JSPS/NRF/NSFC A3 Foresight Program "Nuclear Physics in the 21st Century"
Session: Nuclear Equation of State, 3rd talk (15:40-16:10)
2022 Annual Meeting, Feb. 17-18

Entrainment Effects in Neutron Stars: Overview and Progress

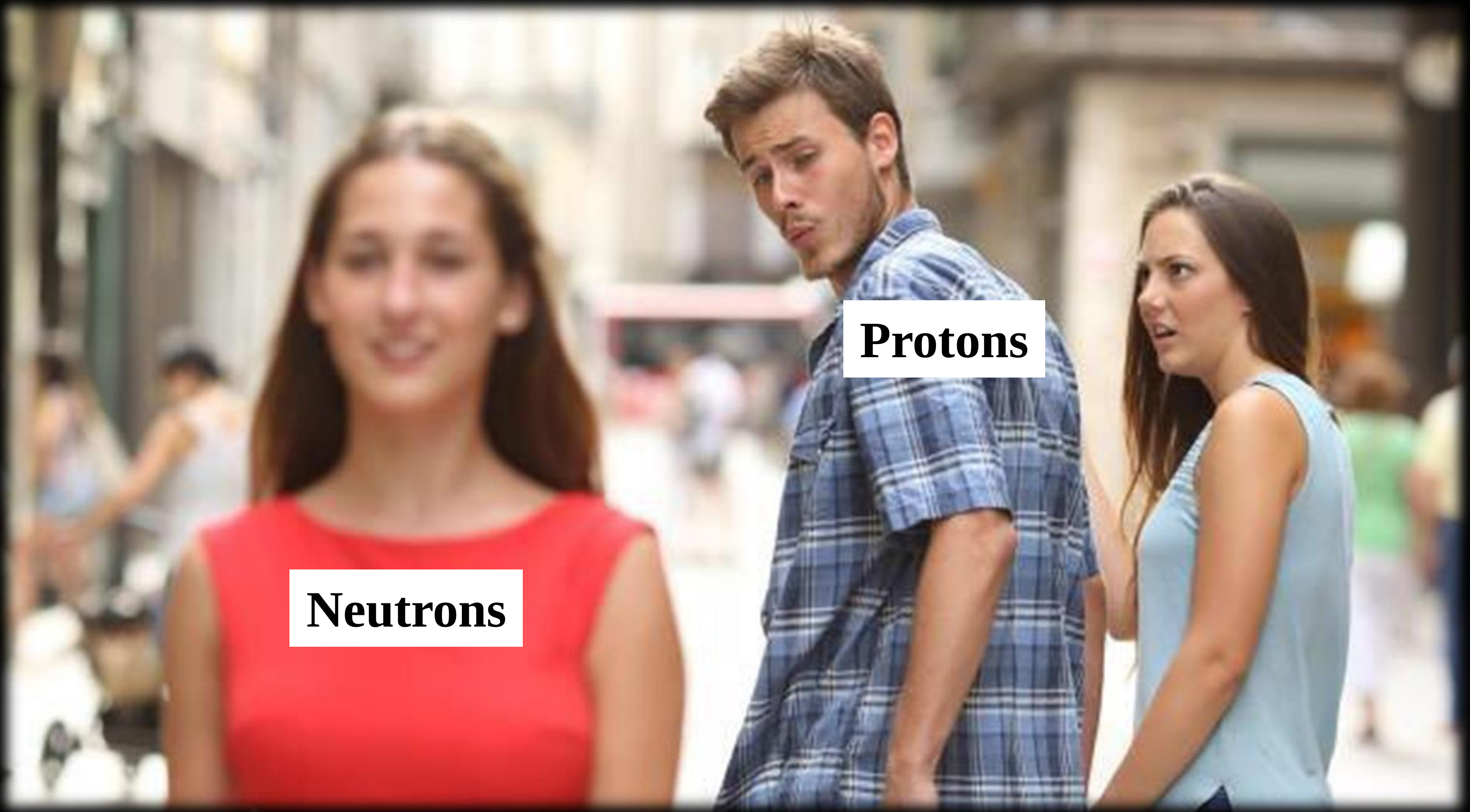
Kazuyuki Sekizawa

Department of Physics, School of Science
Tokyo Institute of Technology

東京工業大学
Tokyo Institute of Technology

“Entrainment” is something more than EoS!

“Entrainment” is a phenomenon between two species (particles, gases, fluids, etc.), where a motion of one component attracts the other.



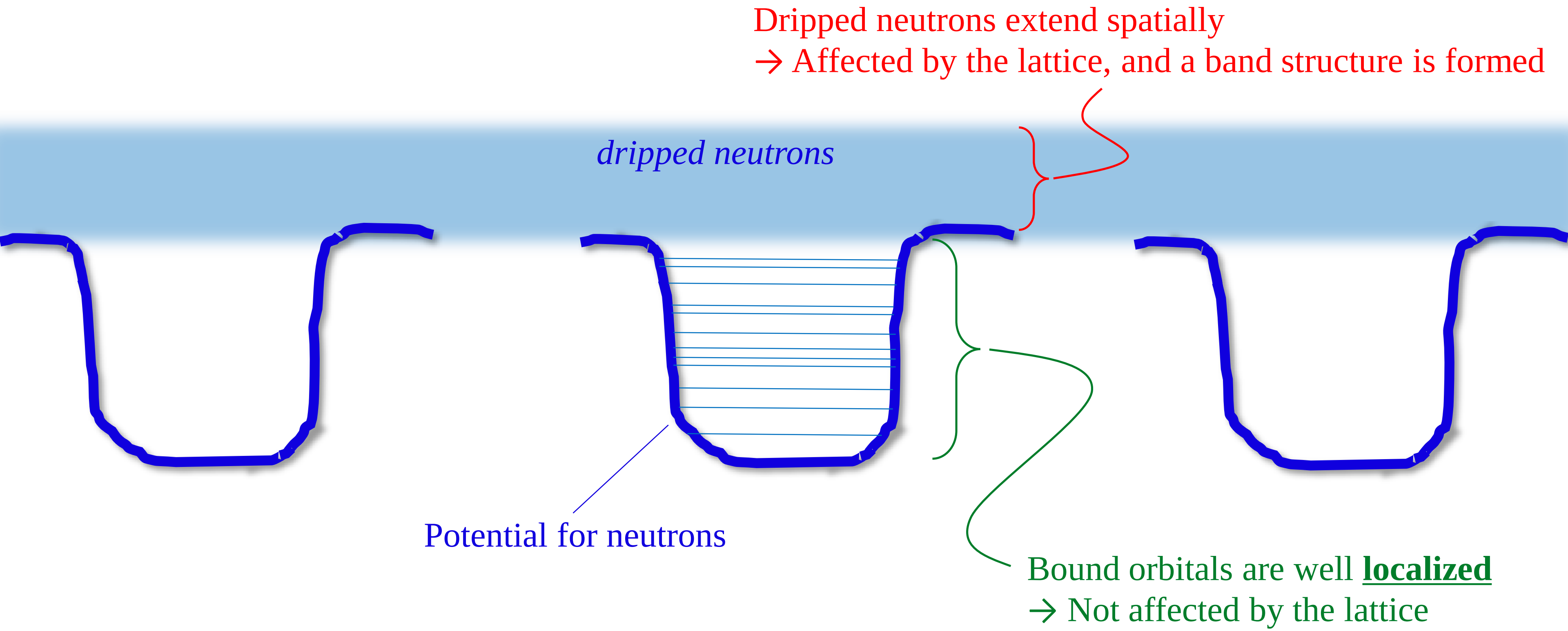
➢ **Part of dripped neutrons can be "effectively bound" (immobilized) by the periodic structure (due to Bragg scatterings), resulting in a larger effective mass**

Entrainment
→ reduction of n_c
→ enhancement of m^*

$$m_{\rm n} n_{\rm n}^{\rm f} = m_{\rm n}^{\star} n_{\rm n}^{\rm c}$$

$n_{\rm n}^{\rm c}$: Conduction neutron number density (neutrons that can actually flow)

$m_{\rm n}^{\star}$: (Macroscopic) Effective mass



The "entrainment effect" is still a debatable problem

- The first consideration for 1D, square-well potential — K. Oyamatsu and Y. Yamada, NPA**578**(1994)184
- Band calculations for slab (1D) and rod (2D) phases — B. Carter, N. Chamel, and P. Haensel, NPA**748**(2005)675
  - Entrainment effects are **weak** for the slab & rod phases:

$$\frac{m^\star}{m} \sim \begin{cases} 1.02-1.03 & \text{for the slab phase} \\ 1.11-1.40 & \text{for the rod phase} \end{cases}$$

- Band calculations for cubic-lattice (3D) phases

  N. Chamel, NPA**747**(2005)109 (2005); NPA**773**(2006)263; PRC**85**(2012)035801; J. Low Temp. Phys. **189**, 328 (2017)
  - **Significant** entrainment effects were found in a low-density region:

$$\frac{m^\star}{m} \gtrsim 10 \text{ or more!} \quad \text{for the cubic lattice}$$

- The first *self-consistent* band calculation for the slab phase (based on DFT with a BCPM EDF)
  - ***"Reduction"*** of the effective mass was observed:

$$\frac{m^\star}{m} \sim 0.65-0.75 \quad \text{for the slab phase}$$

  Yu Kashiwaba and T. Nakatsukasa, PRC**100**(2019)035804

- **Time-dependent extension of the self-consistent band theory (based on TDDFT with a Skyrme EDF)**
  - ***"Reduction"*** was observed, consistent with the Tsukuba group.

  K. Sekizawa, S. Kobayashi, and M. Matsuo, arXiv:2112.14350 (2021)

Furthermore, possible competing effects present:

Pairing correlations and **disorder** of the crustal structure

Superfluid effects

G. Watanabe and C.J. Pethick, Phys. Rev. Lett. **119**, 062701 (2017).

➢ **The band structure effects are suppressed** when the pairing gap is comparable to or greater than the strength of the lattice potential.

✓ Mean field approximation (i.e. BdG) with a **1D periodic (sinusoidal) potential**

✓ **3D case is also evaluated**, using a realistic potential based on ETFSI:
[J.M. Pearson, N. Chamel, A. Pastore, and S. Goriely, Phys. Rev. C **91**, 018801 (2015)]

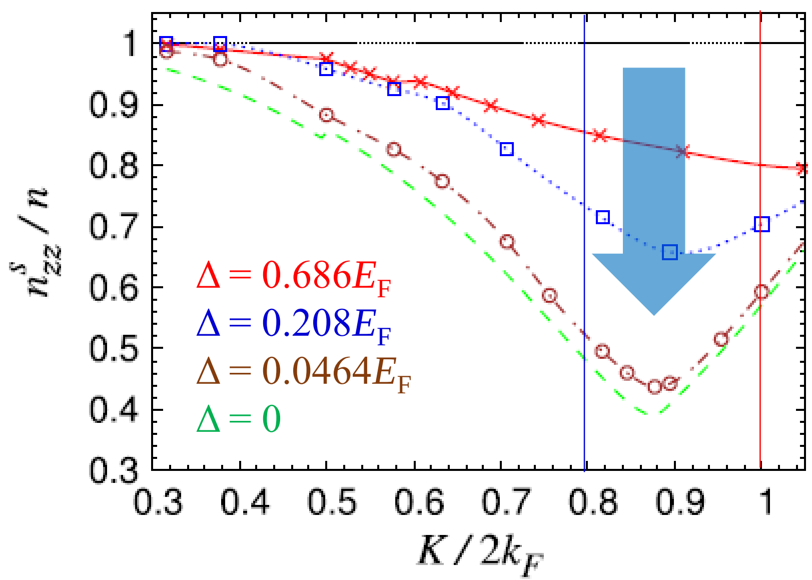
For a 3D system:

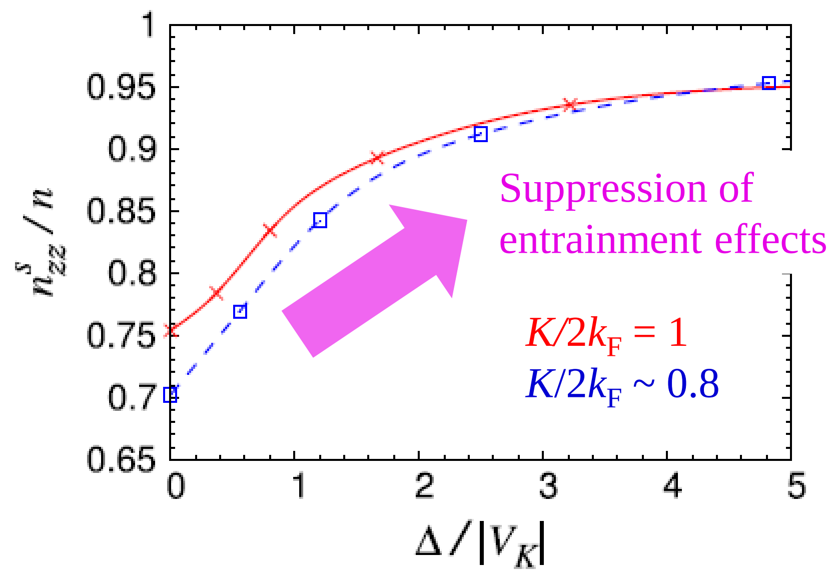
$n_s/n \sim 0.20$ for $\Delta = 0$ → $m^*/m \sim 5.00$ for $\Delta = 0$
$n_s/n \sim 0.64$ for $\Delta = 1$ MeV → $m^*/m \sim 1.41$ for $\Delta = 1$ MeV
$n_s/n \sim 0.71$ for $\Delta = 1.5$ MeV → $m^*/m \sim 1.56$ for $\Delta = 1.5$ MeV

$$V_{\rm ext}(\boldsymbol{r}) = V_K(e^{iKz} + e^{-iKz})$$

Slab period: $a = 2\pi/K$

Entrainment in a 1D system





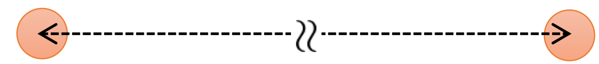
Disorder effects

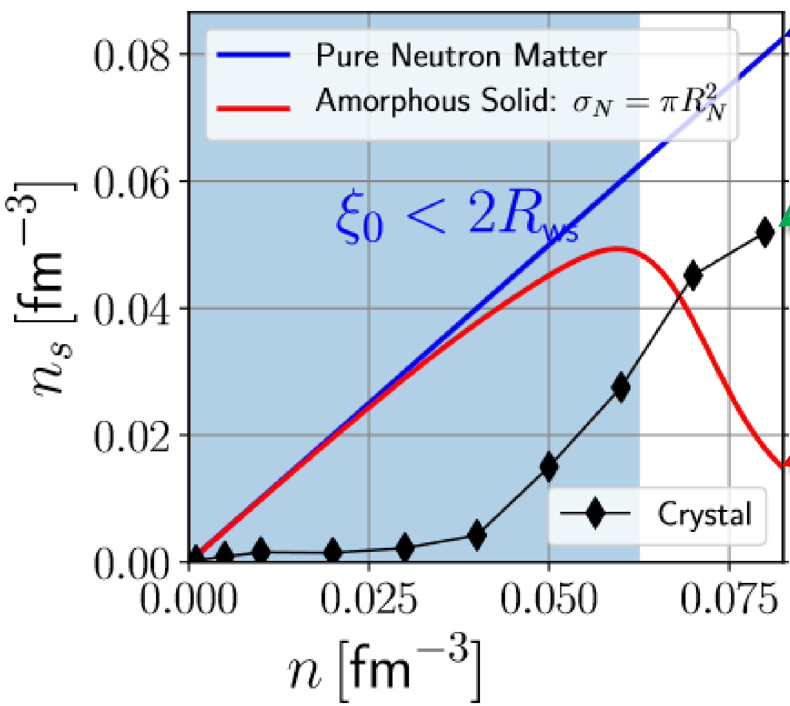
J.A. Sauls, N. Chamel, and M.A. Alpar, arXiv:2001.09959 (appeared in Jan. 2020)

➢ In the case of **amorphous crust** (i.e. **no crystalline order**), there is enough superfluid neutrons to explain large glitches.

- ✓ Superfluid neutron density n_s is reduced due to pair breaking by scatterings off disordered nuclear solid.
- ✓ A theory of "metallic alloys," "amorphous metals," and "dirty superconductors" is applied.
- ✓ Except a bottom layer ($n > 0.06$ fm^{-3}), **the effect is weak**.

For $n \lesssim 0.06\ \mathrm{fm}^{-3}$, $2R_{\mathrm{WS}} \gtrsim 40\ \mathrm{fm}$, while $R \approx 6\ \mathrm{fm}$; i.e.





Pure Neutron Matter at $T = 0$

$$n_{\mathrm{s}} = n_{\mathrm{n}}$$

Results of band calculations for **perfect crystals** (BCC)
N. Chamel, Nucl. Phys. **A747**, 109 (2005)

$$n_{\mathrm{c}} = n_{\mathrm{n}} \times \frac{m_{\mathrm{n}}}{m_{\mathrm{n}}^{\star}}$$

Large effective mass
$\Longrightarrow$ Less conduction neutrons

Amorphous crust (no crystalline order)

$$n_{\mathrm{s}} \approx \begin{cases} n_{\mathrm{n}}\left(1 - \dfrac{\pi^2}{8}\dfrac{\xi_0}{l}\right) & \text{for } \alpha \ll 1 \\ n_{\mathrm{n}}\dfrac{l}{\xi_0} & \text{for } \alpha \gg 1 \end{cases}$$

at low densities $\xi_0 << l$

at high densities $l < \xi_0$

$\xi_0 = \hbar v_{\mathrm{F}}/\pi\Delta$: coherence length in PNM

$l = 1/n_{\mathrm{imp}}\sigma_{\mathrm{tr}}$: mean free path

$$n_{\mathrm{imp}} = \frac{1}{V_{\mathrm{WS}}} \qquad \sigma_{\mathrm{tr}} = \pi R^2$$

The current debatable situation about the entrainment effects

➢ **The purpose is to clarify the actual effects of entrainment in the inner crust of neutron stars!**

Band calculation (Thomas-Fermi approx., w/o pairing)

$$\frac{m^\star}{m} \sim \begin{cases} 1.02-1.03 & \text{for the slab phase} \\ 1.11-1.40 & \text{for the rod phase} \end{cases}$$

$$\frac{m^\star}{m} \gtrsim 10 \text{ or more!} \quad \text{for the cubic lattice}$$

Band calculation [Self-consistent (TD)DFT, w/o pairing]

$$\frac{m^\star}{m} \sim 0.65-0.75 \quad \text{for the slab phase}$$

We consider those numbers should be corrected.

Band calculation (Mean-Field approx., with pairing)

$$\frac{m^\star}{m} \sim 1-2 \quad \text{for the slab phase}$$

$$\frac{m^\star}{m} \sim 1.41-1.56 \quad \text{for the cubic phase (at most)}$$

Disorder effects (w/o band structure effects)

$$\frac{m^\star}{m} \sim 1-1.2 \quad \text{for the cubic phase}$$

Current members of our “Entrainment” group
(consist of Japan & China sides)

Kochi Univ.

K. Iida

EoS, Pasta, QPO
Color superconductivity
Ultracold atomic gases

+some students

Univ. Tsukuba

T. Nakatsukasa

Nuclear DFT
Pairing correlations
Large-amplitude motions

+1 PhD students

Niigata Univ.

M. Matsuo

Nuclear DFT
Pairing correlations
Linear response theory (QRPA)

+1 PhD student (graduated)

Zhejiang Univ.

G. Watanabe

Nuclear pasta
Superfluid phenomena
Ultracold atomic gases

+1 postdoc (Y. Minami)

Tokyo Tech (April 2021~)

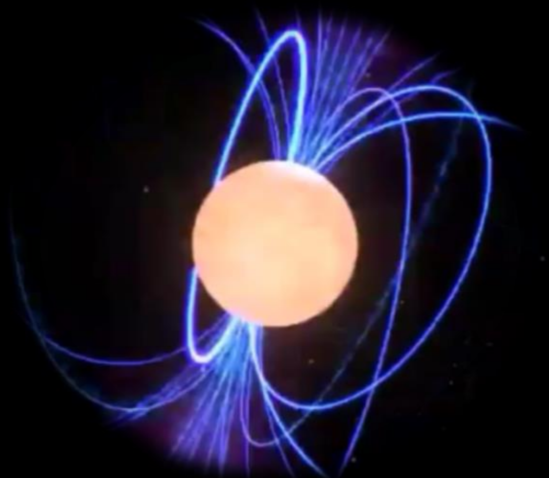
K. Sekizawa

Nuclear (TD)DFT
Superfluid dynamics, Glitches
Ultracold atomic gases

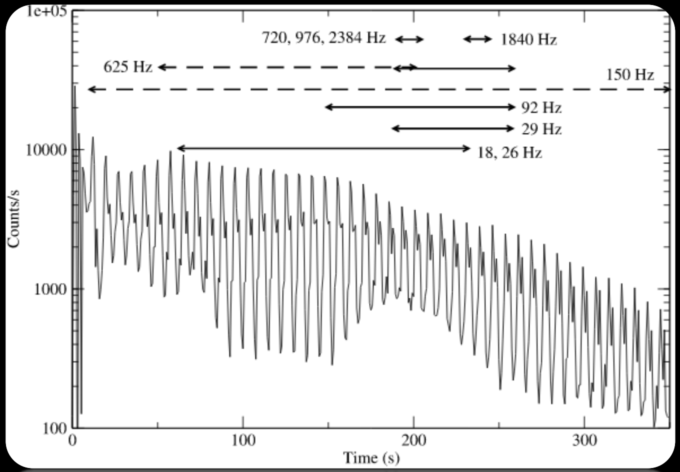
+2 MSc (graduated), 1 BSc students

It may affect interpretation of various phenomena, e.g.:

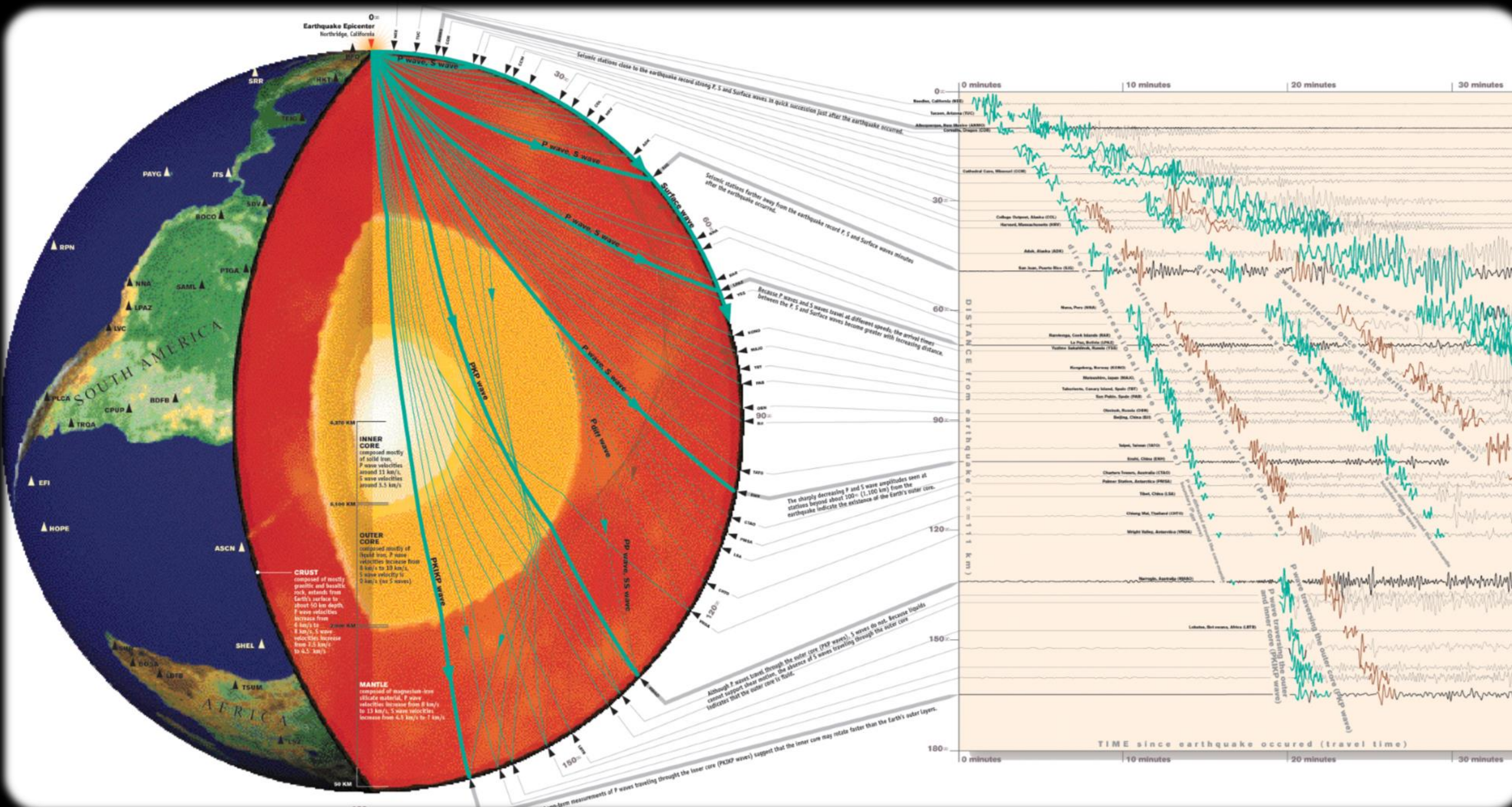
Neutron-star glitch



Quasi-periodic oscillation



Seismology (地震学): Studying inside of the Earth from earthquakes and their propagation



Picture taken from AusPass by Australian National University

QPOs as "asteroseismology"

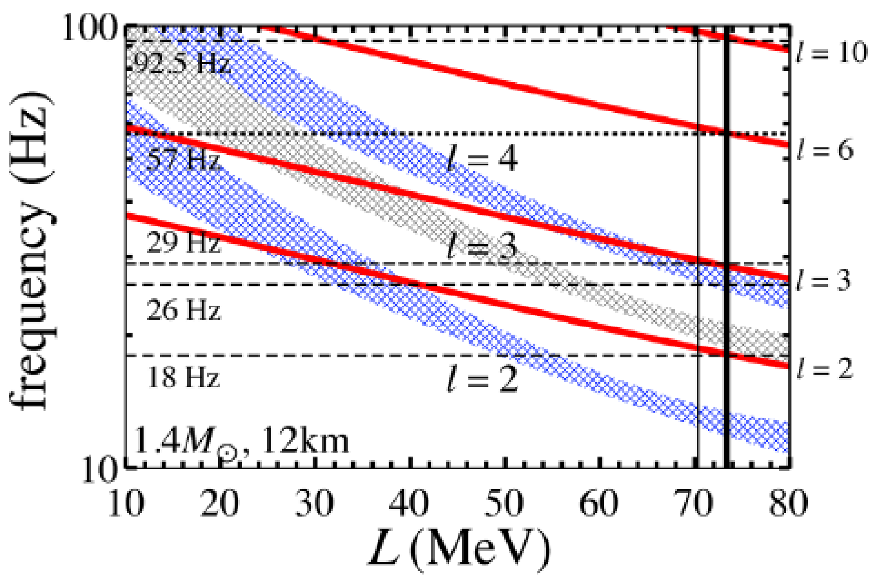
Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS 489, 3022–3030 (2019) doi:10.1093/mnras/stz2385
Advance Access publication 2019 August 29

Astrophysical implications of double-layer torsional oscillations in a neutron star crust as a lasagna sandwich

Hajime Sotani,[1★] Kei Iida[2] and Kazuhiro Oyamatsu[3]

[1] *Division of Science, National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan*
[2] *Department of Mathematics and Physics, Kochi University, 2-5-1 Akebono-cho, Kochi 780-8520, Japan*
[3] *Department of Human Informatics, Aichi Shukutoku University, 2-9 Katahira, Nagakute, Aichi 480-1197, Japan*



- Many (~30) observed QPO frequencies, and prediction by a Bayesian analysis, have been nicely explained by torsional oscillations of tube–bubble or sphere–cylinder layer

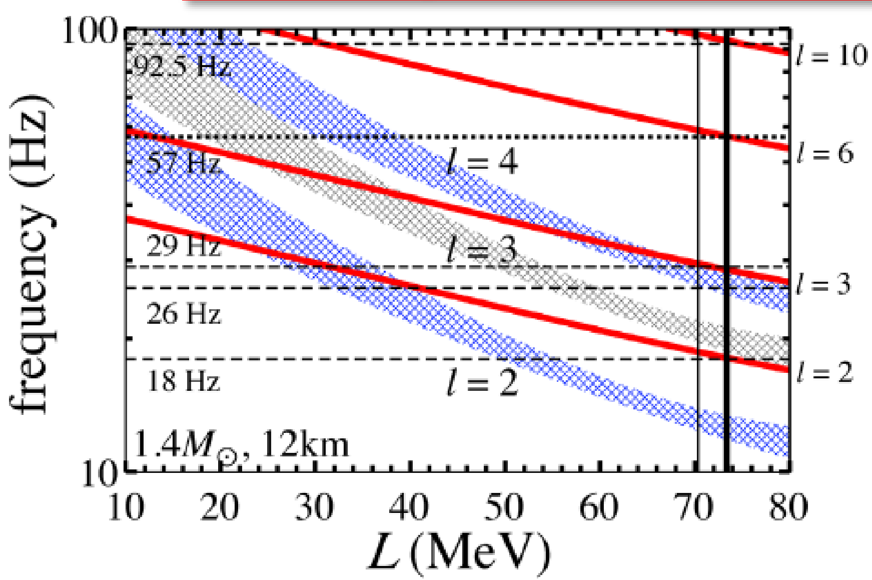
Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS 489, 3022–3030 (2019)
Advance Access publication 2019 August 29
doi:10.1093/mnras/stz2385

Astrophysical implications of double-layer torsional oscillations in a neutron star crust as a lasagna sandwich

Hajime Sotani[1★], Kei Iida[2] and Kazuhiro Oyamatsu[3]
[1] *Division of Science, National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo 181-8588, Japan*
[2] *Department of Mathematics and Physics, Kochi University, 2-5-1 Akebono-cho, Kochi 780-8520, Japan*
[3] *Department of Human Informatics, Aichi Shukutoku University, 2-9 Katahira, Nagakute, Aichi 480-1197, Japan*

The interpretation will be affected by the entrainment effects!

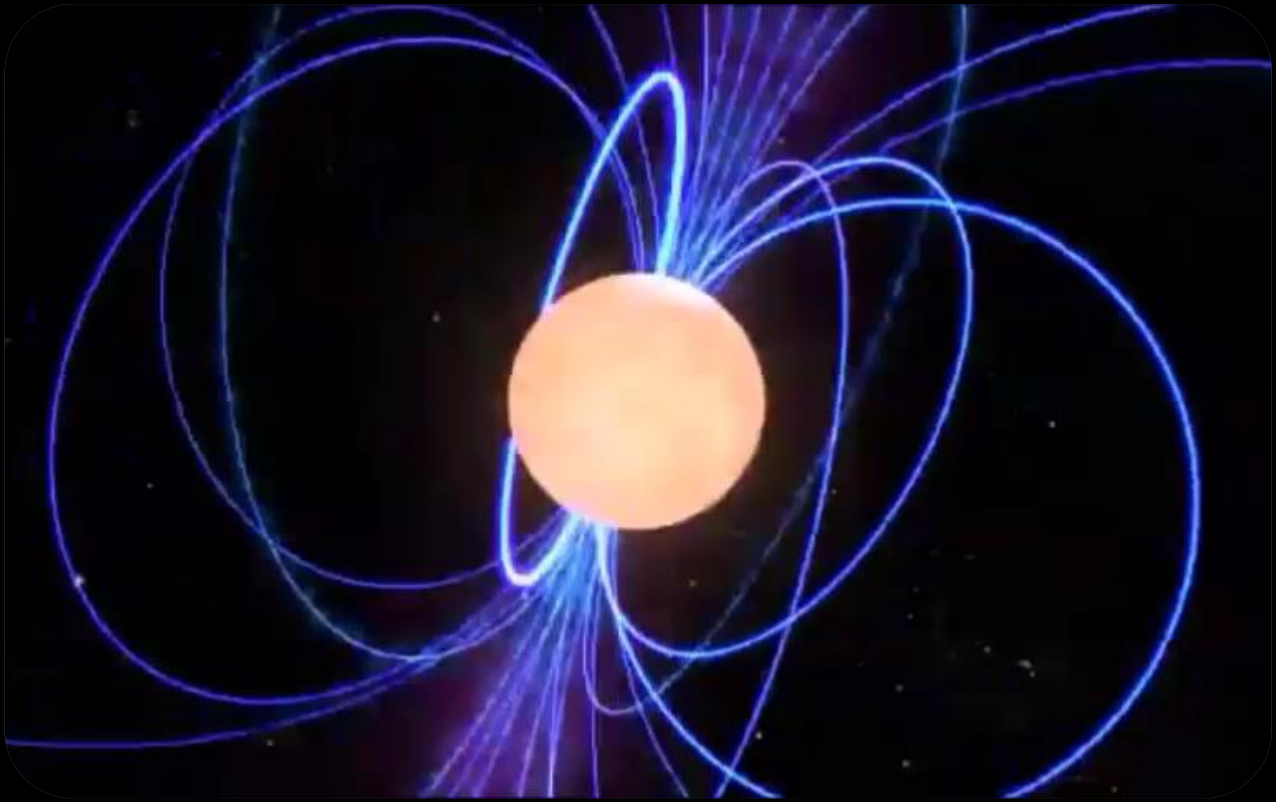


- Many (~30) observed QPO frequencies, and prediction by a Bayesian analysis, have been nicely explained by torsional oscillations of tube–bubble or sphere–cylinder layer

What is the glitch?

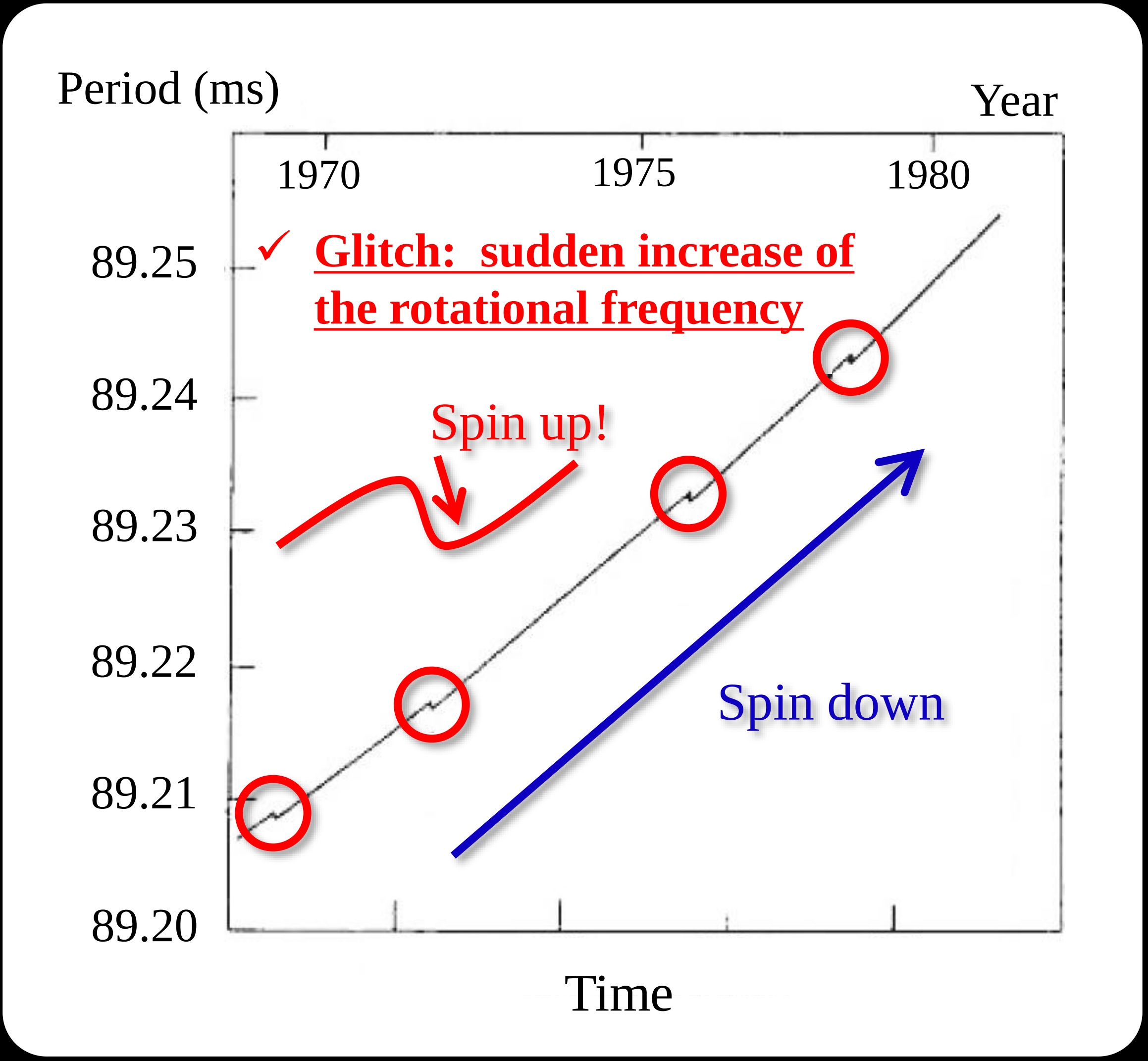
Pulsar - a rotating neutron star

- ✓ First discovery in August 1967 → “Little Green Man” LGM-1 → PSR B1919+21
- ✓ Since then, more than 2650 pulsars have been observed
- ✓ It gradually spins down due to the EM radiation



Typical example: the Vela pulsar

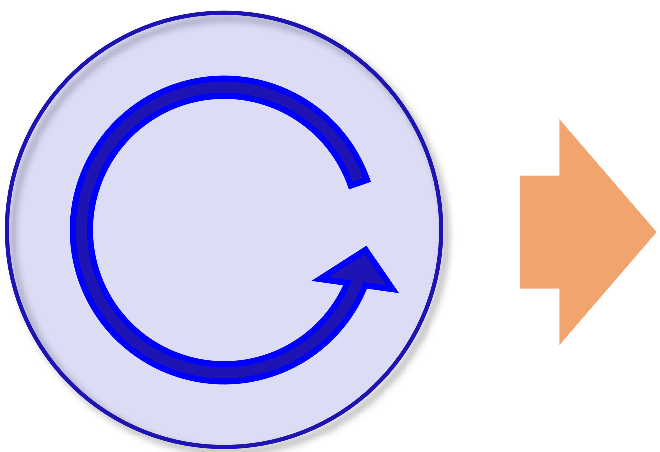
- *Irregularity* has been observed from continuous monitoring of the pulsation period



Quantum vortices in a neutron star

In rotating superfluid, an array of quantum vortices is generated

- Observation in ultra-cold atomic gases

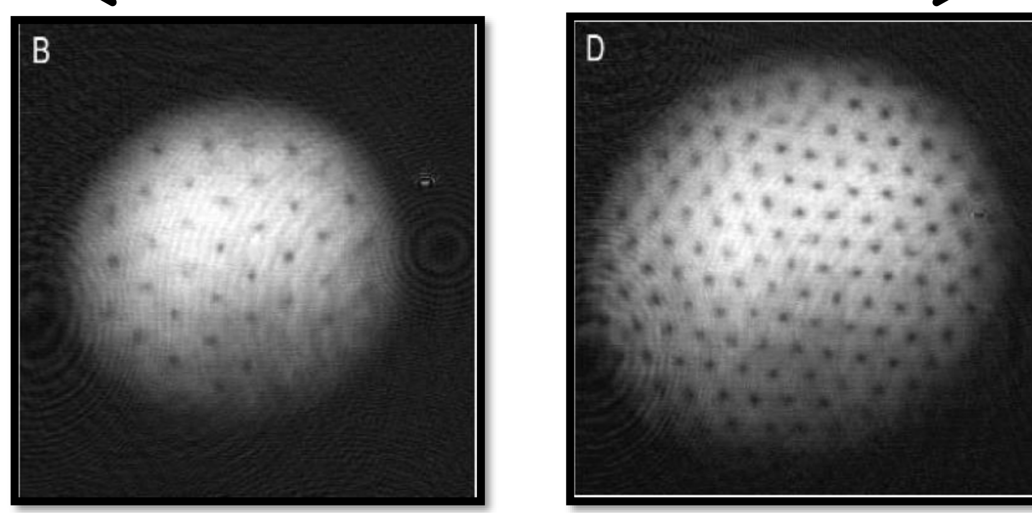


Vortex-number density

$\propto$

Rotation speed

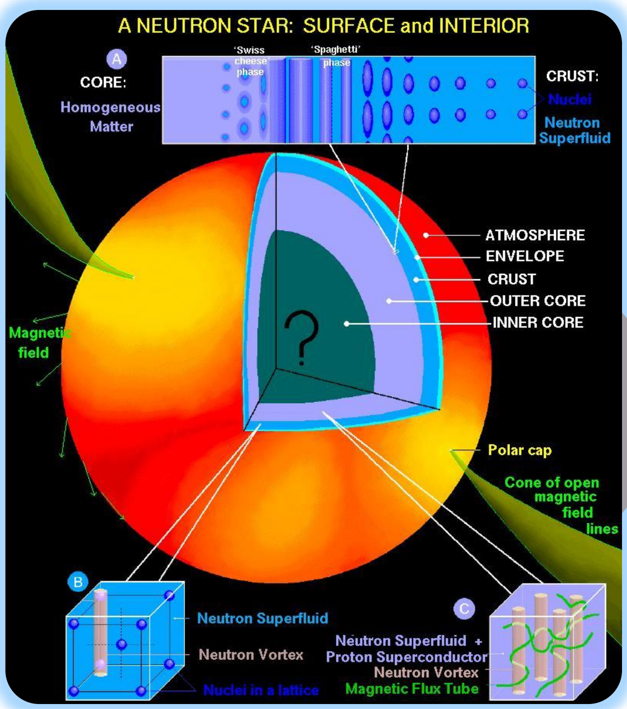
Low ⟷ High



W. Ketterle, MIT Physics Annual. 2001

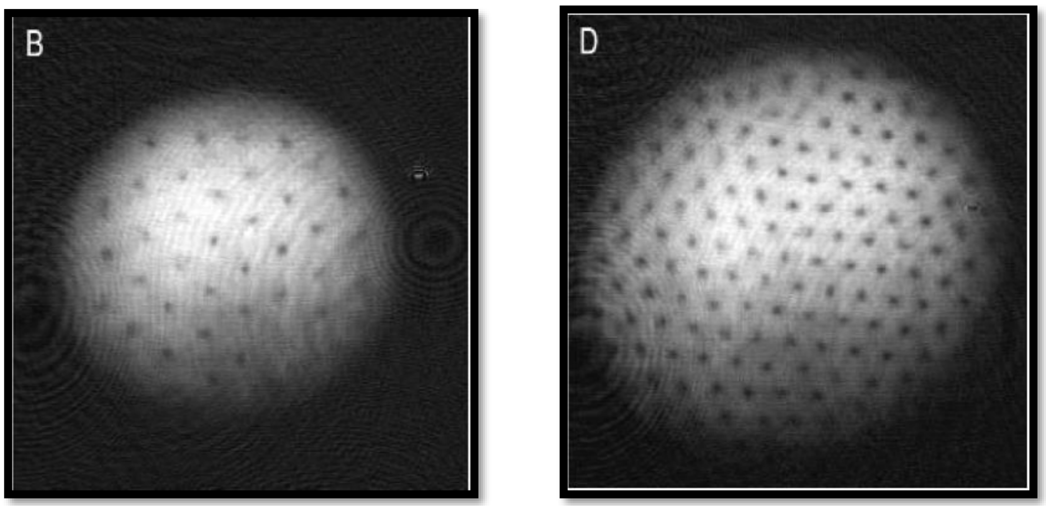
In rotating superfluid, an array of quantum vortices is generated

- Observation in ultra-cold atomic gases



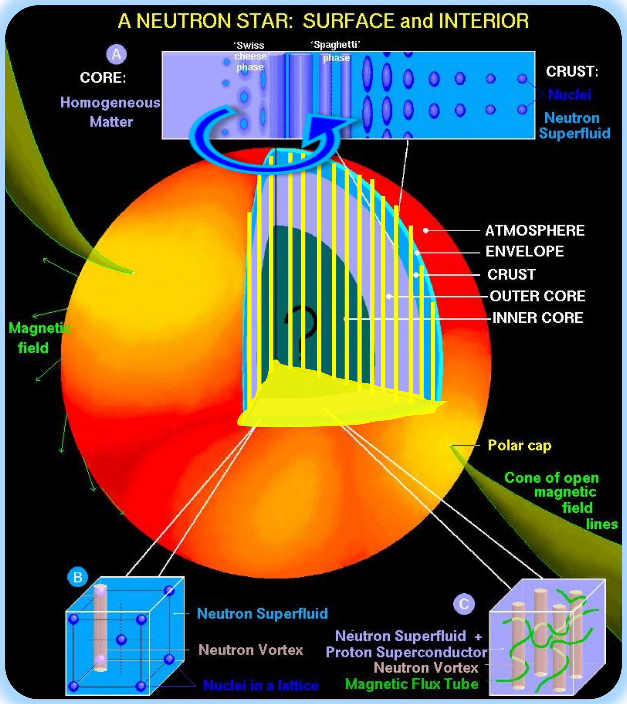
Vortex-number density
$\propto$
Rotation speed

Low ⟷ High



W. Ketterle, MIT Physics Annual. 2001

There must be a huge number (~10^{18}) of vortices inside a neutron star!!

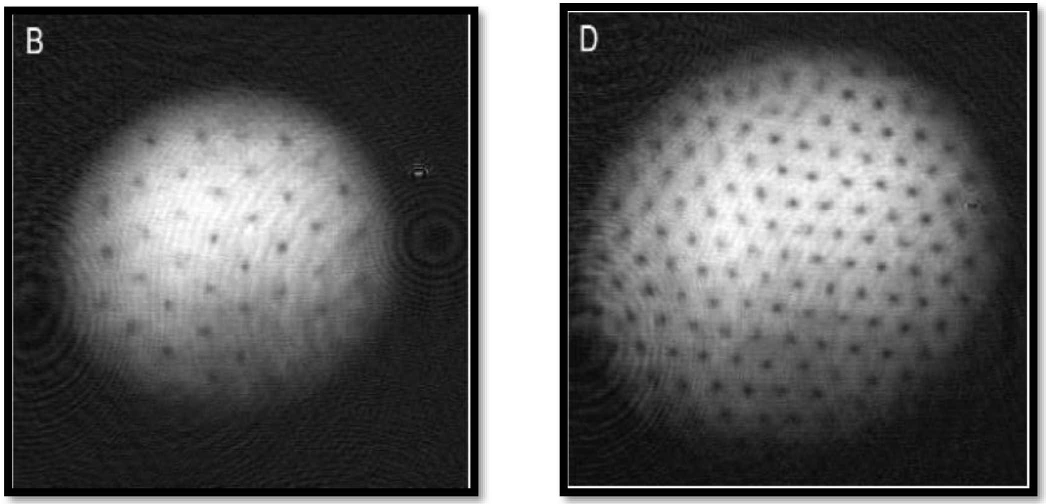


Vortex-number density

$\propto$

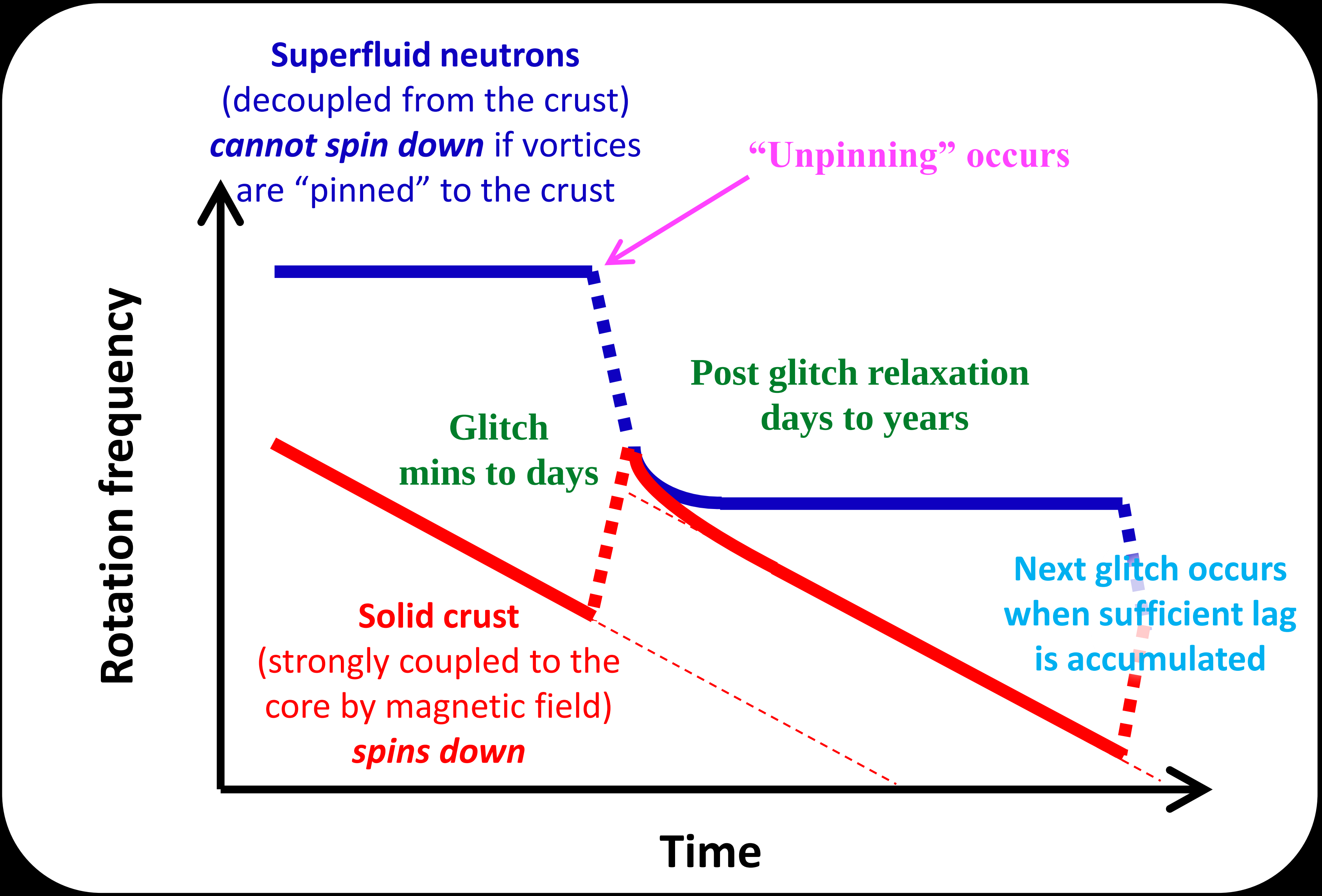
Rotation speed

Low ⟷ High

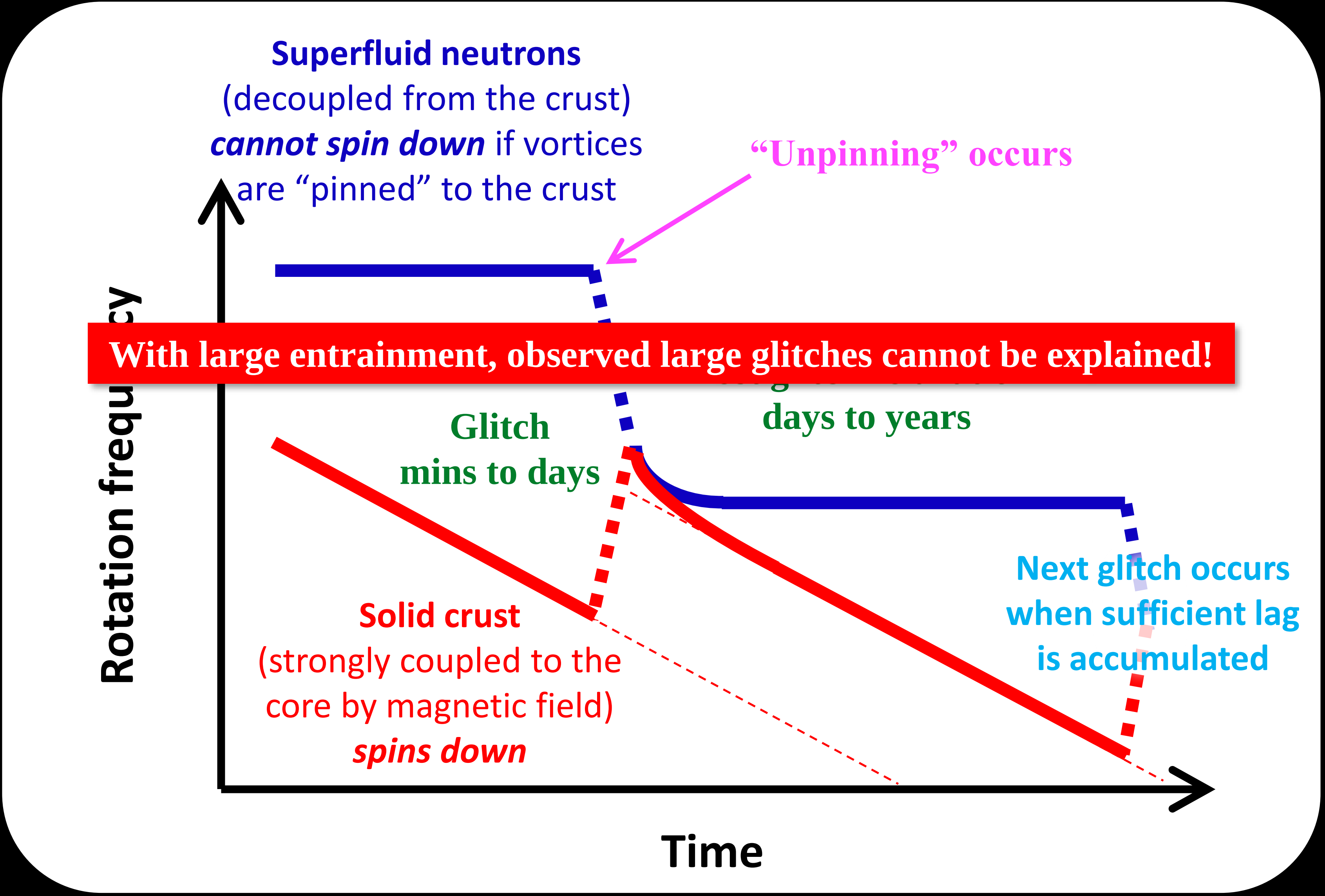


W. Ketterle, MIT Physics Annual. 2001

The vortex mediated glitch: Naive picture

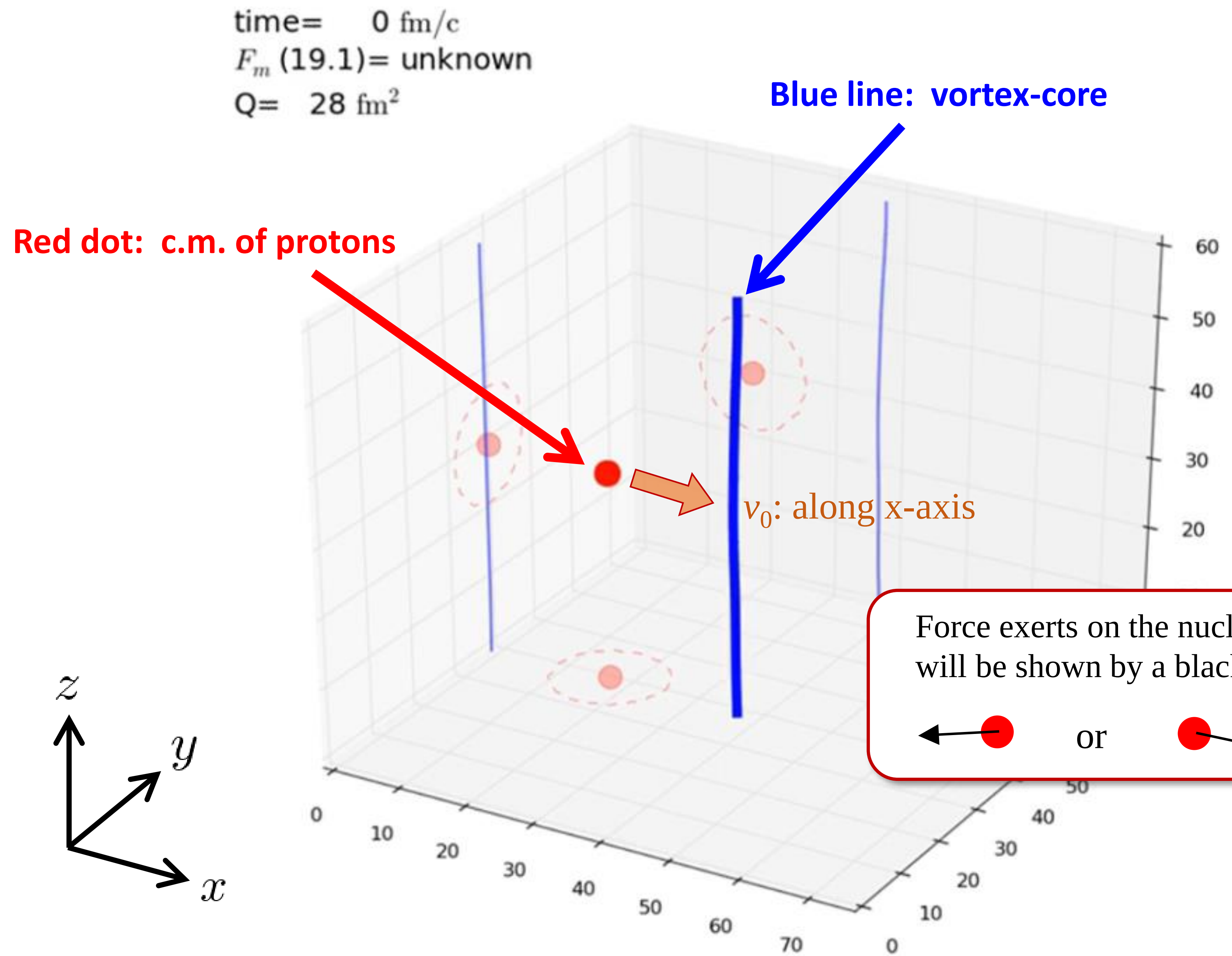


The vortex mediated glitch: Naive picture



With large entrainment, observed large glitches cannot be explained!

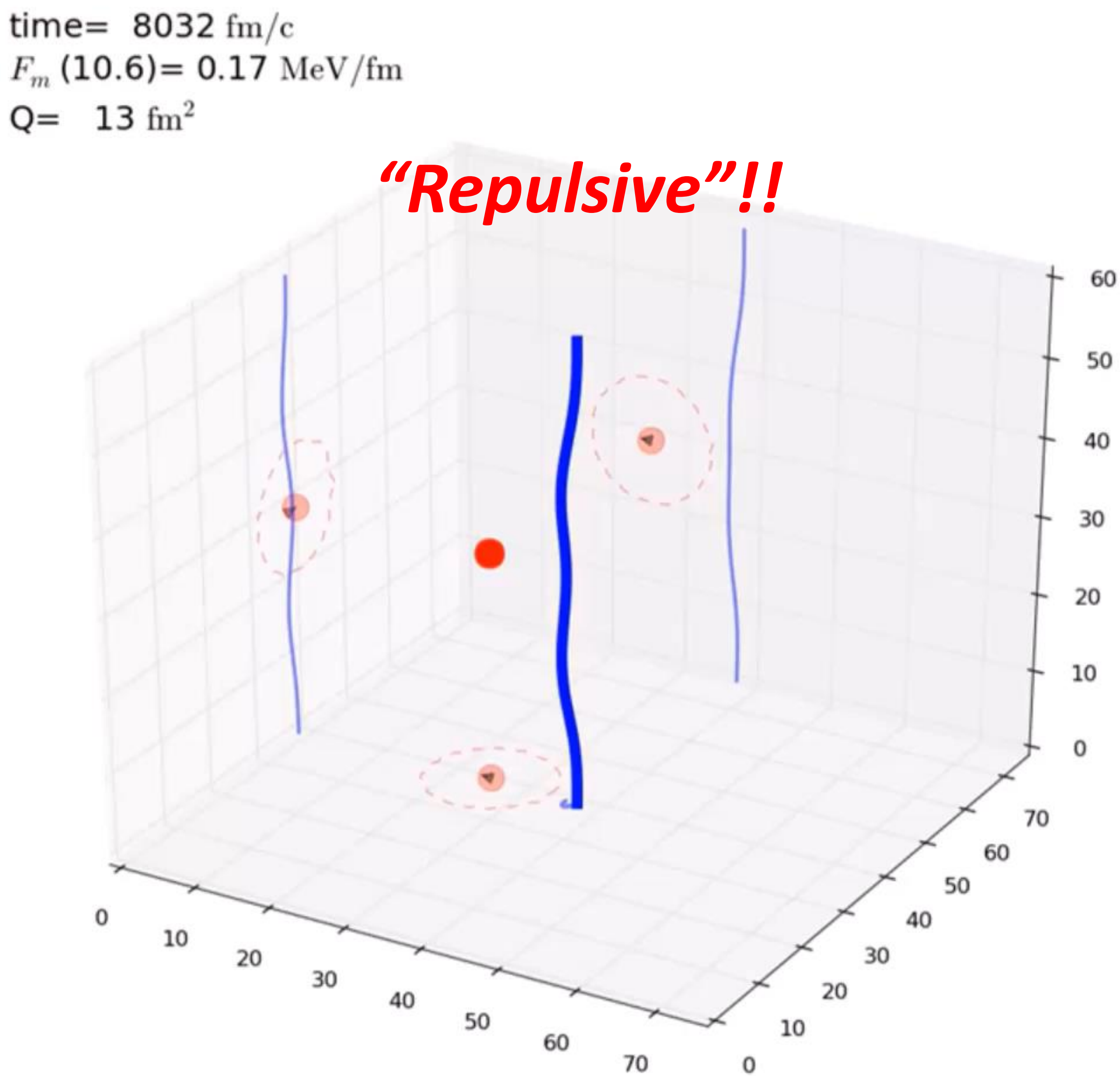
Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$



Force exerts on the nucleus will be shown by a black arrow

or ?

Results of TDSLDA calculation: $\rho_n \simeq 0.014\ \mathrm{fm}^{-3}$



Vortex pinning/unpinning dynamics within 3D-TDGPE simulations

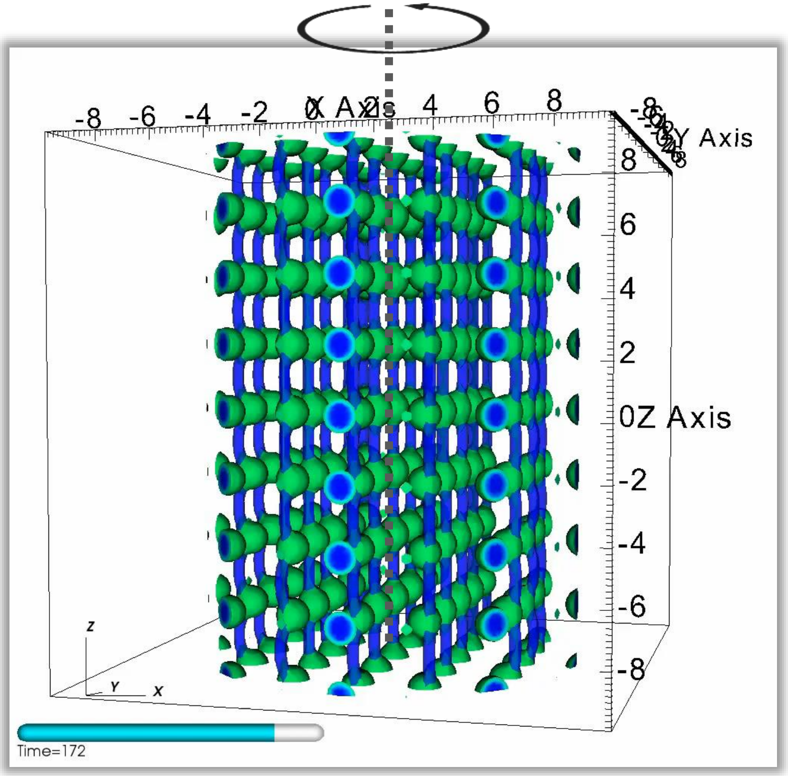
Simulations by Teppei Sasaki (MSc student, will be graduated in Mar. 2022)

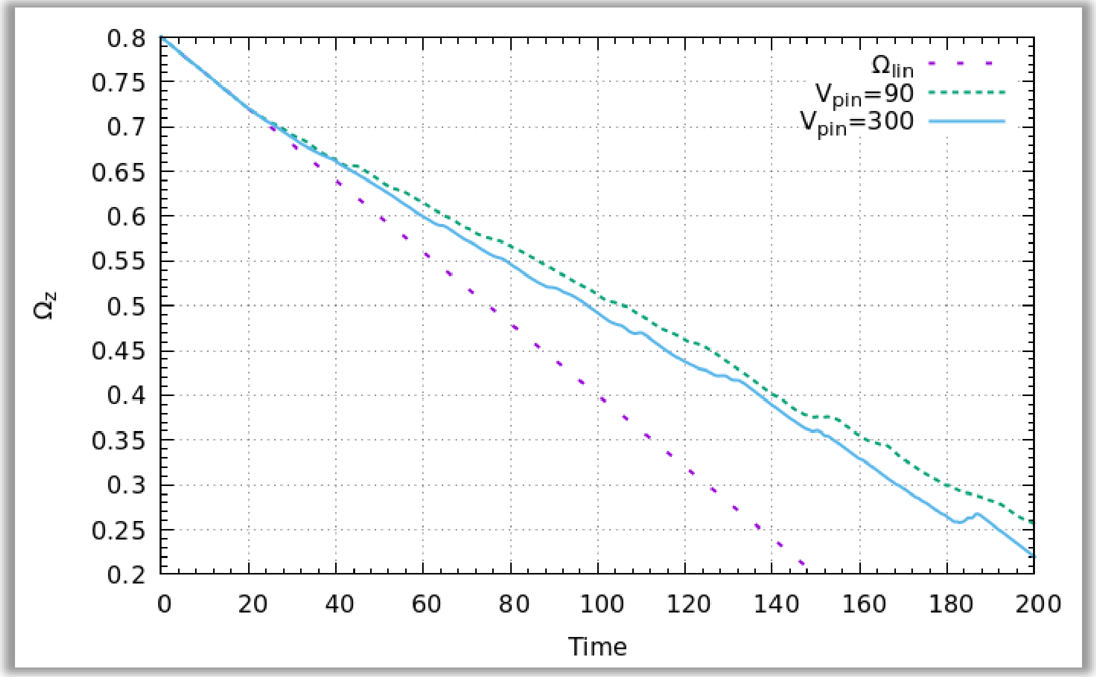
- 3D TDGPE (Time-Dependent Gross-Pitaevskii Eqution):

$$(i-\gamma)\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{x},t)=\left[-\frac{\hbar^2}{2m}\nabla^2+V(\boldsymbol{x})-\mu-\Omega_z\hat{L}_z+gN|\psi(\boldsymbol{x},t)|^2\right]\psi(\boldsymbol{x},t)$$

- Equation of motion for the container:

$$I_c\frac{d\Omega_z}{dt}=-\frac{d\langle L_z\rangle}{dt}-N_{\mathrm{ext}}$$





Vortex pinning/unpinning dynamics within 3D-TDGPE simulations

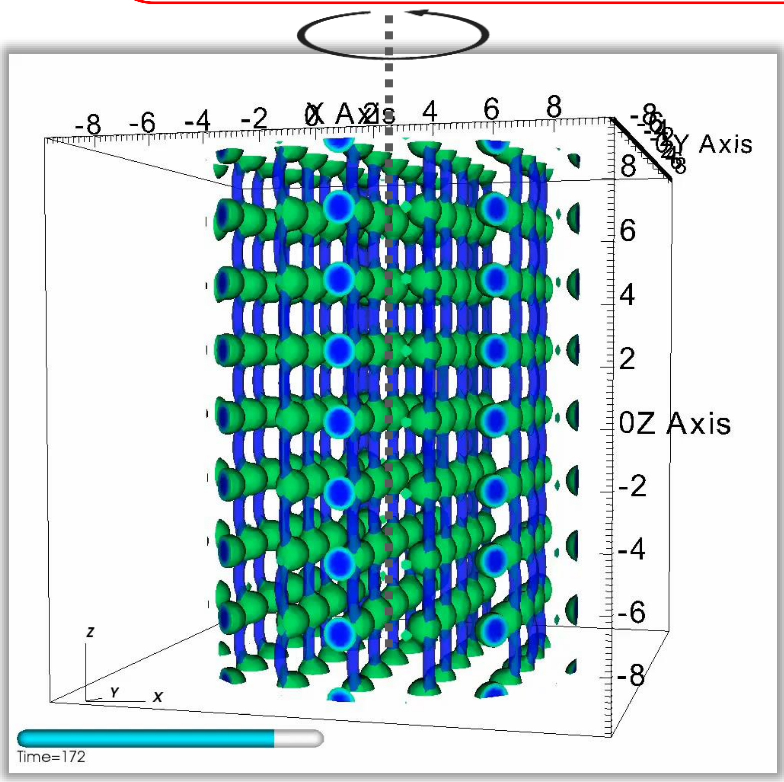
Simulations by Teppei Sasaki (MSc student, will be graduated in Mar. 2022)

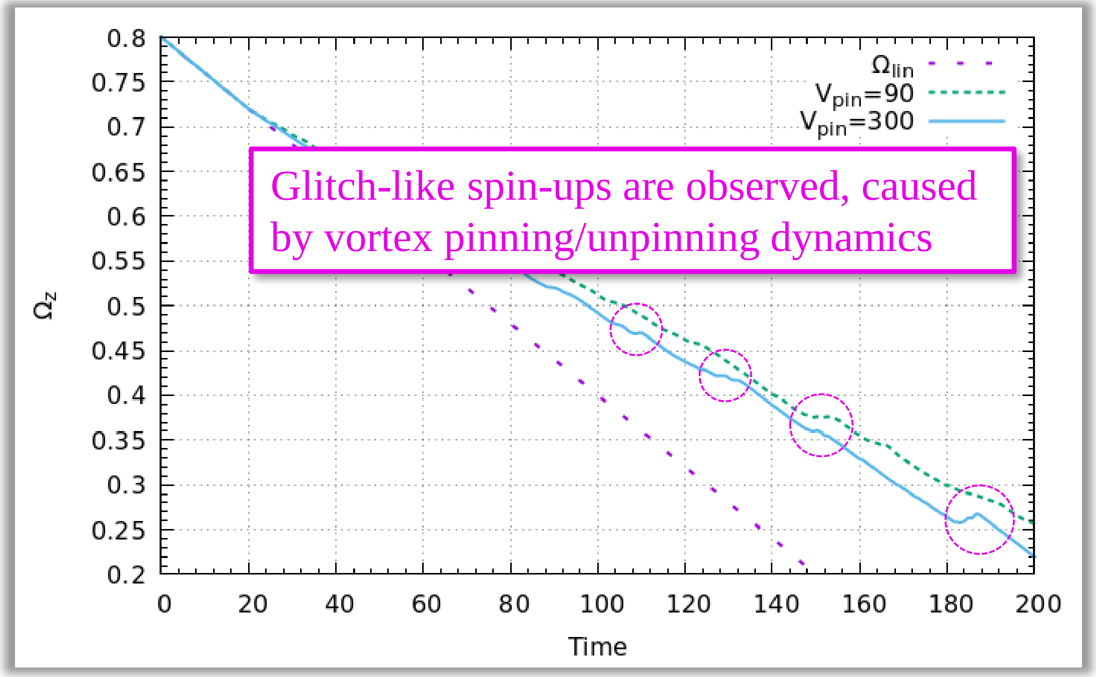
- 3D TDGPE (Time-Dependent Gross-Pitaevskii Eqution):

$$(i-\gamma)\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{x},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{x}) - \mu - \Omega_z\hat{L}_z + gN|\psi(\boldsymbol{x},t)|^2\right]\psi(\boldsymbol{x},t)$$

- Equation of motion for the container:

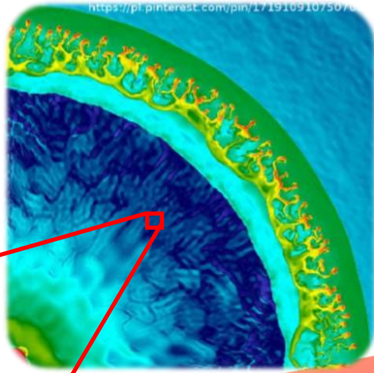
$$I_c\frac{d\Omega_z}{dt} = -\frac{d\langle L_z\rangle}{dt} - N_{\text{ext}}$$

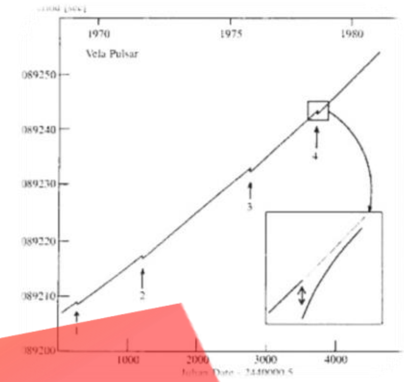


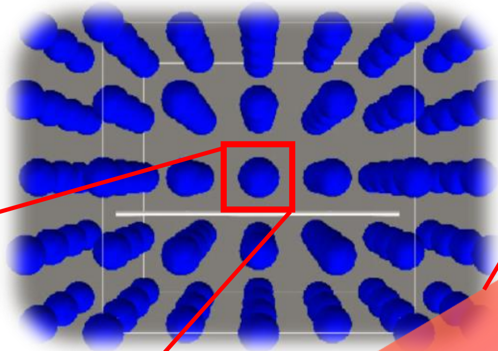


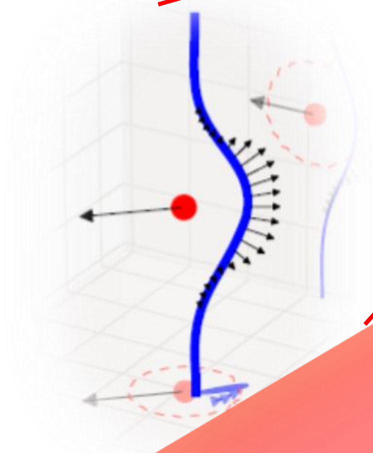
Our goal and strategy

Goal: Unveil the mechanism of glitches









10^4m

Macroscopic

- observations
- hydrodynamics

$\sim 10^{-10}$m

Mesoscopic

- dynamics of *vortices* in a lattice of *nuclei* (e.g. filament model)

Provide model ingredients

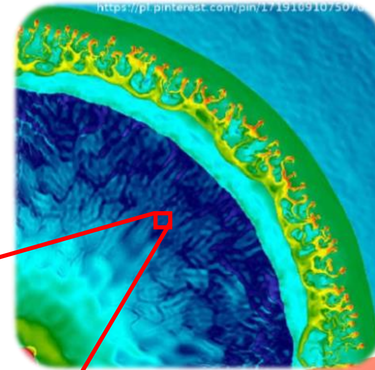
10^{-15}-10^{-13}m

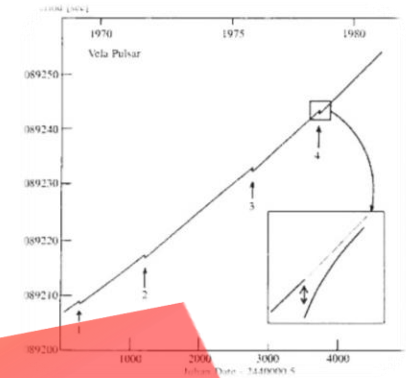
Microscopic

- vortex-nucleus dynamics from *neutrons and protons*

Our goal and strategy

Goal: Unveil the mechanism of glitches

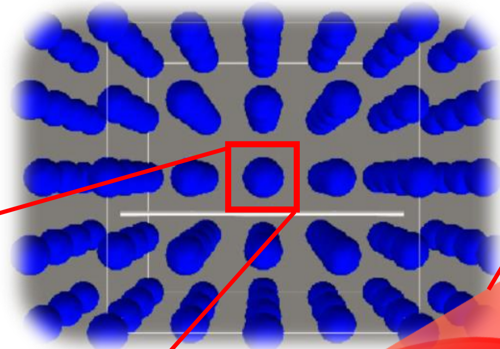




10^4m

Macroscopic

- observations
- hydrodynamics

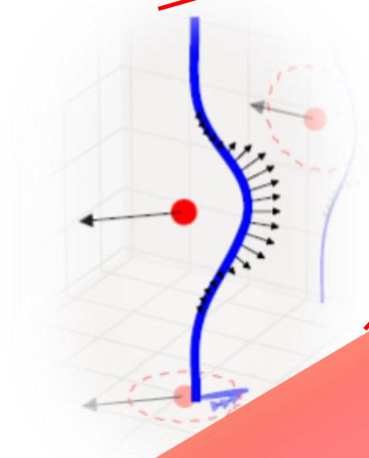


~10^{-10}m

Mesoscopic

- dynamics of *vortices* in a lattice of *nuclei* (e.g. filament model)

Provide model ingredients



10^{-15}-10^{-13}m

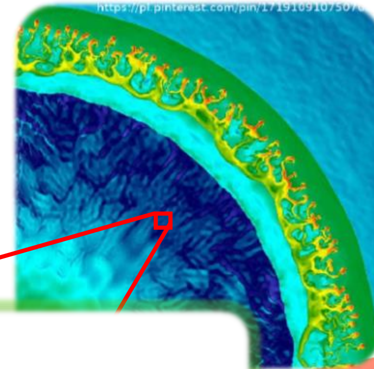
Microscopic

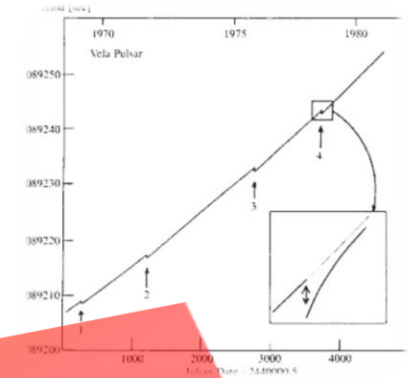
Nuclear Physics!!

- vortex-nucleus dynamics from *neutrons and protons*

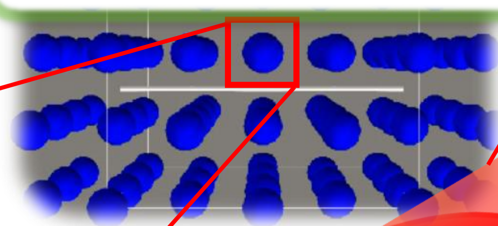
Our goal and strategy

Goal: Unveil the mechanism of glitches





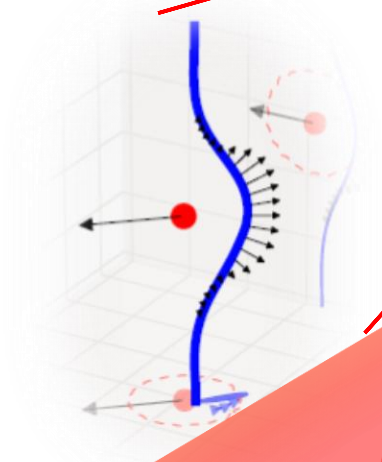
New collaboration started:
Nicolaus Copernicus Astronomical Centre
B. Haskell et al.



10^4m

Macroscopic

- observations
- hydrodynamics



~10^{-10}m

Mesoscopic

- dynamics of *vortices* in a lattice of *nuclei* (e.g. filament model)

Provide model ingredients

10^{-15}-10^{-13}m

Microscopic

Nuclear Physics!!

- vortex-nucleus dynamics from *neutrons and protons*

Current members of our "Entrainment" group
(consist of Japan & China sides)

Kochi Univ.

K. Iida

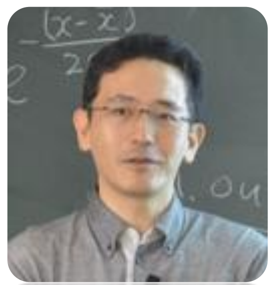
EoS, Pasta, QPO
Color superconductivity
Ultracold atomic gases

+some students

Univ. Tsukuba

T. Nakatsukasa

Nuclear DFT
Pairing correlations
Large-amplitude motions



+1 PhD students

Niigata Univ.

M. Matsuo

Nuclear DFT
Pairing correlations
Linear response theory (QRPA)

+1 PhD student (graduated)

Zhejiang Univ.

G. Watanabe

Nuclear pasta
Superfluid phenomena
Ultracold atomic gases

+1 postdoc (Y. Minami)

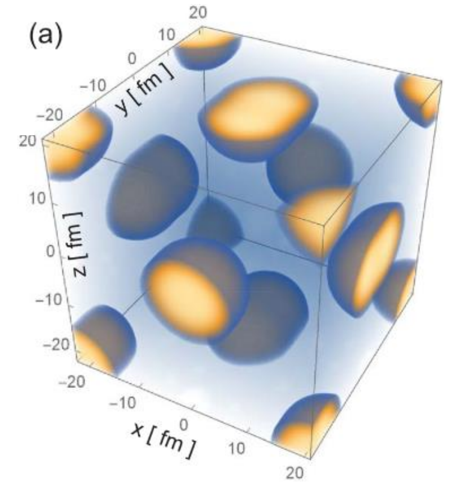
Tokyo Tech (April 2021~)

K. Sekizawa

Nuclear (TD)DFT
Superfluid dynamics, Glitches
Ultracold atomic gases

+2 MSc (graduated), 1 BSc students

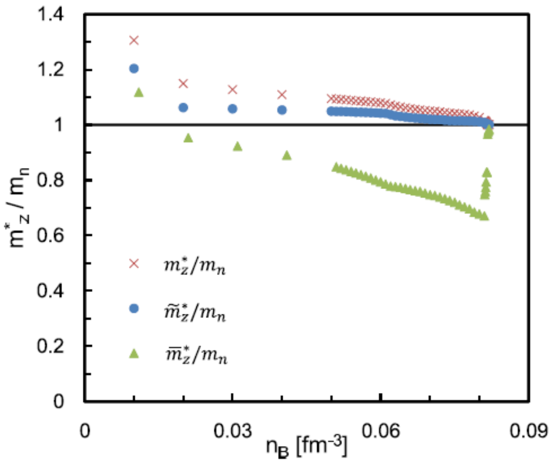
Progress form the Tsukuba group



1. Development of 3D, finite-temperature HFB solver

Yu Kashiwaba and T. Nakatsukasa, Phys. Rev. C **101**, 045804 (2020):
Coordinate-space solver for finite-temperature Hartree-Fock-Bogoliubov calculations using the shifted Krylov method

➢ Densities are calculated by Green's functions, avoiding diagonalizations of HFB matrices



2. First fully self-consistent band theory based on DFT

Yu Kashiwaba and T. Nakatsukasa, Phys. Rev. C **100**, 035804 (2019):
Self-consistent band calculation of the slab phase in the neutron-star crust

3. Development of a polynomial expansion method

T. Nakatsukasa, arXiv:2202.04448:
Self-consistent energy density functional approaches to the crust of neutron stars

➢ 3D, finite-temperature Skyrme HF method is developed, using a Fermion operator expansion method.

$$\hat{\rho}_T \approx \sum_{j=0}^{M} a_j T_j(\hat{H})$$

One-body density, with Fermi-Dirac distribution function is expanded by Chebyshev polynomials
→ offers a possible order-N approach for finite temperatures

Recent advances with TDDFT

Formulation: Band theory for the slab phase

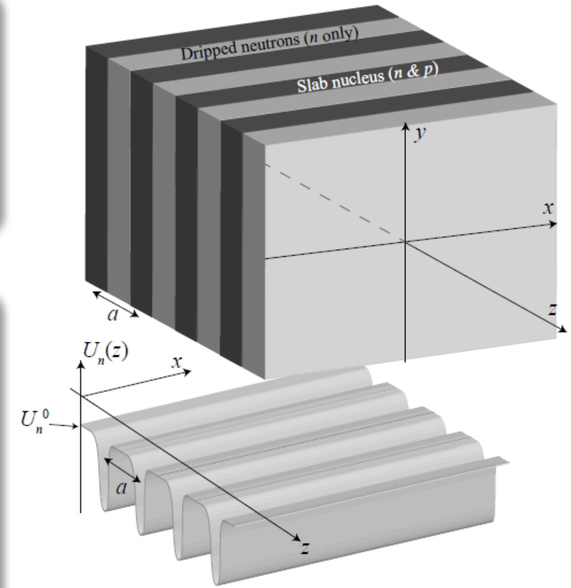
We employ the Skyrme-Kohn-Sham DFT with the Bloch boundary condition

✓ The Bloch boundary condition for single-particle orbitals

$$\psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) = \frac{1}{\sqrt{V}} u^{(q)}_{\alpha\boldsymbol{k}}(z) e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \qquad u^{(q)}_{\alpha\boldsymbol{k}}(z+na) = u^{(q)}_{\alpha\boldsymbol{k}}(z)$$

Periodicity of the slabs

α: Band index $\boldsymbol{k}$: Bloch wave vector q: Isospin (n or p) a: Period of the slabs



Picture from PRC**100**(2019)035804

✓ Skyrme EDF

$$\frac{E}{A} = \frac{1}{N_\mathrm{b}} \int_0^a \left(\frac{\hbar^2}{2m}\tau(z) + \sum_{t=0,1} \left[C^\rho_t[n] n^2_t(z) + C^{\Delta\rho}_t n_t(z) \partial^2_z n_t(z) + C^\tau_t \left(n_t(z)\tau_t(z) - \boldsymbol{j}^2_t(z) \right) \right] + \mathcal{E}^{(p)}_{\mathrm{Coul}}(z) \right) dz$$

Number density:

$$n_q(z) = 2 \sum^{\mathrm{occ.}}_{\alpha,\boldsymbol{k}} \left| \psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) \right|^2$$

Kinetic density:

$$\tau_q(z) = 2 \sum^{\mathrm{occ.}}_{\alpha,\boldsymbol{k}} \left| \nabla \psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) \right|^2$$

Current (momentum) density:

$$\boldsymbol{j}_q(z) = 2 \sum^{\mathrm{occ.}}_{\alpha,\boldsymbol{k}} \mathrm{Im} \left[\psi^{(q)*}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) \nabla \psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) \right]$$

*Uniform background electrons are assumed for the charge neutrality condition: $n_e = \bar{n}_p$

✓ Skyrme-Kohn-Sham equations

Note: While we deal with 3D slabs, the equations to be solved are 1D!

$$\hat{h}^{(q)}(z) \psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) = \varepsilon^{(q)}_{\alpha\boldsymbol{k}} \psi^{(q)}_{\alpha\boldsymbol{k}}(\boldsymbol{r}) \quad \Rightarrow \quad \left(\hat{h}^{(q)}(z) + \hat{h}^{(q)}_{\boldsymbol{k}}(z) \right) u^{(q)}_{\alpha\boldsymbol{k}}(z) = \varepsilon^{(q)}_{\alpha\boldsymbol{k}} u^{(q)}_{\alpha\boldsymbol{k}}(z)$$

Ordinary single-particle Hamiltonian:

$$\hat{h}^{(q)}(z) = -\nabla \cdot \frac{\hbar^2}{2m^\oplus_q(z)} \nabla + U^{(q)}(z) + \frac{1}{2i} \left[\nabla \cdot \boldsymbol{I}^{(q)}(z) + \boldsymbol{I}^{(q)}(z) \cdot \nabla \right]$$

Additional (k-dependent) term:

$$\hat{h}^{(q)}_{\boldsymbol{k}}(z) = \frac{\hbar^2 \boldsymbol{k}^2}{2m^\oplus_q(z)} + \hbar\boldsymbol{k} \cdot \hat{\boldsymbol{v}}^{(q)}(z)$$

Velocity operator:

$$\hat{\boldsymbol{v}}^{(q)}(z) \equiv \frac{1}{i\hbar} \left[\boldsymbol{r}, \hat{h}^{(q)}(z) \right]$$

K.S., S. Kobayashi, and M. Matsuo, arXiv:2112.14350.

Proton fraction:

$$Y_{\rm p} = \frac{\bar{n}_{\rm p}}{\bar{n}_{\rm n} + \bar{n}_{\rm p}}$$

Average nucleon density:

$$\bar{n}_q = \frac{1}{a}\int_0^a n_q(z)dz$$

Single-particle energy:

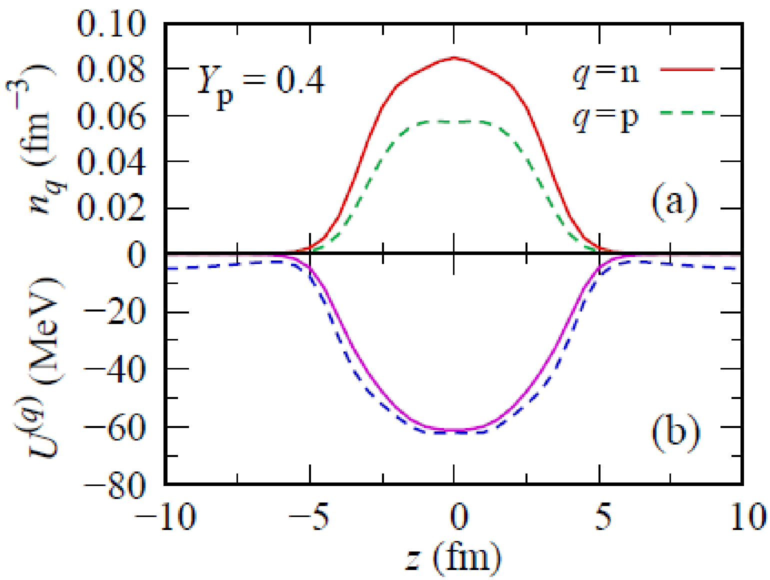
$$\varepsilon^{(q)}_{\alpha\boldsymbol{k}} = e^{(q)}_{\alpha\boldsymbol{k}} + \varepsilon^{(q)}_{\text{kin-}xy,\alpha\boldsymbol{k}} \approx \frac{\hbar^2 k_\parallel^2}{2m} \qquad k_\parallel = \sqrt{k_x^2 + k_y^2}$$

z-component

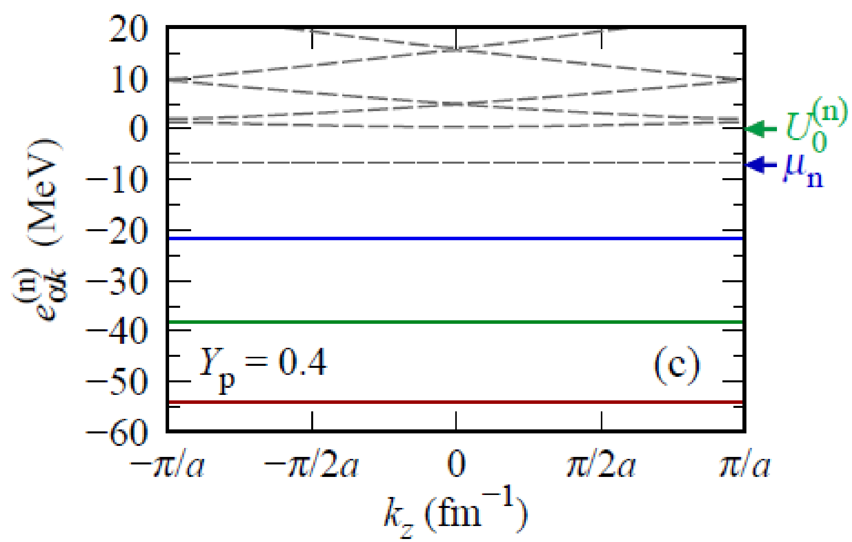
✓ Bound orbitals do not show band structure (k_z dependence)

$Y_{\rm p}$ = 0.4: Isolated slab (no neutron drip)

Density and potential



Neutron single-particle energies



K.S., S. Kobayashi, and M. Matsuo, arXiv:2112.14350.

Proton fraction:

$$Y_\mathrm{p} = \frac{\bar{n}_\mathrm{p}}{\bar{n}_\mathrm{n} + \bar{n}_\mathrm{p}}$$

Average nucleon density:

$$\bar{n}_q = \frac{1}{a}\int_0^a n_q(z)dz$$

Single-particle energy:

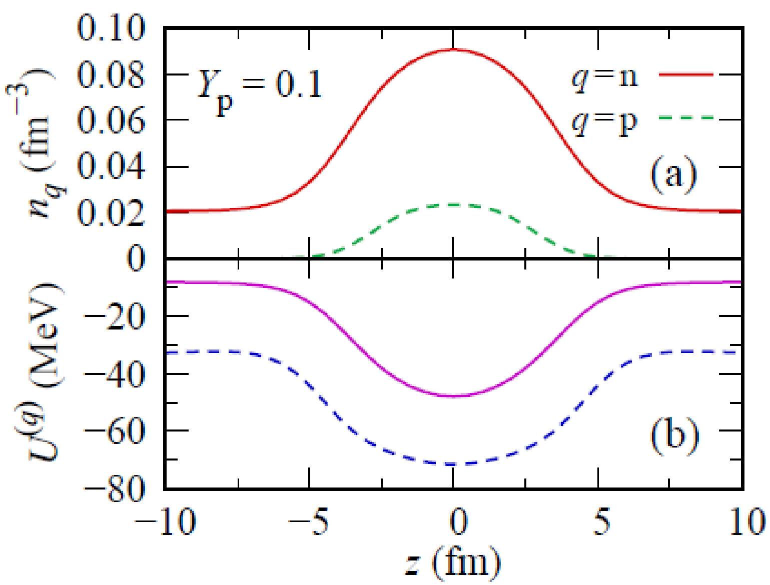
$$\varepsilon^{(q)}_{\alpha\boldsymbol{k}} = e^{(q)}_{\alpha\boldsymbol{k}} + \varepsilon^{(q)}_{\mathrm{kin}\text{-}xy,\alpha\boldsymbol{k}} \approx \frac{\hbar^2 k_\parallel^2}{2m} \qquad k_\parallel = \sqrt{k_x^2 + k_y^2}$$

z-component

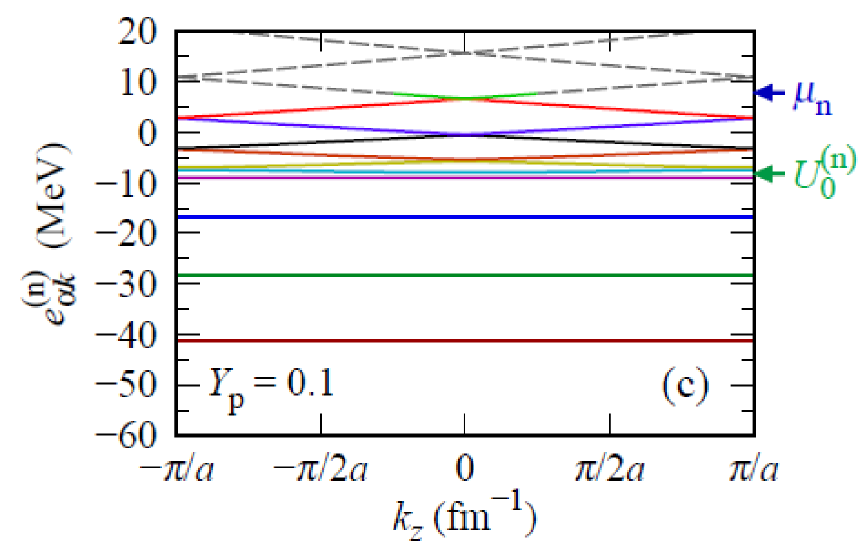
✓ Dripped neutrons show band structure (k_z dependence)

Y_p = 0.1: Neutron-dripped slab

Density and potential

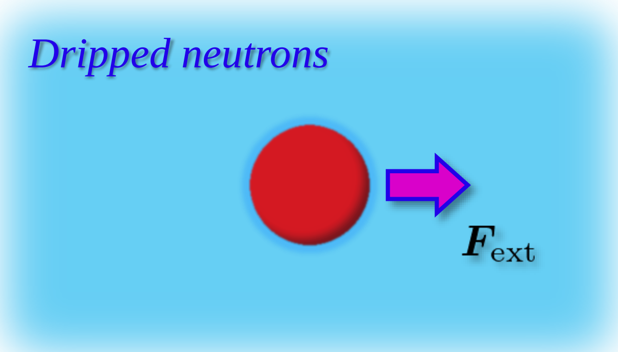


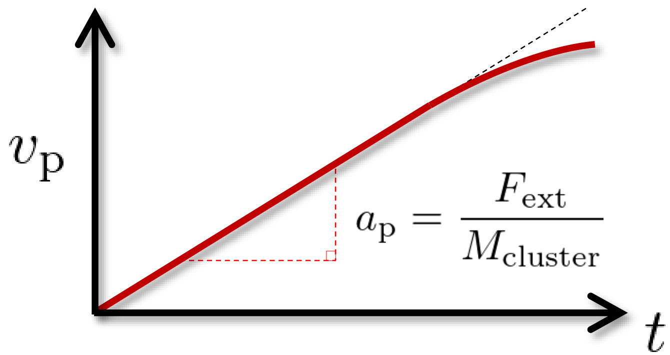
Neutron single-particle energies



✓ The collective mass is extracted from **acceleration motion under constant force**

The real-time method: Idea





How to introduce spatially-uniform electric field

✓ TDKS equation in a "velocity gauge"

$$i\hbar \frac{\partial \tilde{u}^{(q)}_{\alpha\boldsymbol{k}}(z,t)}{\partial t} = \left(\hat{h}^{(q)}(z,t) + \hat{h}^{(q)}_{\boldsymbol{k}(t)}(z,t)\right)\tilde{u}^{(q)}_{\alpha\boldsymbol{k}}(z,t) \qquad \boldsymbol{k}(t) = \boldsymbol{k} + \frac{e}{\hbar c}A_z(t)\hat{\boldsymbol{e}}_z$$

Spatially-uniform Vector potential

Gauge transformation for the Bloch orbitals:

$$\tilde{u}^{(q)}_{\alpha\boldsymbol{k}}(z,t) = \exp\left[-\frac{ie}{\hbar c}A_z(t)z\right]u^{(q)}_{\alpha\boldsymbol{k}}(z,t)$$

Electric field:

$$E_z(t) = -\frac{1}{c}\frac{dA_z}{dt}$$

k-dependent term:

$$\hat{h}^{(q)}_{\boldsymbol{k}}(z) = \frac{\hbar^2\boldsymbol{k}^2}{2m^{\oplus}_q(z)} + \hbar\boldsymbol{k}\cdot\hat{\boldsymbol{v}}^{(q)}(z)$$

Velocity operator:

$$\hat{\boldsymbol{v}}^{(q)}(z) \equiv \frac{1}{i\hbar}\left[\boldsymbol{r}, \hat{h}^{(q)}(z)\right]$$

cf. K. Yabana and G.F. Bertsch, Phys. Rev. B **54**, 4484 (1996); G.F. Bertch *et al.*, Phys. Rev. B **62**, 7998 (2000)

Results: The collective mass

Acceleration:

$$a_p = \frac{d^2 Z}{dt^2}$$

C.m. position of protons:

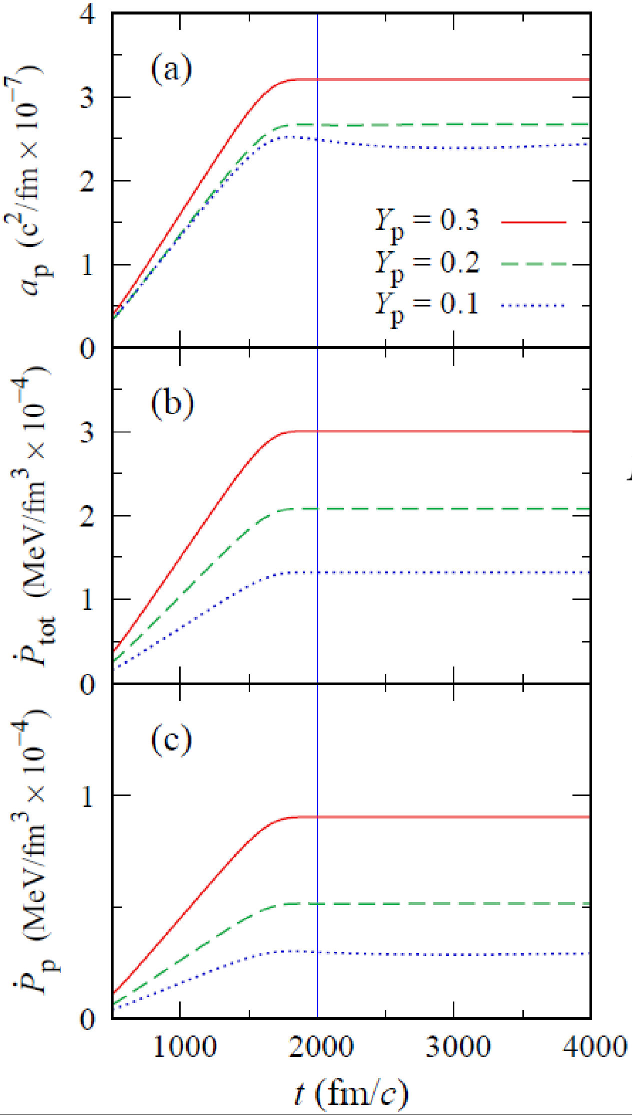
$$Z(t) = \frac{1}{a}\int_0^a z\, n_p(z,t)\, dz$$

Momentum of nucleons:

$$P_q(t) = \hbar \int_0^a j_q(z,t)\, dz$$

Total momentum:

$$P_{tot}(t) = P_n(t) + P_p(t)$$

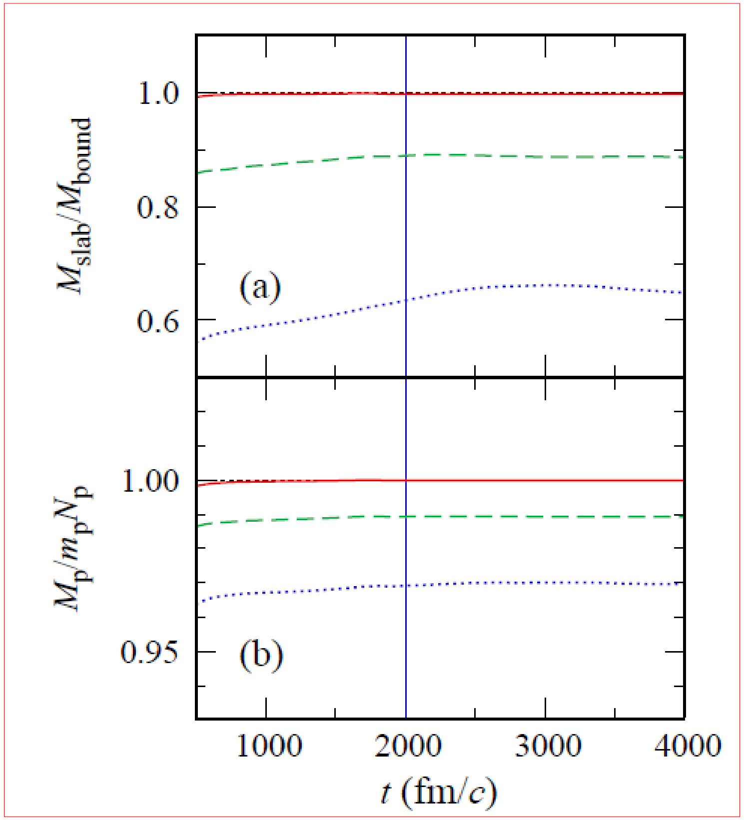


$$M_{slab} = \dot{P}_{tot}/a_p$$

$$M_p = \dot{P}_p/a_p$$

✓ For neutron-dripped slabs, we find significant ***reduction*** of the collective mass!

➢ **What is the origin of reduction?**



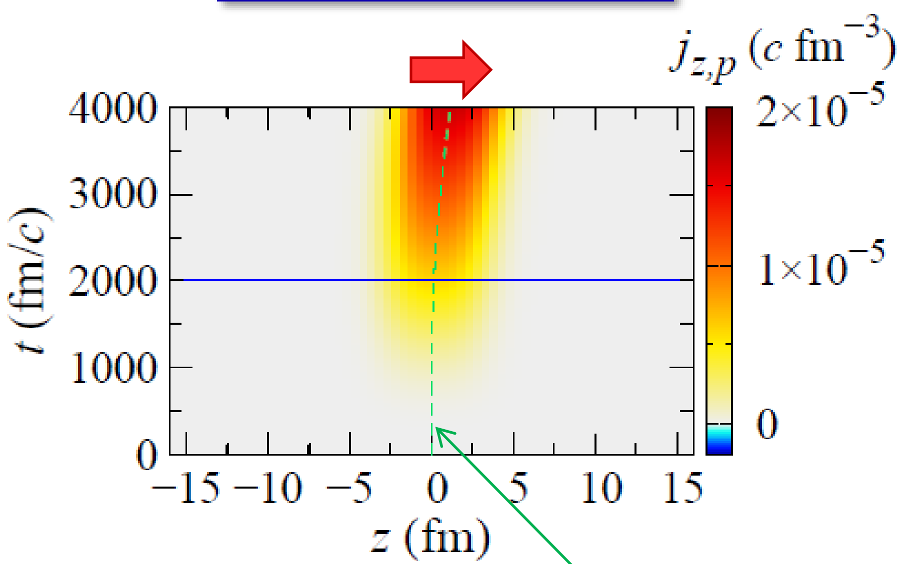
K.S., S. Kobayashi, and M. Matsuo, arXiv:2112.14350.

Current density:

$$j_{z,q}(z,t) = \frac{\hbar}{m_q} \sum_{\alpha,\boldsymbol{k}}^{\text{occ.}} \mathrm{Im}\big[\psi_{\alpha\boldsymbol{k}}^{(q)*}(\boldsymbol{r},t)\nabla\psi_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r},t)\big] = \frac{\hbar}{m_q}\frac{1}{aN_{k_z}}\sum_{\alpha,k_z}\int\frac{k_\parallel}{\pi}\,\mathrm{Im}\big[u_{\alpha\boldsymbol{k}}^{(q)*}(z,t)(\partial_z + ik_z)u_{\alpha\boldsymbol{k}}^{(q)}(z,t)\big]\,\theta(\mu_q - \varepsilon_{\alpha\boldsymbol{k}}^{(q)})\,dk_\parallel$$

✓ Protons inside the slab move toward the direction of the external force, as expected.

Proton current density



C.m. position of protons

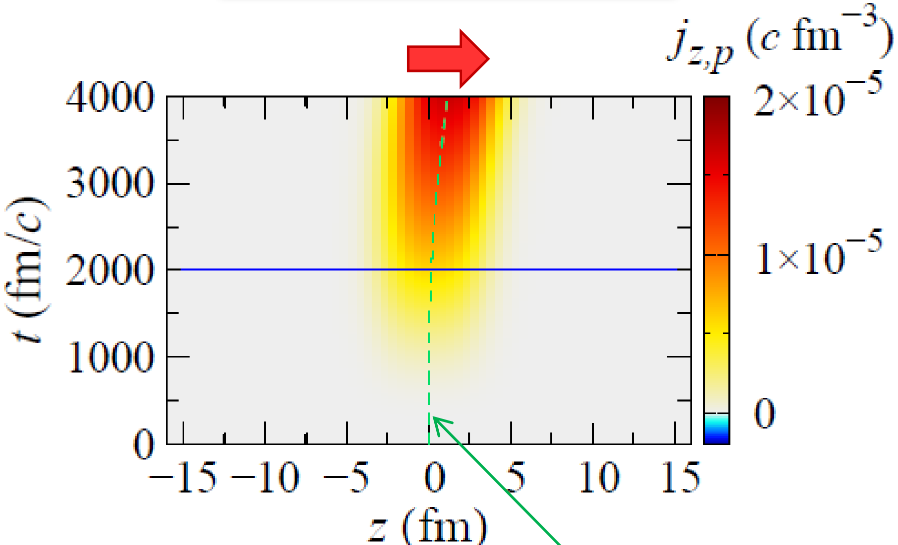
Current density:

$$j_{z,q}(z,t) = \frac{\hbar}{m_q}\sum_{\alpha,\boldsymbol{k}}^{\text{occ.}} \mathrm{Im}\big[\psi_{\alpha\boldsymbol{k}}^{(q)*}(\boldsymbol{r},t)\nabla\psi_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r},t)\big] = \frac{\hbar}{m_q}\frac{1}{aN_{k_z}}\sum_{\alpha,k_z}\int\frac{k_\parallel}{\pi}\,\mathrm{Im}\big[u_{\alpha\boldsymbol{k}}^{(q)*}(z,t)(\partial_z + ik_z)u_{\alpha\boldsymbol{k}}^{(q)}(z,t)\big]\,\theta(\mu_q - \varepsilon_{\alpha\boldsymbol{k}}^{(q)})\,dk_\parallel$$

✓ Dripped neutrons outside the slab move toward the opposite direction!

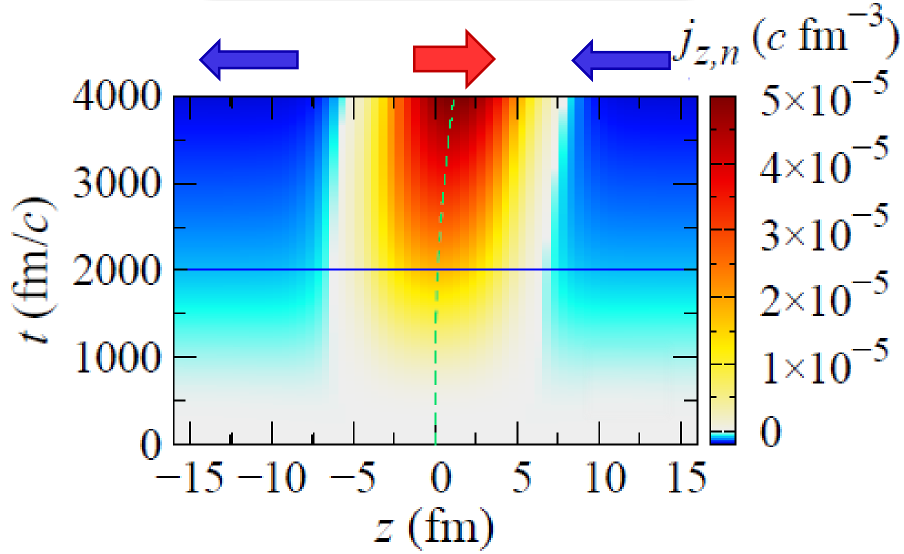
Since it reduces P_{tot} and $\dot{P}_{\text{tot}}$, $M_{\text{slab}} = \dot{P}_{\text{tot}}/a_{\text{p}}$ is reduced

Proton current density



C.m. position of protons

Neutron current density



$$\left(m_{\text{n},\alpha\boldsymbol{k}}^{\star\,-1}\right)_{\mu\nu} = \frac{1}{\hbar^2}\frac{\partial^2\varepsilon_{\alpha\boldsymbol{k}}^{(\text{n})}}{\partial k_\mu \partial k_\nu}$$

K.S., S. Kobayashi, and M. Matsuo, arXiv:2112.14350.

Current density:

$$j_{z,q}(z,t) = \frac{\hbar}{m_q} \sum_{\alpha,\boldsymbol{k}}^{\text{occ.}} \mathrm{Im}\left[\psi_{\alpha\boldsymbol{k}}^{(q)*}(\boldsymbol{r},t)\nabla\psi_{\alpha\boldsymbol{k}}^{(q)}(\boldsymbol{r},t)\right] = \frac{\hbar}{m_q}\frac{1}{aN_{k_z}}\sum_{\alpha,k_z}\int \frac{k_\parallel}{\pi}\mathrm{Im}\left[u_{\alpha\boldsymbol{k}}^{(q)*}(z,t)(\partial_z + ik_z)u_{\alpha\boldsymbol{k}}^{(q)}(z,t)\right]\theta(\mu_q - \varepsilon_{\alpha\boldsymbol{k}}^{(q)})\,dk_\parallel$$

✓ Dripped neutrons outside the slab move toward the opposite direction!

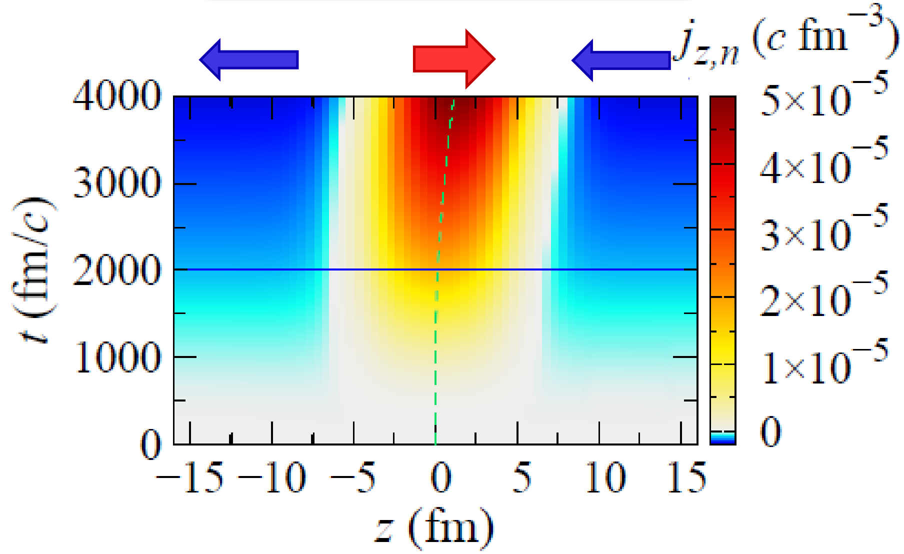
Since it reduces $P_{\rm tot}$ and $\dot{P}_{\rm tot}$, $M_{\rm slab} = \dot{P}_{\rm tot}/a_{\rm p}$ is reduced

Reduction of $M_{\rm slab}$
→ enhancement of $n_{\rm c}$
→ reduction of m^*

We interpret it as an "anti-entrainment" effect

$Y_{\rm p}$	$n_{\rm n}^{\rm f}/\bar{n}_{\rm n}$	Static		Dynamic
		$n_{\rm n}^{\rm c}/\bar{n}_{\rm n}$	$m_{\rm n}^{\star}/m_{\rm n}$	$n_{\rm n}^{\rm c}/\bar{n}_{\rm n}$
0.3	2.09×10^{-4}	0.005	0.040	0.005
0.2	0.127	0.256	0.496	0.229
0.1	0.362	0.630	0.574	0.586

Neutron current density



$$(m_{{\rm n},\alpha\boldsymbol{k}}^{\star\,-1})_{\mu\nu} = \frac{1}{\hbar^2}\frac{\partial^2\varepsilon_{\alpha\boldsymbol{k}}^{(\rm n)}}{\partial k_\mu \partial k_\nu}$$

Summary

The current debatable situation about the entrainment effects

➢ **The purpose is to clarify the actual effects of entrainment in the inner crust of neutron stars!**

Band calculation (Thomas-Fermi approx., w/o pairing)

$$\frac{m^\star}{m} \sim \begin{cases} 1.02-1.03 & \text{for the slab phase} \\ 1.11-1.40 & \text{for the rod phase} \end{cases}$$

$$\frac{m^\star}{m} \gtrsim 10 \text{ or more!} \quad \text{for the cubic lattice}$$

Band calculation [Self-consistent (TD)DFT, w/o pairing]

$$\frac{m^\star}{m} \sim 0.65-0.75 \quad \text{for the slab phase}$$

We consider those numbers should be corrected.

Band calculation (Mean-Field approx., with pairing)

$$\frac{m^\star}{m} \sim 1-2 \quad \text{for the slab phase}$$

$$\frac{m^\star}{m} \sim 1.41-1.56 \quad \text{for the cubic phase (at most)}$$

Disorder effects (w/o band structure effects)

$$\frac{m^\star}{m} \sim 1-1.2 \quad \text{for the cubic phase}$$

Current members of our "Entrainment" group
(consist of Japan & China sides)

Kochi Univ.

K. Iida

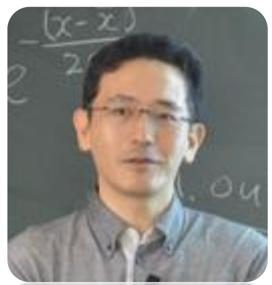
EoS, Pasta, QPO
Color superconductivity
Ultracold atomic gases

+some students

Univ. Tsukuba

T. Nakatsukasa

Nuclear DFT
Pairing correlations
Large-amplitude motions



+1 PhD students

Niigata Univ.

M. Matsuo

Nuclear DFT
Pairing correlations
Linear response theory (QRPA)

+1 PhD student (graduated)

Zhejiang Univ.

G. Watanabe

Nuclear pasta
Superfluid phenomena
Ultracold atomic gases

+1 postdoc (Y. Minami)

If you are interested to join, you are very welcome!!

Tokyo Tech (April 2021~)

K. Sekizawa

Nuclear (TD)DFT
Superfluid dynamics, Glitches
Ultracold atomic gases

+2 MSc (graduated), 1 BSc students

Kazuyuki Sekizawa

Associate Professor

Department of Physics, School of Science

Tokyo Institute of Technology

2-12-1 O-Okayama, Meguro, Tokyo 152-8551, Japan

sekizawa@phys.titech.ac.jp

http://sekizawa.fizyka.pw.edu.pl/english/

See also: