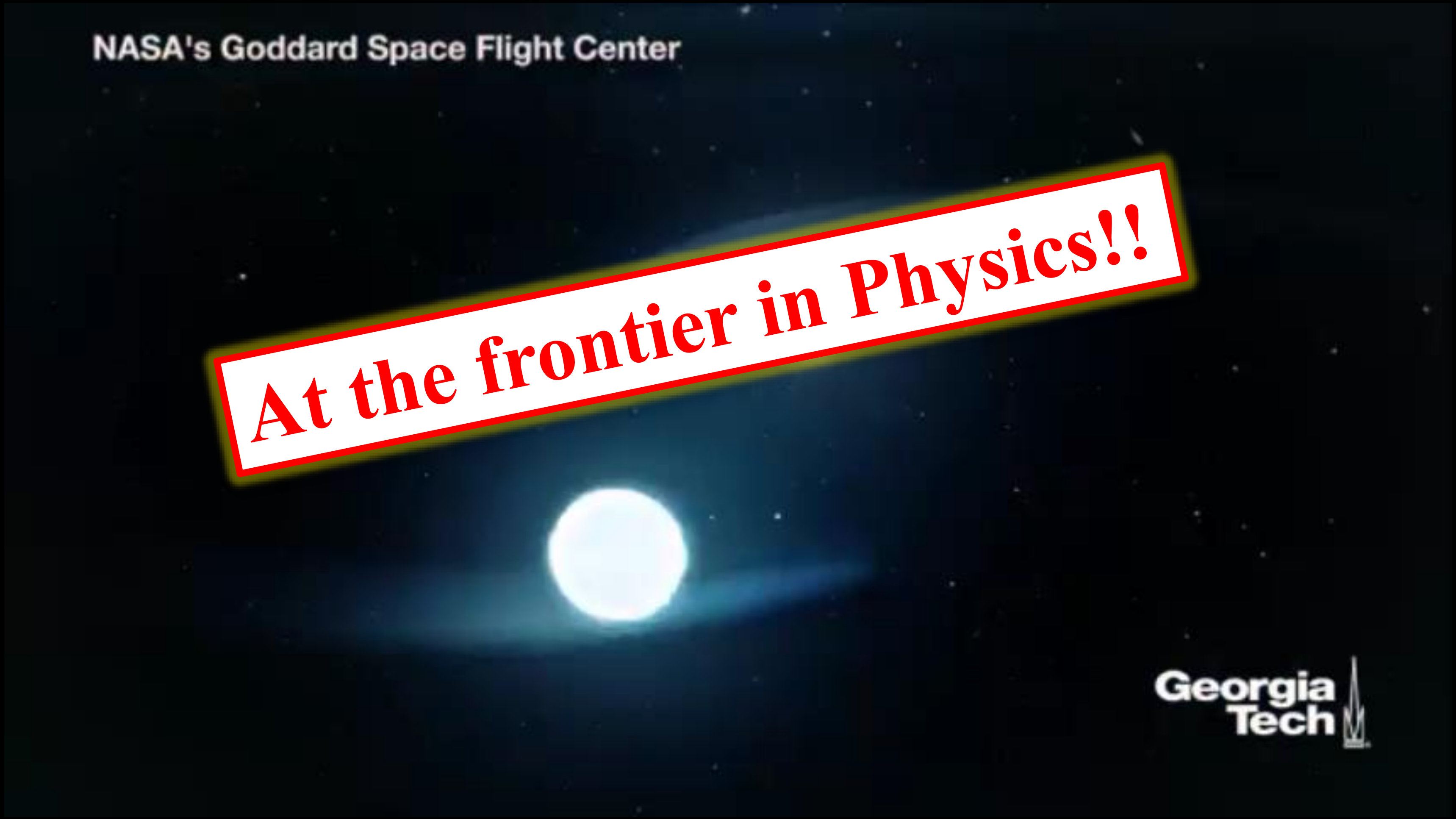
One of the unsolved problems in Physics

but, we have learned a lot!

Neutron Star Merger!!

Relevant to gravitational waves, nucleosynthesis, as well as neutron stars

Gravitational waves and gamma rays were detected on Aug. 17, 2017

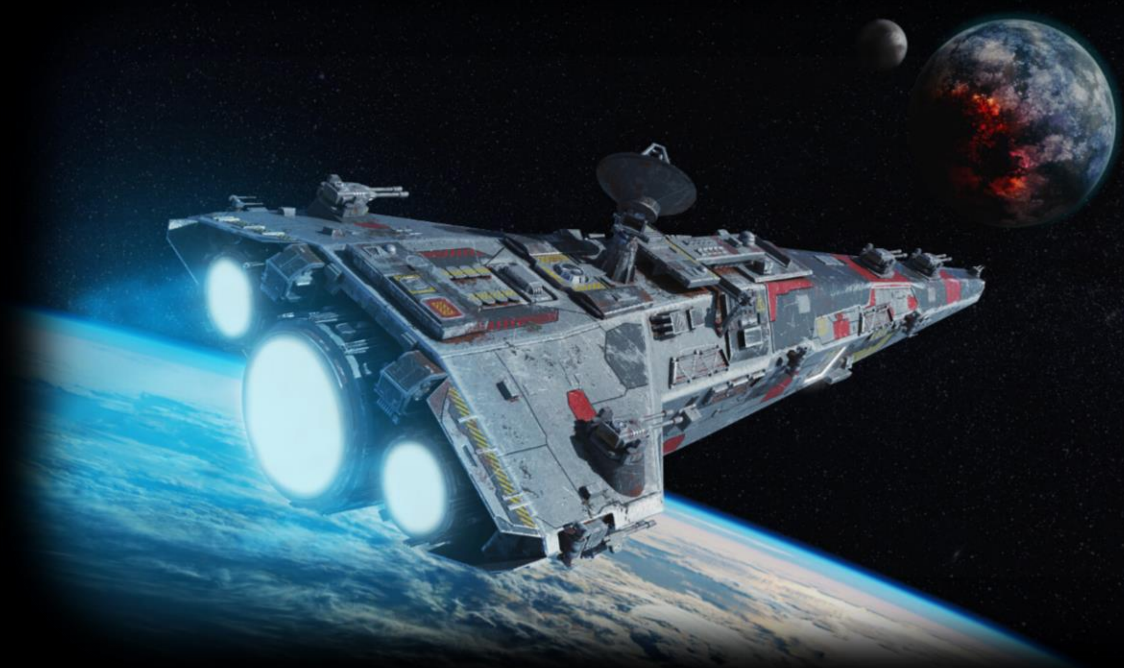


At the frontier in Physics!!

Summary

I hope you enjoyed our exciting adventure!

with a magic of DFT and TDDFT..

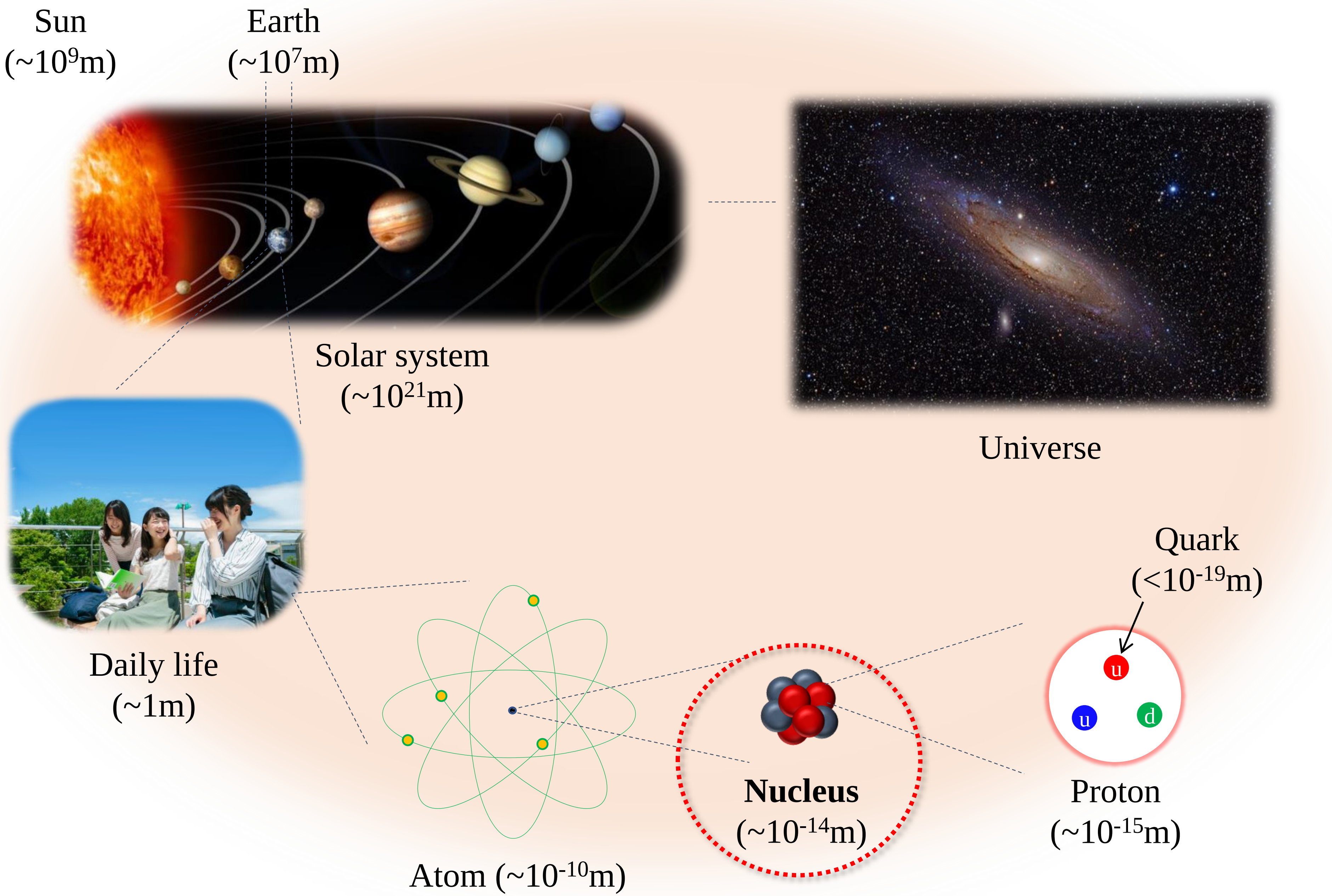


“Flat Earth” by iStock; “Fanciful view of ship sailing over edge of Earth” by Georgia Studies Images; “Star Wars Spaceship” by Valérian Pierret @Artstation

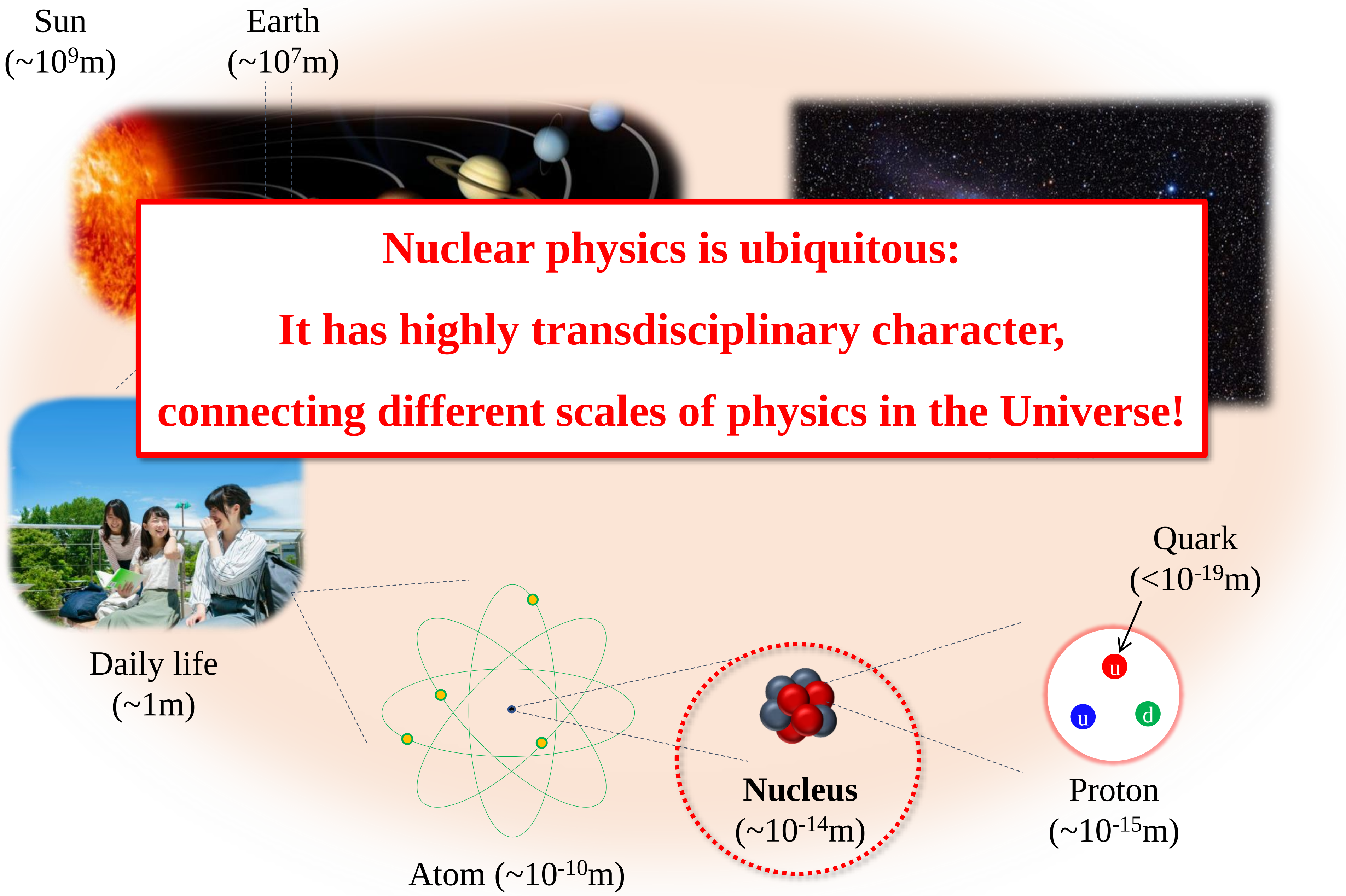
The transdisciplinary character is
one of the fascinating points of Nuclear Physics

What I showed in my lectures are only a tiny part of the huge field! :)

Takeaway message



Takeaway message



Nuclear physics is ubiquitous:

It has highly transdisciplinary character,

connecting different scales of physics in the Universe!

Message #1
for undergraduate course (BSc) students

All the subjects you learn (e.g., classical mechanics, electromagnetism, analytical mechanics, thermodynamics, statistical mechanics, quantum mechanics, etc.) **are indispensable to explore the wonderful world of physics in the universe.** It's like equipment for climbing. When completed, you'll see the breathtaking beauty of the nature that may change the rest of your life! :)

Study hard, be ambitious, and have fun!

Photo by Patrick Hendry on Unsplash

Message #2
for graduate course (MSc and PhD) students

Study what interests you the most. Dig it deeper and deeper.
It doesn’t matter what others say (of course it can be useful as an “opinion” though). Try to reach the deepest ever achieved and find a way to dig in further. This is your thesis work. **It will form your “roots.”** Then, **a thick and high trunk, a wide variety of branches, and abundant fruits of your research “tree” will grow up.** The experience to grow up the tree allows you to plant other trees as well.

Be confident, anything can be interesting!

Kazuyuki Sekizawa
Associate Professor
Department of Physics, School of Science
Tokyo Institute of Technology
2-12-1 O-Okayama, Meguro, Tokyo 152-8551, Japan
sekizawa @ phys.titech.ac.jp
http://sekizawa.fizyka.pw.edu.pl/english/

See also: